Math 141
Exam 2 A
October 23, 2015

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- NO CALCULATORS
- Show all relevant work for full credit.
- No smartphones ANYWHERE in sight!
- #1 - #8: 12 points each. #9: 4 points

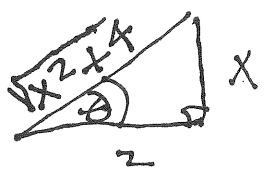
1. Find the indefinite integral: $\int \frac{1}{x\sqrt{x^2+4}}\,dx$

$$\int \frac{2\sec^2\theta\, d\theta}{(2\tan\theta)(2\sec\theta)}$$

$x = 2\tan\theta$

$dx = 2\sec^2\theta\, d\theta$

$\sqrt{x^2+4} = 2\sec\theta$



$$= \frac{1}{2}\int \frac{\sec\theta}{\tan\theta}\,d\theta = \frac{1}{2}\int \frac{\frac{1}{\cos\theta}}{\frac{\sin\theta}{\cos\theta}}\,d\theta$$

$$= \frac{1}{2}\int \frac{1}{\sin\theta}\,d\theta = \frac{1}{2}\int \csc\theta\, d\theta$$

$$= -\frac{1}{2}\ln\left|\csc\theta + \cot\theta\right| + C$$

$$= -\frac{1}{2}\ln\left|\frac{\sqrt{x^2+4}}{x} + \frac{2}{x}\right| + C$$

2. Find the partial fraction decomposition: $\frac{x^2-x-1}{x^2(x-1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-1}$

$x^2 - x - 1 = Ax(x-1) + B(x-1) + Cx^2$

$x = 1 \quad -1 = C$

$x = 0 \quad -1 = -B$

$1 = B$

$x = 2 \quad 1 = 2A + B + 4C$

$1 = 2A + 1 - 4$

$4 = 2A \quad A = 2$

p.f.d. $\frac{2}{x} + \frac{1}{x^2} - \frac{1}{x-1}$

3. Find the limit, if it exists. $\lim_{x \to 0^+} \frac{\ln(2x+1)}{e^x - 1} = \frac{\ln 1}{e^0 - 1} = \frac{0}{0}$

L Hopital's

$$\lim_{x \to 0^+} \frac{\frac{1}{2x+1}(2)}{e^x}$$

$$\lim_{x \to 0^+} \frac{\frac{2}{2x+1}}{e^x} = \frac{\frac{2}{1}}{e^0} = 2$$

4. Determine if the improper integral converges or diverges. If convergent, find its value.

$$\int_1^\infty \frac{1}{x^2+1}\,dx = \lim_{x \to b} \int_1^b \frac{1}{x^2+1}\,dx$$

$$= \lim_{b \to \infty} \left[\arctan x\right]_1^b$$

$$= \lim_{b \to \infty} \arctan b - \arctan 1$$

$$= \arctan \infty - \arctan 1$$

$$= \pi/2 - \pi/4 = \pi/4 \text{ converges}$$

5. For the sequence, find the nth term, a_n. Start with the given value of n.

$$\left\{\frac{1}{1}, -\frac{2}{1}, \frac{5}{2}, -\frac{10}{6}, \frac{17}{24}, -\frac{26}{120}\cdots\right\}, \quad n=0$$

$n=0$ $n=1$ $n=2$ $n=3$ $n=4$ $n=5$

$a_n =$

$$a_n = (-1)^n \frac{n^2+1}{n!}$$

6. Determine the convergence or divergence of the ***sequence*** with given n^{th} term.

a. $a_n = \dfrac{2n}{n^2+1}$

$$\lim_{n\to\infty} \frac{2n}{n^2+1} = \frac{\infty}{\infty}$$

$$\lim_{n\to\infty} \frac{2}{2n} = 0$$

Converges

b. $a_n = \dfrac{2^{n+1}}{2^n+1}$

$$\lim_{n\to\infty} \frac{2^{n+1}}{2^n+1} = \frac{\infty}{\infty}$$

$$\lim_{n\to\infty} \frac{2^{n+1}\cdot \ln 2}{2^n \cdot \ln 2}$$

$$\lim_{n\to\infty} \frac{2}{1} = 2$$

Converges

7. Determine the convergence or divergence of the ***SERIES***:

- Name the test used.
- Support your conclusion.
- Find the sum, whenever possible.

a. $\sum_{n=1}^{\infty} \frac{2n}{n+1}$

n^{th} term test for div.

$\lim_{n\to\infty} \frac{2n}{n+1} = 2 \neq 0$

series diverges

b. $\sum_{n=0}^{\infty} (-1)^n \frac{2}{5^n}$

$\sum_{n=0}^{\infty} 2\left(-\frac{1}{5}\right)^n$

geometric

$r = -1/5 \quad |-1/5| < 1$

converges

$S = \frac{2}{1-(-1/5)} = \frac{2}{6/5} = \frac{10}{6} = \frac{5}{3}$

8. Use the series, $\sum_{n=1}^{\infty}\left(\frac{1}{n+1} - \frac{1}{n+2}\right)$, to find the following.

The type of series: telescoping

$S_4 = \left(\frac{1}{2} - \frac{1}{3}\right) + \left(\frac{1}{3} - \frac{1}{4}\right) + \left(\frac{1}{4} - \frac{1}{5}\right) + \left(\frac{1}{5} - \frac{1}{6}\right) = \frac{1}{2} - \frac{1}{6}$

$S_n = \frac{1}{2} - \frac{1}{n+2}$

S, if convergent. $S = \lim_{n\to\infty}\left(\frac{1}{2} - \frac{1}{n+2}\right) = 1/2$ converges

9. (4 points) Please choose, circle one:

vs.

Answer not unique