Math 141
Quiz 6 not quiz 7
Due November 9, 2015

name G.I. Jane and Joe

- Test the series for convergence or divergence.
- Name the test used, and support your conclusion.
- Whenever possible, find the sum if convergent.

$$\sum_{n=1}^{\infty} \frac{n^2}{n^4+1}$$

Test: LCT to p-series $\sum \frac{1}{n^2}$ conv. $p = 2 > 1$

Apply test / reasoning: $\lim_{n\to\infty} \left(\frac{\frac{n^2}{n^4+1}}{\frac{1}{n^2}} \right) = \lim_{n\to\infty} \left(\frac{n^2}{n^4+1} \cdot \frac{n^2}{1} \right) = \lim_{n\to\infty} \left(\frac{n^4}{n^4+1} \right) = 1$ pos. finite

Converge / diverge:

Sum, if possible: converges both converge

$$\sum_{n=0}^{\infty} \frac{2^n}{n!}$$

$(-1)^n$

$a_n = \frac{2^n}{n!}$ $a_{n+1} = \frac{2^{n+1}}{(n+1)!}$

Test: RATIO

Apply test / reasoning: $\lim_{n\to\infty} \left| \frac{2^{n+1}}{(n+1)!} \cdot \frac{n!}{2^n} \right|$

$\lim_{n\to\infty} \left| \frac{1}{n+1} \cdot \frac{2}{1} \right|$

$\lim_{n\to\infty} \left| \frac{2}{n+1} \right| = 0 < 1$

Converge / diverge: $\lim_{n\to\infty} \left| \frac{n!}{(n+1)!} \cdot \frac{2^{n+1}}{2^n} \right|$

Sum, if possible:

converges absolutely

$$\sum_{n=0}^{\infty} (-1)^n \frac{2}{5^n} = \sum_{n=0}^{\infty} 2\left(-\tfrac{1}{5}\right)^n$$

Test: geometric

Apply test / reasoning: $r = -1/5$, $|-1/5| < 1$, converges

Converge / diverge:

Sum, if possible: $S = \frac{2}{1-(-1/5)} = \frac{2}{6/5} = 5/3$

$$\sum_{n=0}^{\infty}(-1)^{n+1}\frac{1}{2n+1}$$

$a_n = \frac{1}{2n+1}$

$a_{n+1} = \frac{1}{2(n+1)+1} = \frac{1}{2n+3}$

converges

Test: AST

Apply test / reasoning:

1) $\lim a_n = 0$ 2) $a_{n+1} \le a_n$

$\lim_{n\to\infty} \frac{1}{2n+1} = 0$ $\frac{1}{2n+3} \le \frac{1}{2n+1}$

Converge / diverge:

Sum, if possible:

$$\sum_{n=0}^{\infty}\frac{n^2}{2n^2+1}$$

nth term test for div.

Test:

$\lim_{n\to\infty} \frac{n^2}{2n^2+1} = \frac{1}{2} \neq 0$

Apply test / reasoning:

series diverges

Converge / diverge:

Sum, if possible:

$$1+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{3}}+\frac{1}{2}+\frac{1}{\sqrt{5}}+\frac{1}{\sqrt{6}}+\ldots = \sum_{n=1}^{\infty}\frac{1}{\sqrt{n}} = \sum_{n=1}^{\infty}\frac{1}{n^{1/2}}$$

Test:

p-series $p = 1/2 \le 1$

Apply test / reasoning:

diverges

Converge / diverge:

Sum, if possible:

$$\sum_{n=0}^{\infty}\frac{1}{\sqrt{n+1}}$$

~~Integral test~~

LCT to $\sum \frac{1}{\sqrt{n}}$