

# B Integration Tables

## Forms Involving $u^n$

$$1. \int u^n du = \frac{u^{n+1}}{n+1} + C, \quad n \neq -1$$

$$2. \int \frac{1}{u} du = \ln|u| + C$$

## Forms Involving $a + bu$

$$3. \int \frac{u}{a + bu} du = \frac{1}{b^2}(bu - a \ln|a + bu|) + C$$

$$4. \int \frac{u}{(a + bu)^2} du = \frac{1}{b^2} \left( \frac{a}{a + bu} + \ln|a + bu| \right) + C$$

$$5. \int \frac{u}{(a + bu)^n} du = \frac{1}{b^2} \left[ \frac{-1}{(n-2)(a + bu)^{n-2}} + \frac{a}{(n-1)(a + bu)^{n-1}} \right] + C, \quad n \neq 1, 2$$

$$6. \int \frac{u^2}{a + bu} du = \frac{1}{b^3} \left[ -\frac{bu}{2}(2a - bu) + a^2 \ln|a + bu| \right] + C$$

$$7. \int \frac{u^2}{(a + bu)^2} du = \frac{1}{b^3} \left( bu - \frac{a^2}{a + bu} - 2a \ln|a + bu| \right) + C$$

$$8. \int \frac{u^2}{(a + bu)^3} du = \frac{1}{b^3} \left[ \frac{2a}{a + bu} - \frac{a^2}{2(a + bu)^2} + \ln|a + bu| \right] + C$$

$$9. \int \frac{u^2}{(a + bu)^n} du = \frac{1}{b^3} \left[ \frac{-1}{(n-3)(a + bu)^{n-3}} + \frac{2a}{(n-2)(a + bu)^{n-2}} - \frac{a^2}{(n-1)(a + bu)^{n-1}} \right] + C, \quad n \neq 1, 2, 3$$

$$10. \int \frac{1}{u(a + bu)} du = \frac{1}{a} \ln \left| \frac{u}{a + bu} \right| + C$$

$$11. \int \frac{1}{u(a + bu)^2} du = \frac{1}{a} \left( \frac{1}{a + bu} + \frac{1}{a} \ln \left| \frac{u}{a + bu} \right| \right) + C$$

$$12. \int \frac{1}{u^2(a + bu)} du = -\frac{1}{a} \left( \frac{1}{u} + \frac{b}{a} \ln \left| \frac{u}{a + bu} \right| \right) + C$$

$$13. \int \frac{1}{u^2(a + bu)^2} du = -\frac{1}{a^2} \left[ \frac{a + 2bu}{u(a + bu)} + \frac{2b}{a} \ln \left| \frac{u}{a + bu} \right| \right] + C$$

Forms Involving  $a + bu + cu^2$ ,  $b^2 \neq 4ac$ 

$$14. \int \frac{1}{a + bu + cu^2} du = \begin{cases} \frac{2}{\sqrt{4ac - b^2}} \arctan \frac{2cu + b}{\sqrt{4ac - b^2}} + C, & b^2 < 4ac \\ \frac{1}{\sqrt{b^2 - 4ac}} \ln \left| \frac{2cu + b - \sqrt{b^2 - 4ac}}{2cu + b + \sqrt{b^2 - 4ac}} \right| + C, & b^2 > 4ac \end{cases}$$

$$15. \int \frac{u}{a + bu + cu^2} du = \frac{1}{2c} \left( \ln |a + bu + cu^2| - b \int \frac{1}{a + bu + cu^2} du \right)$$

Forms Involving  $\sqrt{a + bu}$ 

$$16. \int u^n \sqrt{a + bu} du = \frac{2}{b(2n + 3)} \left[ u^n (a + bu)^{3/2} - na \int u^{n-1} \sqrt{a + bu} du \right]$$

$$17. \int \frac{1}{u \sqrt{a + bu}} du = \begin{cases} \frac{1}{\sqrt{a}} \ln \left| \frac{\sqrt{a + bu} - \sqrt{a}}{\sqrt{a + bu} + \sqrt{a}} \right| + C, & a > 0 \\ \frac{2}{\sqrt{-a}} \arctan \sqrt{\frac{a + bu}{-a}} + C, & a < 0 \end{cases}$$

$$18. \int \frac{1}{u^n \sqrt{a + bu}} du = \frac{-1}{a(n-1)} \left[ \frac{\sqrt{a + bu}}{u^{n-1}} + \frac{(2n-3)b}{2} \int \frac{1}{u^{n-1} \sqrt{a + bu}} du \right], n \neq 1$$

$$19. \int \frac{\sqrt{a + bu}}{u} du = 2\sqrt{a + bu} + a \int \frac{1}{u \sqrt{a + bu}} du$$

$$20. \int \frac{\sqrt{a + bu}}{u^n} du = \frac{-1}{a(n-1)} \left[ \frac{(a + bu)^{3/2}}{u^{n-1}} + \frac{(2n-5)b}{2} \int \frac{\sqrt{a + bu}}{u^{n-1}} du \right], n \neq 1$$

$$21. \int \frac{u}{\sqrt{a + bu}} du = \frac{-2(2a - bu)}{3b^2} \sqrt{a + bu} + C$$

$$22. \int \frac{u^n}{\sqrt{a + bu}} du = \frac{2}{(2n+1)b} \left( u^n \sqrt{a + bu} - na \int \frac{u^{n-1}}{\sqrt{a + bu}} du \right)$$

Forms Involving  $a^2 \pm u^2$ ,  $a > 0$ 

$$23. \int \frac{1}{a^2 + u^2} du = \frac{1}{a} \arctan \frac{u}{a} + C$$

$$24. \int \frac{1}{u^2 - a^2} du = - \int \frac{1}{a^2 - u^2} du = \frac{1}{2a} \ln \left| \frac{u - a}{u + a} \right| + C$$

$$25. \int \frac{1}{(a^2 \pm u^2)^n} du = \frac{1}{2a^2(n-1)} \left[ \frac{u}{(a^2 \pm u^2)^{n-1}} + (2n-3) \int \frac{1}{(a^2 \pm u^2)^{n-1}} du \right], n \neq 1$$

Forms Involving  $\sqrt{u^2 \pm a^2}$ ,  $a > 0$ 

$$26. \int \sqrt{u^2 \pm a^2} du = \frac{1}{2} (u \sqrt{u^2 \pm a^2} \pm a^2 \ln |u + \sqrt{u^2 \pm a^2}|) + C$$

$$27. \int u^2 \sqrt{u^2 \pm a^2} du = \frac{1}{8} [u(2u^2 \pm a^2) \sqrt{u^2 \pm a^2} - a^4 \ln |u + \sqrt{u^2 \pm a^2}|] + C$$

$$28. \int \frac{\sqrt{u^2 + a^2}}{u} du = \sqrt{u^2 + a^2} - a \ln \left| \frac{a + \sqrt{u^2 + a^2}}{u} \right| + C$$

$$29. \int \frac{\sqrt{u^2 - a^2}}{u} du = \sqrt{u^2 - a^2} - a \operatorname{arcsec} \frac{|u|}{a} + C$$

$$30. \int \frac{\sqrt{u^2 \pm a^2}}{u^2} du = \frac{-\sqrt{u^2 \pm a^2}}{u} + \ln|u + \sqrt{u^2 \pm a^2}| + C$$

$$31. \int \frac{1}{\sqrt{u^2 \pm a^2}} du = \ln|u + \sqrt{u^2 \pm a^2}| + C$$

$$32. \int \frac{1}{u\sqrt{u^2 + a^2}} du = \frac{-1}{a} \ln \left| \frac{a + \sqrt{u^2 + a^2}}{u} \right| + C$$

$$33. \int \frac{1}{u\sqrt{u^2 - a^2}} du = \frac{1}{a} \operatorname{arcsec} \frac{|u|}{a} + C$$

$$34. \int \frac{u^2}{\sqrt{u^2 \pm a^2}} du = \frac{1}{2} (u\sqrt{u^2 \pm a^2} \mp a^2 \ln|u + \sqrt{u^2 \pm a^2}|) + C$$

$$35. \int \frac{1}{u^2 \sqrt{u^2 \pm a^2}} du = \mp \frac{\sqrt{u^2 \pm a^2}}{a^2 u} + C$$

$$36. \int \frac{1}{(u^2 \pm a^2)^{3/2}} du = \frac{\pm u}{a^2 \sqrt{u^2 \pm a^2}} + C$$

Forms Involving  $\sqrt{a^2 - u^2}$ ,  $a > 0$

$$37. \int \sqrt{a^2 - u^2} du = \frac{1}{2} \left( u\sqrt{a^2 - u^2} + a^2 \arcsin \frac{u}{a} \right) + C$$

$$38. \int u^2 \sqrt{a^2 - u^2} du = \frac{1}{8} \left[ u(2u^2 - a^2) \sqrt{a^2 - u^2} + a^4 \arcsin \frac{u}{a} \right] + C$$

$$39. \int \frac{\sqrt{a^2 - u^2}}{u} du = \sqrt{a^2 - u^2} - a \ln \left| \frac{a + \sqrt{a^2 - u^2}}{u} \right| + C$$

$$40. \int \frac{\sqrt{a^2 - u^2}}{u^2} du = \frac{-\sqrt{a^2 - u^2}}{u} - \arcsin \frac{u}{a} + C$$

$$41. \int \frac{1}{\sqrt{a^2 - u^2}} du = \arcsin \frac{u}{a} + C$$

$$42. \int \frac{1}{u\sqrt{a^2 - u^2}} du = \frac{-1}{a} \ln \left| \frac{a + \sqrt{a^2 - u^2}}{u} \right| + C$$

$$43. \int \frac{u^2}{\sqrt{a^2 - u^2}} du = \frac{1}{2} \left( -u\sqrt{a^2 - u^2} + a^2 \arcsin \frac{u}{a} \right) + C$$

$$44. \int \frac{1}{u^2 \sqrt{a^2 - u^2}} du = \frac{-\sqrt{a^2 - u^2}}{a^2 u} + C$$

$$45. \int \frac{1}{(a^2 - u^2)^{3/2}} du = \frac{u}{a^2 \sqrt{a^2 - u^2}} + C$$

Forms Involving  $\sin u$  or  $\cos u$ 

46.  $\int \sin u \, du = -\cos u + C$

48.  $\int \sin^2 u \, du = \frac{1}{2}(u - \sin u \cos u) + C$

50.  $\int \sin^n u \, du = -\frac{\sin^{n-1} u \cos u}{n} + \frac{n-1}{n} \int \sin^{n-2} u \, du$

52.  $\int u \sin u \, du = \sin u - u \cos u + C$

54.  $\int u^n \sin u \, du = -u^n \cos u + n \int u^{n-1} \cos u \, du$

56.  $\int \frac{1}{1 \pm \sin u} \, du = \tan u \mp \sec u + C$

58.  $\int \frac{1}{\sin u \cos u} \, du = \ln|\tan u| + C$

47.  $\int \cos u \, du = \sin u + C$

49.  $\int \cos^2 u \, du = \frac{1}{2}(u + \sin u \cos u) + C$

51.  $\int \cos^n u \, du = \frac{\cos^{n-1} u \sin u}{n} + \frac{n-1}{n} \int \cos^{n-2} u \, du$

53.  $\int u \cos u \, du = \cos u + u \sin u + C$

55.  $\int u^n \cos u \, du = u^n \sin u - n \int u^{n-1} \sin u \, du$

57.  $\int \frac{1}{1 \pm \cos u} \, du = -\cot u \pm \csc u + C$

Forms Involving  $\tan u$ ,  $\cot u$ ,  $\sec u$ ,  $\csc u$ 

59.  $\int \tan u \, du = -\ln|\cos u| + C$

60.  $\int \cot u \, du = \ln|\sin u| + C$

61.  $\int \sec u \, du = \ln|\sec u + \tan u| + C$

62.  $\int \csc u \, du = \ln|\csc u - \cot u| + C \quad \text{or} \quad \int \csc u \, du = -\ln|\csc u + \cot u| + C$

63.  $\int \tan^2 u \, du = -u + \tan u + C$

64.  $\int \cot^2 u \, du = -u - \cot u + C$

65.  $\int \sec^2 u \, du = \tan u + C$

66.  $\int \csc^2 u \, du = -\cot u + C$

67.  $\int \tan^n u \, du = \frac{\tan^{n-1} u}{n-1} - \int \tan^{n-2} u \, du, \quad n \neq 1$

68.  $\int \cot^n u \, du = -\frac{\cot^{n-1} u}{n-1} - \int (\cot^{n-2} u) \, du, \quad n \neq 1$

69.  $\int \sec^n u \, du = \frac{\sec^{n-2} u \tan u}{n-1} + \frac{n-2}{n-1} \int \sec^{n-2} u \, du, \quad n \neq 1$

70.  $\int \csc^n u \, du = -\frac{\csc^{n-2} u \cot u}{n-1} + \frac{n-2}{n-1} \int \csc^{n-2} u \, du, \quad n \neq 1$

71.  $\int \frac{1}{1 \pm \tan u} \, du = \frac{1}{2}(u \pm \ln|\cos u \pm \sin u|) + C$

72.  $\int \frac{1}{1 \pm \cot u} \, du = \frac{1}{2}(u \mp \ln|\sin u \pm \cos u|) + C$

73.  $\int \frac{1}{1 \pm \sec u} \, du = u + \cot u \mp \csc u + C$

74.  $\int \frac{1}{1 \pm \csc u} \, du = u - \tan u \pm \sec u + C$

Forms Involving Inverse Trigonometric Functions

$$75. \int \arcsin u \, du = u \arcsin u + \sqrt{1-u^2} + C$$

$$77. \int \arctan u \, du = u \arctan u - \ln \sqrt{1+u^2} + C$$

$$79. \int \operatorname{arcsec} u \, du = u \operatorname{arcsec} u - \ln|u + \sqrt{u^2-1}| + C$$

$$80. \int \operatorname{arccsc} u \, du = u \operatorname{arccsc} u + \ln|u + \sqrt{u^2-1}| + C$$

$$76. \int \arccos u \, du = u \arccos u - \sqrt{1-u^2} + C$$

$$78. \int \operatorname{arccot} u \, du = u \operatorname{arccot} u + \ln \sqrt{1+u^2} + C$$

Forms Involving  $e^u$ 

$$81. \int e^u \, du = e^u + C$$

$$83. \int u^n e^u \, du = u^n e^u - n \int u^{n-1} e^u \, du$$

$$85. \int e^{au} \sin bu \, du = \frac{e^{au}}{a^2 + b^2} (a \sin bu - b \cos bu) + C$$

$$86. \int e^{au} \cos bu \, du = \frac{e^{au}}{a^2 + b^2} (a \cos bu + b \sin bu) + C$$

$$82. \int u e^u \, du = (u-1)e^u + C$$

$$84. \int \frac{1}{1+e^u} \, du = u - \ln(1+e^u) + C$$

Forms Involving  $\ln u$ 

$$87. \int \ln u \, du = u(-1 + \ln u) + C$$

$$89. \int u^n \ln u \, du = \frac{u^{n+1}}{(n+1)^2} [-1 + (n+1) \ln u] + C, n \neq -1$$

$$90. \int (\ln u)^2 \, du = u[2 - 2 \ln u + (\ln u)^2] + C$$

$$88. \int u \ln u \, du = \frac{u^2}{4} (-1 + 2 \ln u) + C$$

$$91. \int (\ln u)^n \, du = u(\ln u)^n - n \int (\ln u)^{n-1} \, du$$

Forms Involving Hyperbolic Functions

$$92. \int \cosh u \, du = \sinh u + C$$

$$94. \int \operatorname{sech}^2 u \, du = \tanh u + C$$

$$96. \int \operatorname{sech} u \tanh u \, du = -\operatorname{sech} u + C$$

$$93. \int \sinh u \, du = \cosh u + C$$

$$95. \int \operatorname{csch}^2 u \, du = -\coth u + C$$

$$97. \int \operatorname{csch} u \coth u \, du = -\operatorname{csch} u + C$$

Forms Involving Inverse Hyperbolic Functions (in logarithmic form)

$$98. \int \frac{du}{\sqrt{u^2 \pm a^2}} = \ln(u + \sqrt{u^2 \pm a^2}) + C$$

$$100. \int \frac{du}{u\sqrt{a^2 \pm u^2}} = -\frac{1}{a} \ln \frac{a + \sqrt{a^2 \pm u^2}}{|u|} + C$$

$$99. \int \frac{du}{a^2 - u^2} = \frac{1}{2a} \ln \left| \frac{a+u}{a-u} \right| + C$$