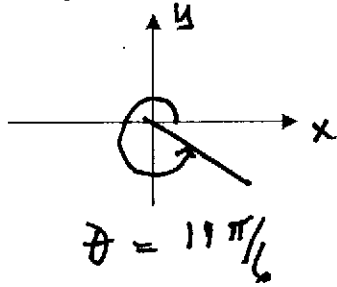


Math 41 Trigonometry Worksheet 1
Spring 2016

1. a. Draw the angle in standard position and convert the angle to degree measure.

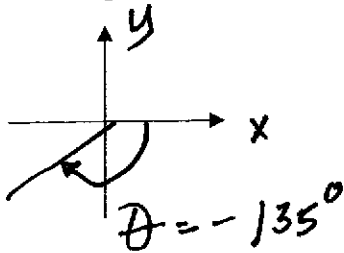
$$\theta = \frac{11\pi}{6}$$



$$11\pi/6 \times \frac{180^\circ}{\pi} = 330^\circ$$

- b. Draw the angle in standard position and convert the angle to radian measure

$$\theta = -135^\circ$$



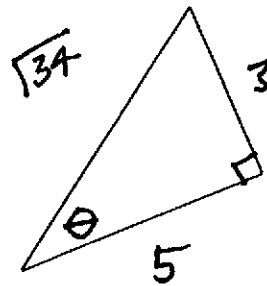
$$-135^\circ \times \frac{\pi}{180^\circ} = -3\pi/4$$

2. Use the right triangle to find each ratio of the angle, θ .

$$a. \cos \theta = \frac{5}{\sqrt{34}} = \frac{5\sqrt{34}}{34}$$

$$b. \cot \theta = \frac{5}{3}$$

$$c. \csc \theta = \frac{\sqrt{34}}{3}$$



$$\text{hyp}^2 = 5^2 + 3^2$$

$$\text{hyp}^2 = 25 + 9$$

$$\text{hyp}^2 = 34$$

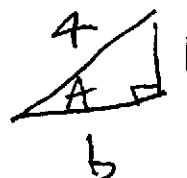
$$\text{hyp} = \sqrt{34}$$

3. Find the exact ratio of the special angle without a calculator:

a. $\sin 45^\circ = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

b. $\sec \frac{\pi}{3} = \frac{1}{\cos \frac{\pi}{3}} = \frac{1}{1/2} = 2$

4. Draw a right triangle find the trig ratios given $\sin A = \frac{1}{4}$.



$\cos A = \frac{\sqrt{15}}{4}$

$\tan A = \frac{1}{\sqrt{15}} = \frac{\sqrt{15}}{15}$

$\cot A = \sqrt{15}$

$\sec A = \frac{4}{\sqrt{15}} = \frac{4\sqrt{15}}{15}$

$$\begin{aligned} 1^2 + b^2 &= 4^2 \\ b^2 &= 15 \\ b &= \sqrt{15} \end{aligned}$$

5. Find θ , $0^\circ < \theta < 90^\circ$, for each equation. Do not use a calculator.

a. $\sin \theta = \frac{1}{2} \quad \theta = 30^\circ$

b. $\sec \theta = \sqrt{2} \rightarrow \cos \theta = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$
 $\theta = 45^\circ$

6. Find θ , $0 < \theta < \frac{\pi}{2}$, for each equation. Do not use a calculator.

a. $\cos \theta = \frac{1}{2} \quad \theta = \pi/3$

b. $\cot \theta = 1 \rightarrow \tan \theta = 1/1 = 1$
 $\theta = \pi/4$

7. Find θ , $0 < \theta < \frac{\pi}{2}$, and $0^\circ < \theta < 90^\circ$ for each equation. Round to two decimal places.

a. $\cos \theta = 0.7528$

$\theta \approx 0.72$ or $\theta \approx 41.17^\circ$

b. $\cot \theta = \frac{8}{5} \rightarrow \tan \theta = 5/8$

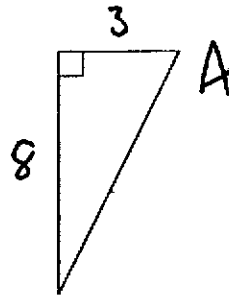
$\theta \approx 0.56$ or $\theta \approx 32.01^\circ$

8. Find angle A of the right triangle.

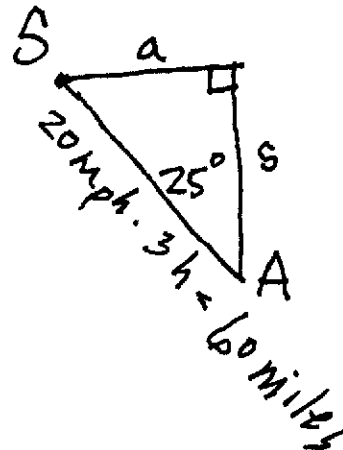
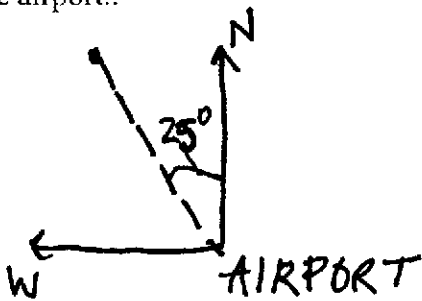
$$\tan A = \frac{8}{3}$$

$$A = \tan^{-1}\left(\frac{8}{3}\right)$$

$$\approx 69.44^\circ$$



9. Patrick and SpongeBob leave Bikini Bottom airport in a ultra-light flying at a bearing of N 25° W flying at 20 mph. Find the distance north and the distance west of the airport Homer is three hours after leaving the airport..



to find side s

$$\cos A = \frac{s}{60}$$

$$60 \cos 25^\circ = s$$

$s \approx 54.4$ miles north
of airport

to find side a

$$\sin A = \frac{a}{60}$$

$$60 \sin 25^\circ = a$$

$a \approx 25.4$ miles
west of airport

10. Find the trigonometric functions of the angle θ whose terminal side passes through the point $(-1, 2)$.

$$\sin \theta = y/r = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$$

$$\cos \theta = x/r = -\frac{1}{\sqrt{5}} = -\frac{\sqrt{5}}{5}$$

$$\tan \theta = y/x = \frac{2}{-1} = -2$$



$$r = \sqrt{(-1)^2 + 2^2}$$

$$r = \sqrt{1+4} = \sqrt{5}$$

11. Find trigonometric functions of θ satisfying the given conditions

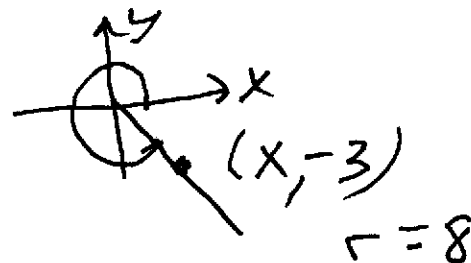
$$\sin \theta = -\frac{3}{8} \text{ and } \cos \theta > 0.$$

$$\sin \theta < 0 \text{ and } \cos \theta > 0$$

Quadrant IV

$$\cos \theta = \frac{\sqrt{55}}{8}$$

$$\tan \theta = \frac{-3}{\sqrt{55}} = -\frac{3\sqrt{55}}{55}$$



Find x

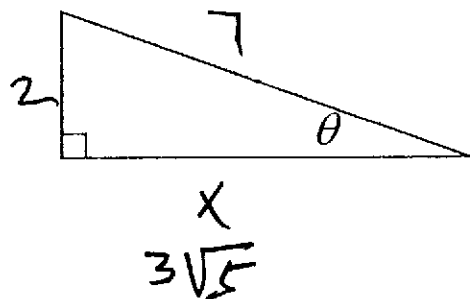
$$8 = \sqrt{x^2 + (-3)^2}$$

$$64 = x^2 + 9$$

$$55 = x^2$$

$$\sqrt{55} = x$$

10. Label the right triangle to evaluate, assume the angle is acute.



Given $\csc \theta = \frac{7}{2}$, find $\cos \theta =$.

$$x^2 + 2^2 = 7^2$$

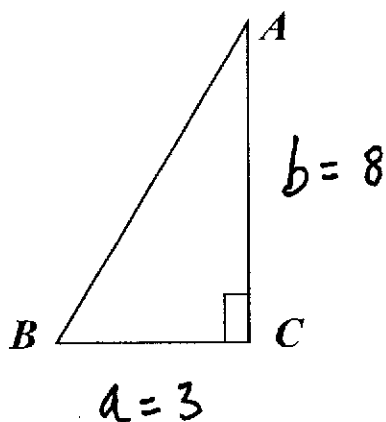
$$x^2 + 4 = 49$$

$$x^2 = 45$$

$$x = \sqrt{45} = 3\sqrt{5}$$

$$\cos \theta = \frac{3\sqrt{5}}{7}$$

11. Given side $a = 3$, and side $b = 8$, find angle A of the right triangle in degrees rounded to one decimal.



$$\tan A = \frac{3}{8}$$

$$A = \tan^{-1}\left(\frac{3}{8}\right)$$

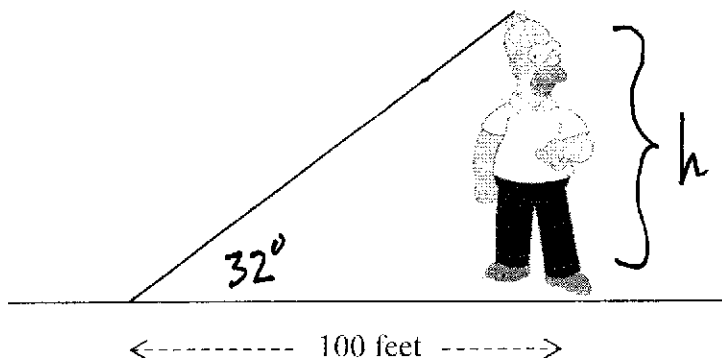
$$A \approx 20.6^\circ$$

12. The angle of elevation from a point 100 feet from the base to the top of a Homer Simpson Macy's Thanksgiving Day parade balloon is approximately 32° . Find the approximate height of the balloon.

$$\tan 32^\circ = \frac{h}{100}$$

$$100 \tan 32^\circ = h$$

$$h \approx 62.5 \text{ feet}$$



13. Find $\csc \theta$ given the terminal side of θ passes through the point $(2, -3)$.



$$r = \sqrt{x^2 + y^2}$$

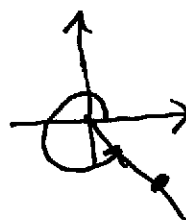
$$r = \sqrt{2^2 + (-3)^2} = \sqrt{13}$$

$$\csc \theta = \frac{r}{y}$$

$$= \frac{\sqrt{13}}{-3} = -\frac{\sqrt{13}}{3}$$

14. Find the $\tan \theta$ given $\sin \theta = -\frac{3}{8}$ and $\cos \theta > 0$.

$$\tan \theta = -\frac{3}{\sqrt{55}} = -\frac{3\sqrt{55}}{55}$$



$\sin \theta < 0$
 $\cos \theta > 0$
 Q IV

$$(x, -3) \quad r = 8$$

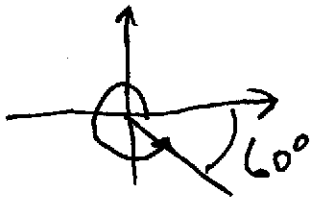
$$8 = \sqrt{x^2 + (-3)^2}$$

$$64 = x^2 + 9 \quad x^2 = 55$$

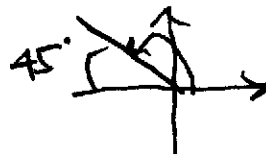
$$x = \sqrt{55}$$

15. Evaluate the trigonometric function without using a calculator:

a. $\sin 300^\circ = -\sin 60^\circ$
 $= -\frac{\sqrt{3}}{2}$



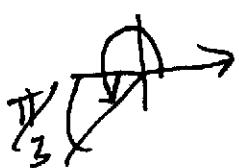
b. $\cos 135^\circ = -\cos 45^\circ = -\frac{\sqrt{2}}{2}$



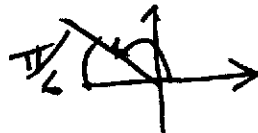
or
 $= -\frac{1}{\sqrt{2}}$

16. Evaluate the trigonometric function without using a calculator:

a. $\tan \frac{4\pi}{3} = +\tan \frac{\pi}{3}$
 $= \sqrt{3}$

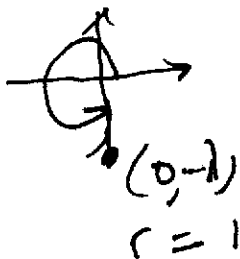


b. $\sin \frac{5\pi}{6} = +\sin \frac{\pi}{6} = \frac{1}{2}$

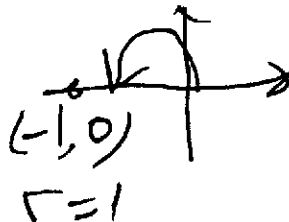


17. If possible, evaluate the trigonometric function of the quadrant angle.

a. $\tan \frac{3\pi}{2} = \text{undefined}$



b. $\sin 180^\circ = \frac{-1}{1} = -1$

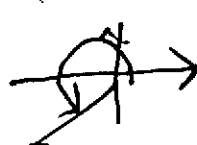


18. Find two solutions in radians ($0 \leq \theta \leq 2\pi$) of the equation without using a calculator:

$\sin \theta = -\frac{1}{2}$

Q III, IV

$\theta' = \frac{\pi}{6}$



$\theta = \frac{7\pi}{6}$



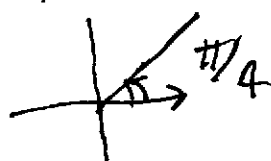
$\theta = \frac{11\pi}{6}$

19. Find two solutions in radians ($0 \leq \theta \leq 2\pi$) of the equation without using a calculator:

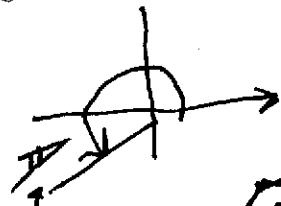
$$\cot \theta = 1 \rightarrow \tan \theta = 1$$

Q I, III

$$\theta' = \pi/4$$



$$\theta = \pi/4$$



$$\theta = 5\pi/4$$

20. Solve for θ , in radians $0 \leq \theta \leq 2\pi$ without using a calculator.

$$\cos^2 \theta - \sin \theta - 1 = 0$$

$$(1 - \sin^2 \theta) - \sin \theta - 1 = 0$$

$$- \sin^2 \theta - \sin \theta = 0$$

$$- \sin \theta (\sin \theta + 1) = 0$$

$$\sin \theta = 0 \quad \sin \theta + 1 = 0$$

$$\theta = 0, \pi, 2\pi \quad \theta = 3\pi/2$$

21. Solve for θ , in radians $0 \leq \theta \leq 2\pi$ without using a calculator..

$$2 \cos^2 \theta - \cos \theta - 1 = 0$$

$$(2 \cos \theta + 1)(\cos \theta - 1) = 0$$

$$2 \cos \theta + 1 = 0 \quad \cos \theta - 1 = 0$$

$$\cos \theta = -1/2$$

$$\cos \theta = 1$$

II, III $\theta' = \pi/3$

$$\theta = 0, 2\pi$$

$$\theta = 2\pi/3, 4\pi/3$$

22. Use a calculator to approximate two values of θ in radians ($0 \leq \theta < 2\pi$) that satisfy the equation. Round your answers to two decimal places

$$5 \tan x - 8 = 0$$

$$\tan x = 5/8$$

I, III

$$x' \approx 0.56$$

Q I

$$x \approx 0.56$$

Q III

$$x \approx 0.56 + \pi$$

$$\approx 3.70$$

23. Sketch one full period of the graph of $y = 4 \sin(2x - \pi)$, by finding all of the following:

amplitude: 4

period: $2\pi/2 = \pi$

step: $\pi/4$

starting point (phase shift): $2x - \pi = 0$
 $x = \pi/2$

key points:

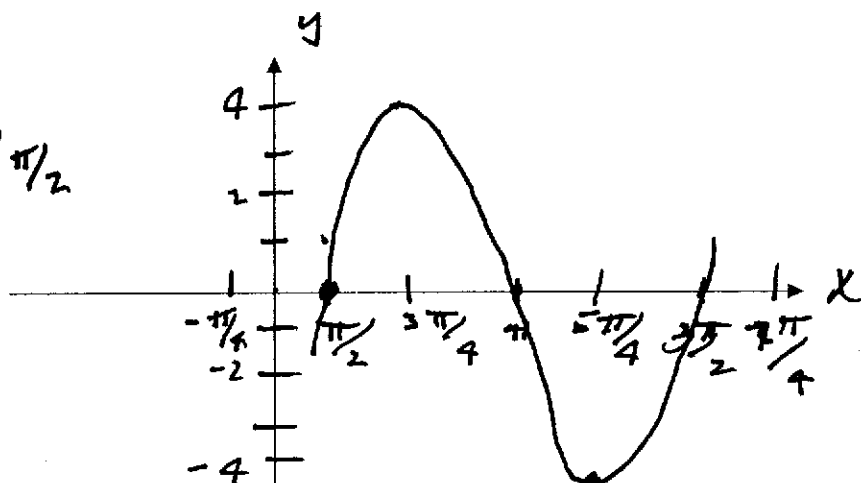
$$(\pi/2, 0)$$

$$(3\pi/4, 4)$$

$$(\pi, 0)$$

$$(5\pi/4, -4)$$

$$(3\pi/2, 0)$$



24. Find each of the following without using a calculator.

a. $\arcsin\left(\frac{1}{2}\right) = \pi/6$

b/c $\sin \pi/6 = 1/2$

b. $\arccos\left(-\frac{\sqrt{2}}{2}\right) = 3\pi/4$

b/c $\cos 3\pi/4 = -\sqrt{2}/2$