Math 141
Exam 3
#1-6:16 pts each. #7: 4 pts.

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show all relevant work for full credit

No calculators.
No pens: -10 pts.
No questions regarding content: -10 pts.
No late exams accepted. -100 pts.
No smart phones anywhere: -100 pts.

1. Test the series for convergence or divergence. Name the test utilized. Support your conclusion. If convergent, find the sum whenever possible.

$$\sum_{n=0}^{\infty} \frac{2n}{n^2+1}$$

Test: LCT to $\sum_{1}^{\infty} \frac{1}{n}$ div. p-series $p=1\le 1$

Apply test / reasoning: $\lim_{n\to\infty} \left(\frac{\frac{2n}{n^2+1}}{\frac{1}{n}}\right) = \lim_{n\to\infty}\left(\frac{2n}{n^2+1}\cdot\frac{n}{1}\right) = \lim_{n\to\infty} \frac{2n^2}{n^2+1} = 2$

pos. finite

Converge / diverge:

Sum, if possible:

both series div.

2. Find the **center**, **radius**, and the (open) **interval of convergence** for the power series. You need ***NOT*** test endpoints.

$$f(x) = \sum_{n=1}^{\infty} \frac{(-1)^n x^n}{(n-1)!}$$

$u_n = \frac{x^n}{(n-1)!}$ $u_{n+1} = \frac{x^{n+1}}{n!}$

Center: $c = 0$

$\lim_{n\to\infty} \left| \frac{x^{n+1}}{n!} \cdot \frac{(n-1)!}{x^n} \right| = \lim_{n\to\infty} \left| \frac{x}{1} \cdot \frac{1}{n} \right|$

$= |x| \lim_{n\to\infty} \left| \frac{1}{n} \right|$

$= |x|(0) = 0 < 1$

conv. for all x

Radius: $R = \infty$

(open) Interval of Convergence Radius: $(-\infty, \infty)$

3. Find the **center**, **radius** , and the (open) **interval of convergence** for the power series,

$f(x)=\sum_{n=1}^{\infty}\frac{(-1)^n(x-2)^n}{5^n\cdot n^2}$. You need ***NOT*** test endpoints.

Center: $c=2$ $\quad u_n=\frac{(x-2)^n}{5^n\cdot n^2}$ $\quad u_{n+1}=\frac{(x-2)^{n+1}}{5^{n+1}(n+1)^2}$

$$\lim_{n\to\infty}\left|\frac{(x-2)^{n+1}}{5^{n+1}(n+1)^2}\cdot\frac{5^n\cdot n^2}{(x-2)^n}\right|$$

$$=\lim_{n\to\infty}\left|\frac{x-2}{1}\cdot\frac{1}{5}\cdot\frac{n^2}{(n+1)^2}\right|$$

$$=\left|\frac{x-2}{5}\right|\lim_{n\to\infty}\left|\frac{n^2}{(n+1)^2}\right|$$

or n^2+2n+1

$$=\left|\frac{x-2}{5}\right|(1)=\left|\frac{x-2}{5}\right|<1 \Rightarrow |x-2|<5$$

$$|x-c|<R$$

Radius: $R=5$

(open) Interval of Convergence Radius: $(-3,7)$

4. Given the power series, $f(x)=\sum_{n=1}^{\infty}\frac{(-1)^{n+1}(x-1)^n}{n}$, and the fact that the radius of convergence is $R=1$, to find the interval of convergence, ***including*** testing endpoints, if necessary.

$x=0$

TEST LEFT ENDPOINT:

$$f(0)=\sum_{1}^{\infty}\frac{(-1)^{n+1}(-1)^n}{n}$$

$$=\sum_{1}^{\infty}\frac{(-1)^{2n+1}}{n}$$

$$=\sum_{1}^{\infty}-\frac{1}{n}$$

$$=-\sum_{1}^{\infty}\frac{1}{n}$$

div. p-series

$p=1\leq 1$

$x=2$

TEST RIGHT ENDPOINT:

$$f(2)=\sum_{1}^{\infty}\frac{(-1)^{n+1}(2-1)^n}{n}$$

$$=\sum_{1}^{\infty}\frac{(-1)^{n+1}}{n}$$

AST $a_n=\frac{1}{n}$ $a_{n+1}=\frac{1}{n+1}$

1) $\lim_{n\to\infty}a_n=\lim_{n\to\infty}\frac{1}{n}=0$

2) $a_{n+1}\leq a_n$

$\frac{1}{n+1}\leq\frac{1}{n}$ Conv.

FINAL interval of Convergence Radius: (after testing endpoints) $\boxed{(0,2]}$

5. For the power series, $f(x)=\sum_{n=0}^{\infty}\frac{(-1)^n(x-2)^n}{n!}$ find ***AND*** simplify or rewrite n^{th} term for each of the following. You need ***NOT*** find interval of convergence, nor test endpoints.

$$f(x)=\sum_{n=0}^{\infty}\frac{(-1)^n(x-2)^n}{n!}=1-\frac{1}{1!}(x-2)+\frac{1}{2!}(x-2)^2-\frac{1}{3!}(x-2)^3+\frac{1}{4!}(x-2)^4-\frac{1}{5!}(x-2)^5+\ldots$$

a. $f'(x)=\sum_{n=1}^{\infty}\frac{(-1)^n\, n\,(x-2)^{n-1}}{n!}$

$$=\sum_{n=1}^{\infty}\frac{(-1)^n(x-2)^{n-1}}{(n-1)!}$$

$$f(x)=\sum_{n=0}^{\infty}\frac{(-1)^n(x-2)^n}{n!}=1-\frac{1}{1!}(x-2)+\frac{1}{2!}(x-2)^2-\frac{1}{3!}(x-2)^3+\frac{1}{4!}(x-2)^4-\frac{1}{5!}(x-2)^5+\ldots$$

b. $\int f(x)dx=C+\sum_{n=0}^{\infty}\frac{(-1)^n(x-2)^{n+1}}{n!\,(n+1)}$

$$=C+\sum_{n=0}^{\infty}\frac{(-1)^n(x-2)^{n+1}}{(n+1)!}$$

6. Use the definition of the Taylor/Maclaurin Polynomial and Taylor/Maclaurin Series to find the 4th Maclaurin Polynomial for $f(x) = e^{-2x}$ centered at $c = 0$. Then use your polynomial to write the Maclaurin Series for $f(x) = e^{-2x}$ centered at $c = 0$.

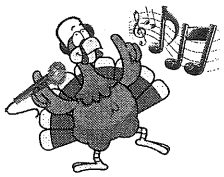
$n = 0$	$f(x) = e^{-2x}$	$f(0) = 1$	$a_0 = 1/0! = 1$
$n = 1$	$f'(x) = -2e^{-2x}$	$f'(0) = -2$	$a_1 = -2/1! = -2$
$n = 2$	$f''(x) = 4e^{-2x}$	$f''(0) = 4$	$a_2 = 4/2! = 2$
$n = 3$	$f'''(x) = -8e^{-2x}$	$f'''(0) = -8$	$a_3 = -8/3! = -4/3$
$n = 4$	$f^{(4)}(x) = 16e^{-2x}$	$f^{(4)}(0) = 16$	$a_4 = 16/4! = 2/3$

4th Maclaurin polynomial: $P_4(x) = 1 - 2x + 2x^2 - \frac{4}{3}x^3 + \frac{2}{3}x^4$

or $P_4(x) = 1 - \frac{2}{1!}x + \frac{4}{2!}x^2 - \frac{8}{3!}x^3 + \frac{16}{4!}x^4$

Maclaurin Series: $P(x) = \sum_{n=0}^{\infty} \frac{(-1)^n 2^n x^n}{n!}$

7. (4 points): In your opinion the greatest song (or recording) of all time (in any musical genre).

Answer not unique.

There is no wrong answer.