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$$\int \frac{1}{x^2-25}\,dx$$

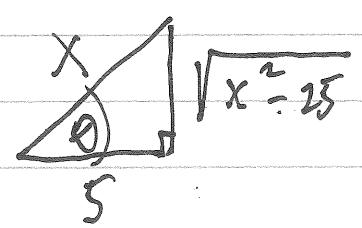
1) trig sub.

$$\int \frac{1}{\left(\sqrt{x^2-25}\right)^2}\,dx$$

$x = 5\sec\theta$

$dx = 5\sec\theta\tan\theta\,d\theta$

$\sqrt{x^2-25} = 5\tan\theta$



$$= \int \frac{5\sec\theta\tan\theta\,d\theta}{(5\tan\theta)^2}$$

$$= \int \frac{5\sec\theta\tan\theta\,d\theta}{25\tan^2\theta}$$

$$= \frac{1}{5}\int \frac{\sec\theta}{\tan\theta}\,d\theta$$

$$= \frac{1}{5}\int \frac{\frac{1}{\cos\theta}}{\frac{\sin\theta}{\cos\theta}}\,d\theta$$

$$= \frac{1}{5}\int \frac{1}{\cos\theta}\cdot\frac{\cos\theta}{\sin\theta}\,d\theta$$

$$= \frac{1}{5}\int \frac{1}{\sin\theta}\,d\theta$$

$$= \frac{1}{5}\int \csc\theta\,d\theta$$

$$= -\frac{1}{5}\ln\left|\csc\theta + \cot\theta\right| + C$$

$$= -\frac{1}{5}\ln\left|\frac{x}{\sqrt{x^2-25}} + \frac{5}{\sqrt{x^2-25}}\right| + C$$

$$= -\frac{1}{5}\ln\left|\frac{x+5}{\sqrt{x^2-25}}\right| + C$$

$$\int \frac{1}{x^2-25}\,dx$$

$$\frac{1}{x^2-25} = \frac{1}{(x+5)(x-5)}$$

$$\frac{1}{(x+5)(x-5)} = \frac{A}{x+5} + \frac{B}{x-5}$$

$$(x+5)(x-5)\left(\frac{1}{(x+5)(x-5)}\right) = \left(\frac{A}{x+5} + \frac{B}{x-5}\right)(x+5)(x-5)$$

$$1 = A(x-5) + B(x+5)$$

let $x=5$ $\quad 1 = 10B$ $\quad \frac{1}{10} = B$ $\qquad x=-5$ $\quad 1 = -10A$ $\quad -\frac{1}{10} = A$

$$\frac{-\frac{1}{10}}{x+5} + \frac{\frac{1}{10}}{x-5}$$

$$\int\left(\frac{-\frac{1}{10}}{x+5} + \frac{\frac{1}{10}}{x-5}\right)dx$$

$$-\frac{1}{10}\int\left(\frac{1}{x+5} - \frac{1}{x-5}\right)dx = -\frac{1}{10}\left[\ln|x+5| - \ln|x-5|\right] + C$$

$$\int \frac{1}{x^2-25}\,dx = \frac{1}{2(5)} \ln\left|\frac{x-5}{x+5}\right| + C$$

$$= \frac{1}{10} \ln\left|\frac{x-5}{x+5}\right| + C$$

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$$\int \frac{1}{u^2-a^2}\,du = \frac{1}{2a} \ln\left|\frac{u-a}{u+a}\right| + C$$

$u = x \quad a = 5 \quad du = dx$