Math 141
Exam 1 A
September 25, 2015

name Ophelia Payne

*no calculators
*show all relevant work to receive full credit
*any evidence of academic dishonesty = 0 grade
*only pencils, pens = -5 from grade
*late for exam= -5 from grade

1. ***SET UP*** the definite integral to find the area of the region bounded by the graphs of $x = 1 - y^2$ and $x = y - 1$.

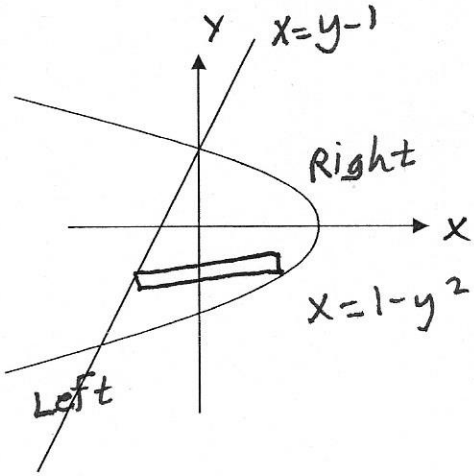
limits:

$y - 1 = 1 - y^2$

$y^2 + y - 2 = 0$

$(y+2)(y-1) = 0$

$y = -2, 1$



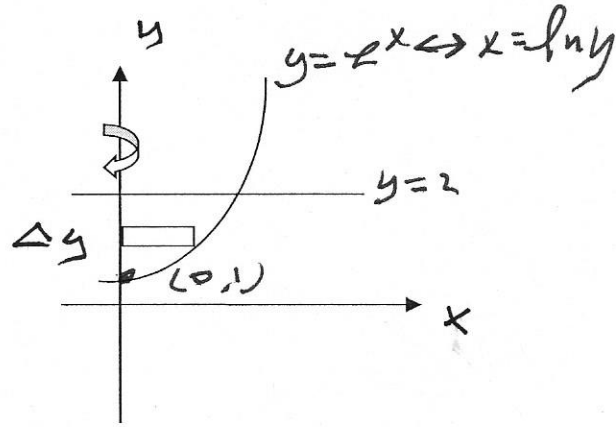
$$A = \int_{-2}^{1} [(1-y^2) - (y-1)]\, dy$$

2. ***SET UP*** the integral to find the volume of the solid generated by rotating the region bounded by the graphs of $y = e^x$, $x = 0$, $y = 2$ about the y-axis using horizontal rectangles. ← Δy

DISK $y = e^x$ $R = \ln y$

$\ln y = x$

$$V = \pi \int_{1}^{2} [\ln y]^2\, dy$$



Math 141
Exam 1 B
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1. ***SET UP*** the definite integral to find the area of the region bounded by the graphs of $x = y^2 - 1$ and $x = y + 1$.

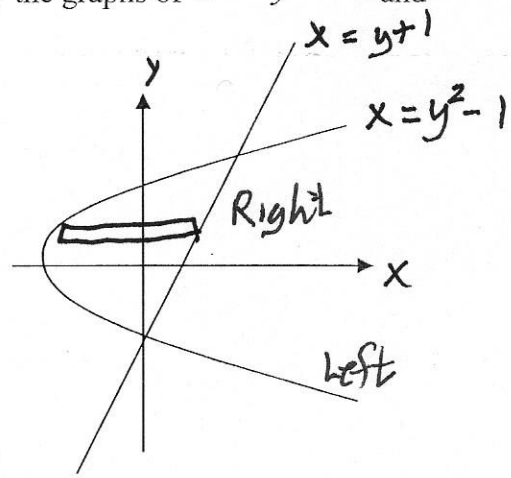
limits:

$y^2 - 1 = y + 1$

$y^2 - y - 2 = 0$

$(y-2)(y+1) = 0$

$y = 2, -1$

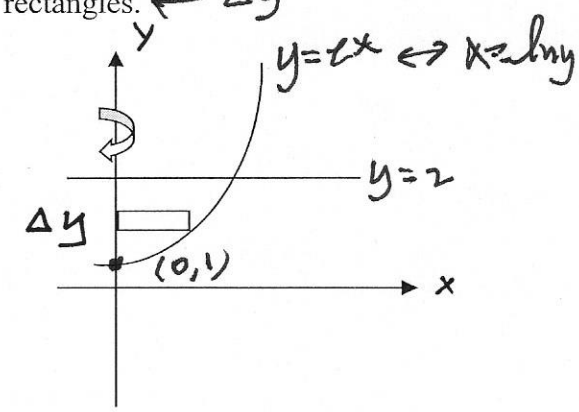


$$A = \int_{-1}^{2} \left[(y+1) - (y^2-1)\right] dy$$

2. ***SET UP*** the integral to find the volume of the solid generated by rotating the region bounded by the graphs of $y = e^x$, $x = 0$, $y = 2$ about the y-axis using horizontal rectangles. ← Δy

DISK $y = e^x$ $R = \ln y$

$\ln y = x$



$$V = \pi \int_{1}^{2} [\ln y]^2 \, dy$$

3. ***SET UP*** the integral to find the volume of the solid generated by rotating the region bounded by the graphs of $y = 2x - x^2$ and $y = x^2$ about the x-axis using vertical rectangles.

WASHER $R = 2x - x^2$ $r = x^2$

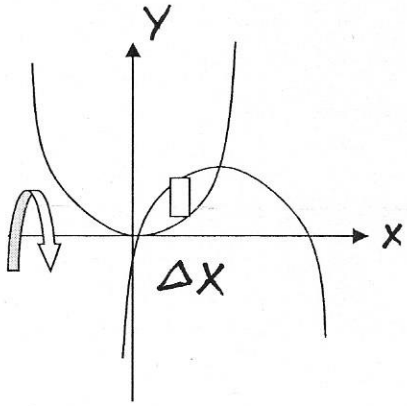
Limits:

$2x - x^2 = x^2$

$2x - 2x^2 = 0$

$2x(1-x) = 0$

$x = 0, 1$



$$V = \pi \int_0^1 \left[(2x - x^2)^2 - (x^2)^2\right] dx$$

4. ***SET UP*** the integral to find the volume of the solid generated by rotating the region bounded by the graphs of $y = \frac{1}{x^2}$, $x = 1$, $x = 2$, and $y = 0$ about $x = 2$.

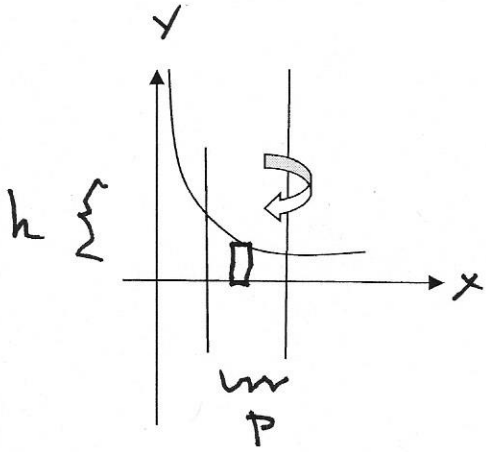
Indicate your choice of method by circling one of the following:

DISK ***WASHER*** ***SHELL.***

$p = 2 - x$

$h = 1/x^2$

$$V = 2\pi \int_1^2 \left[(2-x)\, 1/x^2\right] dx$$



5. ***SET UP*** the integral to find the volume of the solid with <u>*square*</u> cross-sections taken perpendicular to the x-axis whose base is bounded by the graphs of $y = x^2$ and $y = 2x$.

$V = \int A(x)\,dx$

side $= 2x - x^2$

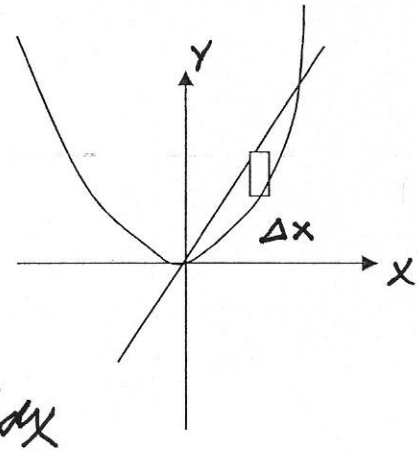
$A = (2x - x^2)^2$

limits: $2x = x^2$

$2x - x^2 = 0$

$x(2-x) = 0$

$x = 0, 2$



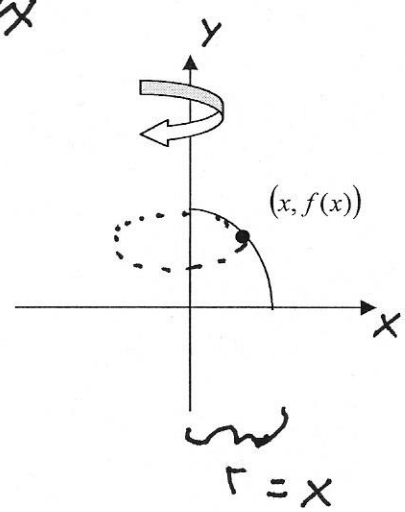
$$V = \int_0^2 (2x - x^2)^2\,dx$$

6. ***SET UP*** the integral to find the area of the *surface* formed by rotating the graph of the curve $y = 1 - x^2$ over $[0,1]$ about the y-axis.

$$S = 2\pi \int_a^b r(x)\sqrt{1 + [f'(x)]^2}\,dx$$

$y' = -2x$

$[y']^2 = (-2x)^2 = 4x^2$



$$S = 2\pi \int_0^1 x\sqrt{1 + 4x^2}\,dx$$

7. Find the indefinite integral: $\int x \sin 2x \, dx$

$$\int u\,dv = uv - \int v\,du$$

$u = x \qquad dv = \int \sin 2x\,dx$

$du = dx \qquad v = -\frac{1}{2}\cos 2x$

$$\int x \sin 2x\,dx = -\frac{1}{2} x \cos 2x - \int -\frac{1}{2}\cos 2x\,dx$$

$$= -\frac{1}{2} x \cos 2x + \frac{1}{2}\int \cos 2x\,dx$$

$$= -\frac{1}{2} x \cos 2x + \frac{1}{4}\sin 2x + C$$

8. Find the indefinite integral: $\int \ln x \, dx$

$u = \ln x \qquad dv = dx$

$du = \frac{1}{x}\,dx \qquad v = \int dx = x$

$$\int \ln x\,dx = x \ln x - \int x\left(\frac{1}{x}\right)dx$$

$$= x \ln x - \int dx$$

$$= x \ln x - x + C$$

9. Find the indefinite integral: $\int \sin^5 x \cos^3 x \, dx$

$$\int \sin^5 x \cos^2 x \cos x \, dx$$

$$\int \sin^5 x (1-\sin^2 x) \cos x \, dx$$

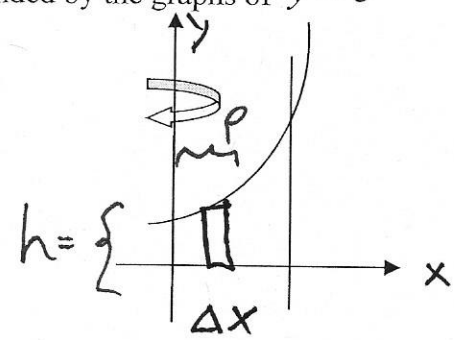
$u = \sin x \, dx$

$du = \cos x \, dx$

$$\int (\sin^5 x - \sin^7 x) \cos x \, dx$$

$$\int (u^5 - u^7) du = \frac{1}{6} u^6 - \frac{1}{8} u^8 + C \rightarrow \frac{1}{6} \sin^6 x - \frac{1}{8} \sin^8 x + C$$

10. ***FIND*** (don't just set it up) the volume of the solid, if the region bounded by the graphs of $y = e^x$ $x = 0$, $y = 0$, and $x = 1$ is rotated about the y-axis.



$p = x$

$h = e^x$

shell

$p = x$

$h = e^x$

$$V = 2\pi \int_0^1 x e^x \, dx$$

$$\int x e^x \, dx$$

$u = x \quad dv = e^x dx$

$du = dx \quad v = e^x$

$$\int x e^x dx = x e^x - \int e^x dx$$

$$= x e^x - e^x + C$$

$$= 2\pi \left[x e^x - e^x \right]_0^1$$

$$= 2\pi \left[(e - e) - (0 - 1) \right] = 2\pi \text{ units}^3$$