Chapter 4 Trigonometry

Section 4.1 Radian Measure and Degree Measure

Terminal and Initial Side of an Angle

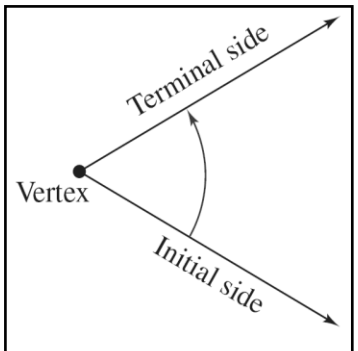


Figure 4.1

Standard Position of an Angle

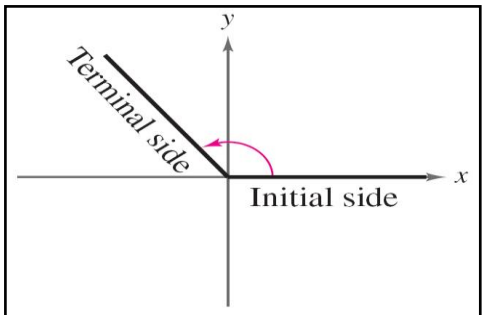


Figure 4.2

Positive and Negative Angles

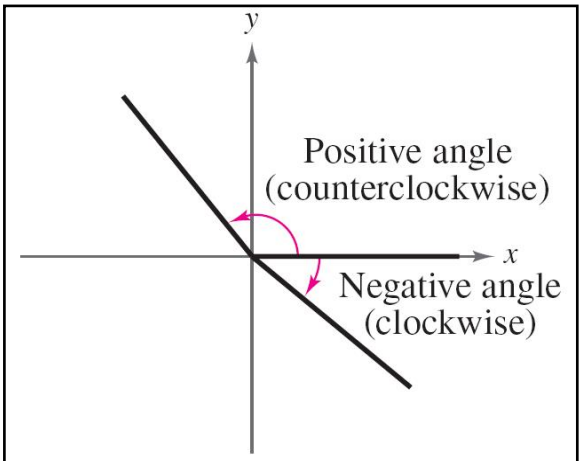


Figure 4.3

Coterminal Angles

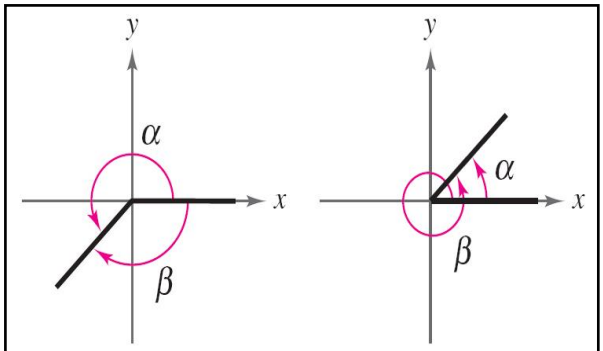
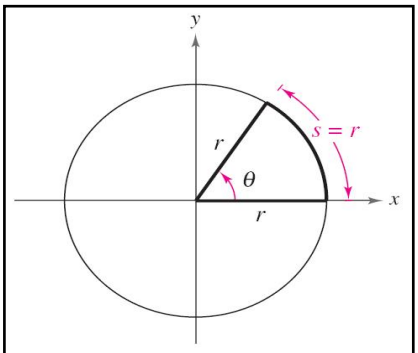


Figure 4.4

Illustration of Arc Length



Arc length = radius when θ = 1 radian.
Figure 4.5

Definition of Radian

One **radian** (rad) is the measure of a central angle θ that intercepts an arc s equal in length to the radius r of the circle. See Figure 4.5. Algebraically this means that

$$\theta = \frac{s}{r}$$

where θ is measured in radians.

Illustration of Six Radian Lengths

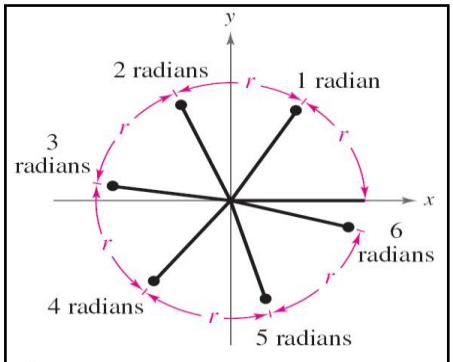


Figure 4.6

Common Radian Angles

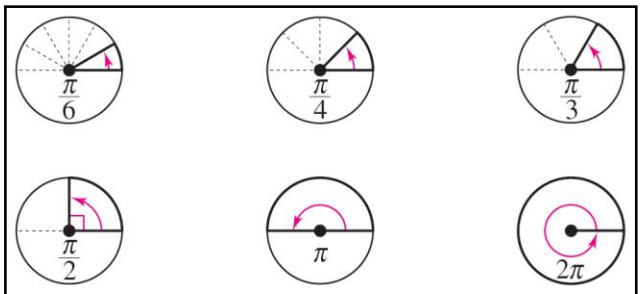


Figure 4.7

Section 4.2 The Unit Circle

Common Degree Measures on the Unit Circle

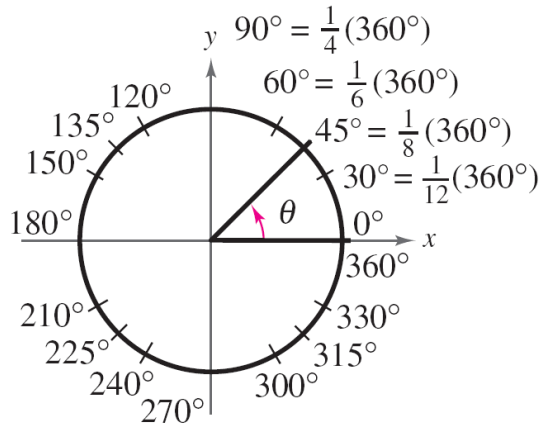


Figure 4.13

Common Values on a Unit Circle

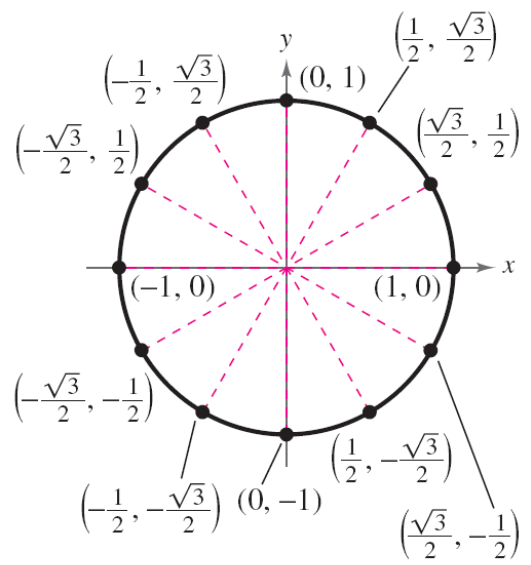


Figure 4.21

The Unit Circle

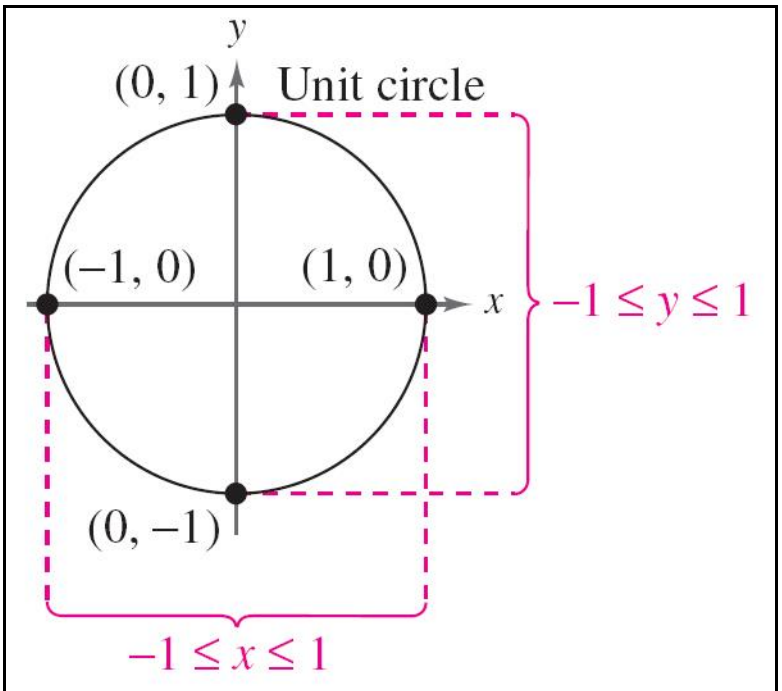


Figure 4.22

Unit Circle and Periodic Behavior

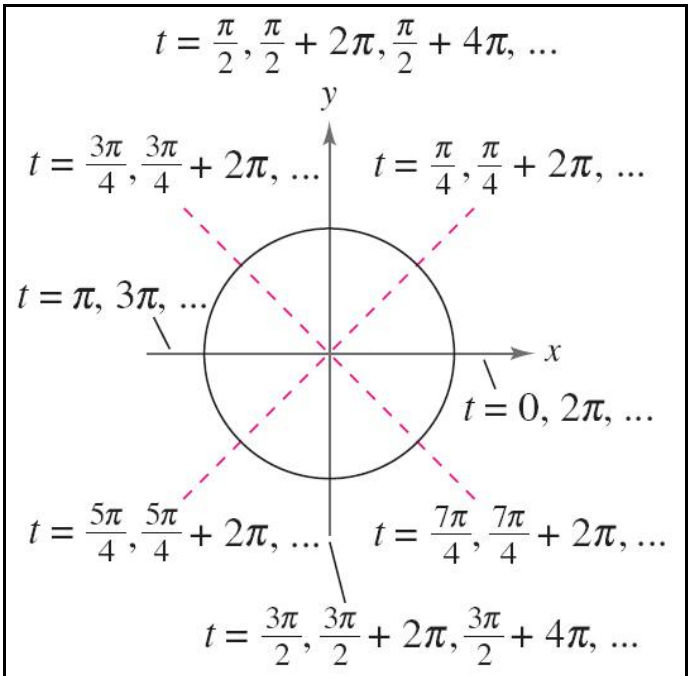


Figure 4.23

Section 4.3 Right Triangle Trigonometry

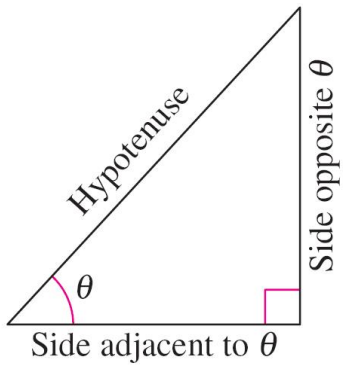


Figure 4.24

Right Triangle Definitions of Trigonometric Functions

Let θ be an *acute* angle of a right triangle. Then the six trigonometric functions *of the angle* θ are defined as follows. (Note that the functions in the second row are the *reciprocals* of the corresponding functions in the first row.)

$$\sin\theta = \frac{\text{opp}}{\text{hyp}} \qquad \cos\theta = \frac{\text{adj}}{\text{hyp}} \qquad \tan\theta = \frac{\text{opp}}{\text{adj}}$$

$$\csc\theta = \frac{\text{hyp}}{\text{opp}} \qquad \sec\theta = \frac{\text{hyp}}{\text{adj}} \qquad \cot\theta = \frac{\text{adj}}{\text{opp}}$$

The abbreviations "opp," "adj," and "hyp" represent the lengths of the three sides of a right triangle.

opp = the length of the side *opposite* θ

adj = the length of the side *adjacent* to θ

hyp = the length of the *hypotenuse*

Fundamental Trigonometric Identities

Reciprocal Identities

$$\sin\theta = \frac{1}{\csc\theta} \qquad \cos\theta = \frac{1}{\sec\theta} \qquad \tan\theta = \frac{1}{\cot\theta}$$

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

Quotient Identities

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

Pythagorean Identities

$$\sin^2\theta + \cos^2\theta = 1$$

$$1 + \tan^2\theta = \sec^2\theta$$

$$1 + \cot^2\theta = \csc^2\theta$$

Signs of the Trigonometric Functions

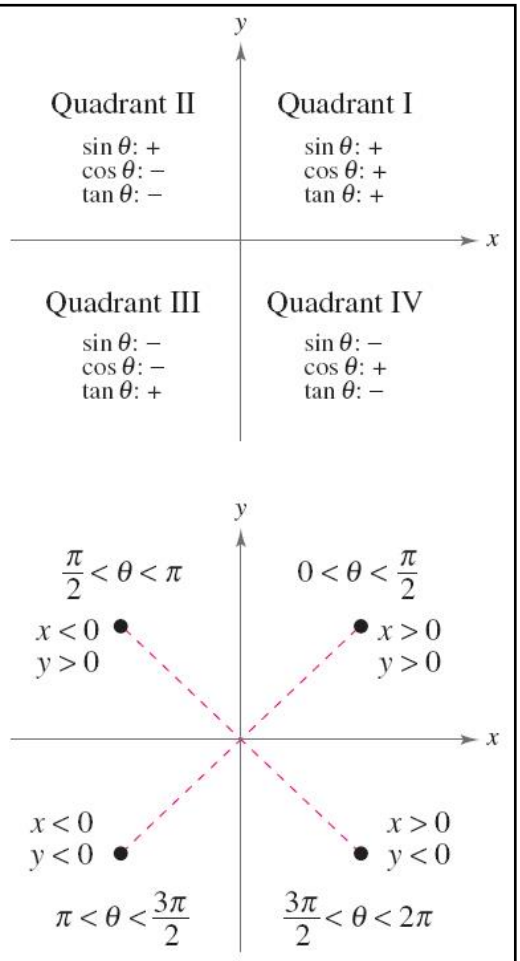


Figure 4.33

Trigonometric Values of Common Angles

θ (degrees)	$0°$	$30°$	$45°$	$60°$	$90°$	$180°$	$270°$
θ (radians)	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$
$\sin\theta$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	0	-1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	-1	0
$\tan\theta$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	Undef.	0	Undef.

Section 4.5 Graphs of Sines and Cosines

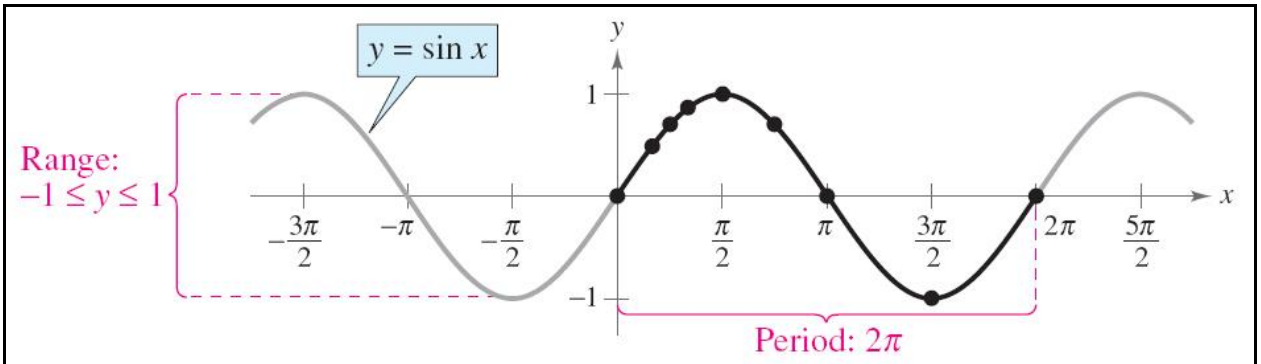


Figure 4.42

Graph of Cosine Curve

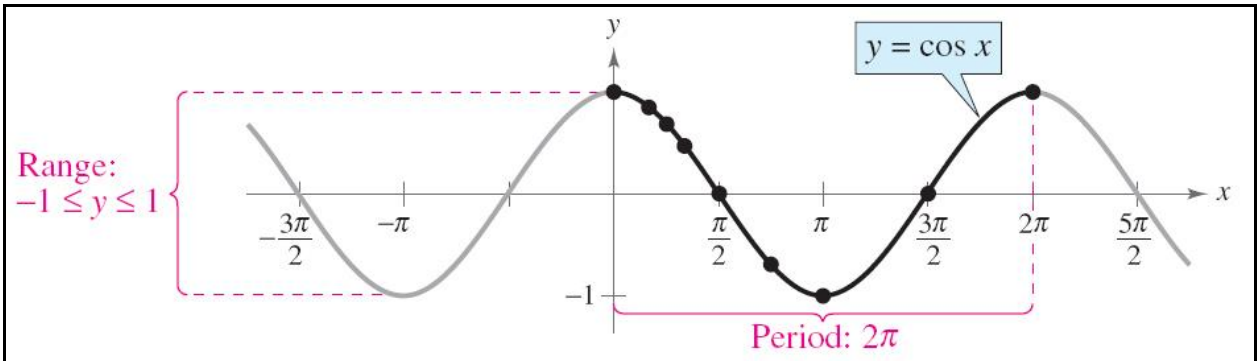


Figure 4.43

Key Points in the Sine and Cosine Curves

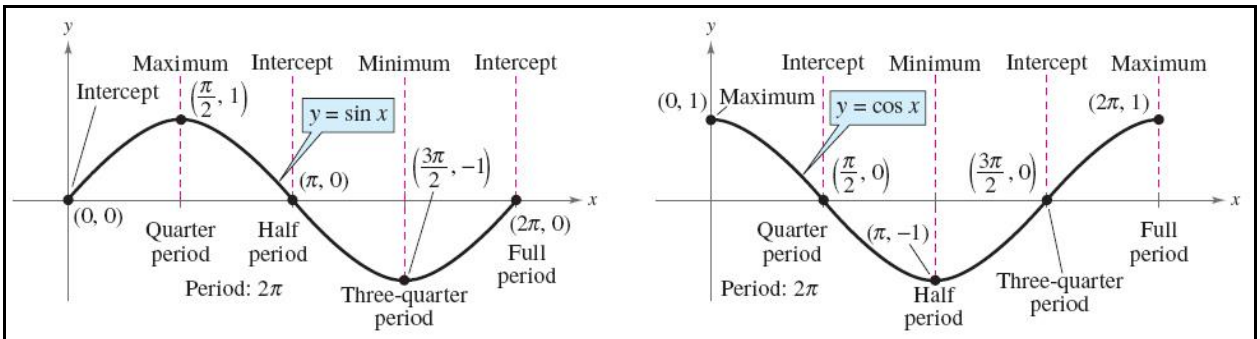
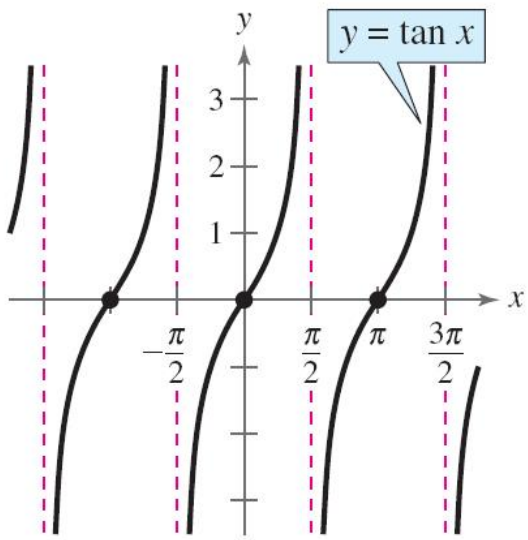


Figure 4.44

Graph of Tangent Function



Period: π

Domain: *all real numbers x,*

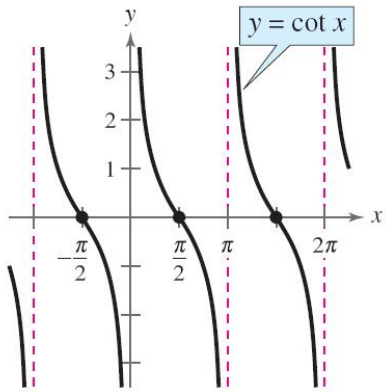
except $x = \frac{\pi}{2} + n\pi$

Range: $(-\infty, \infty)$

Vertical asymptotes: $x = \frac{\pi}{2} + n\pi$

Figure 4.55

Graph of Cotangent Function



Period: π

Domain: *all real numbers x,*

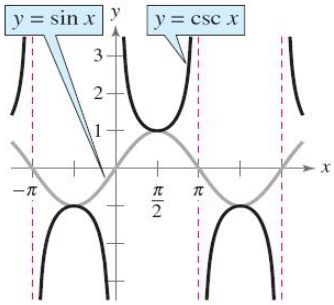
except $x = n\pi$

Range: $(-\infty, \infty)$

Vertical asymptotes: $x = n\pi$

Figure 4.59

Graphs of Cosecant and Secant Functions



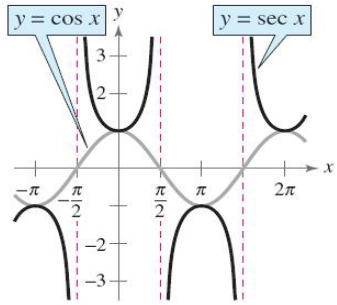
Period: 2π

Domain: *all real numbers x, except* $x = n\pi$

Range: $(-\infty, -1] \cup [1, \infty)$

Vertical asymptotes: $x = n\pi$

Symmetry: *origin*



Period: 2π

Domain: *all real numbers x, except* $x = \frac{\pi}{2} + n\pi$

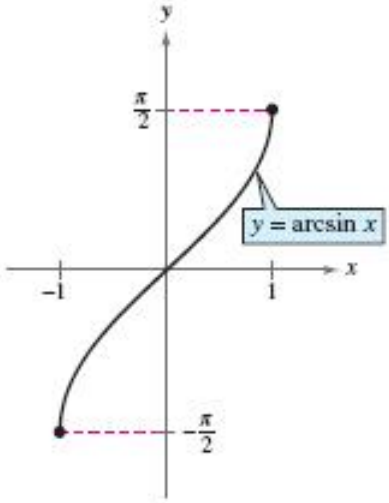
Range: $(-\infty, -1] \cup [1, \infty)$

Vertical asymptotes: $x = \frac{\pi}{2} + n\pi$

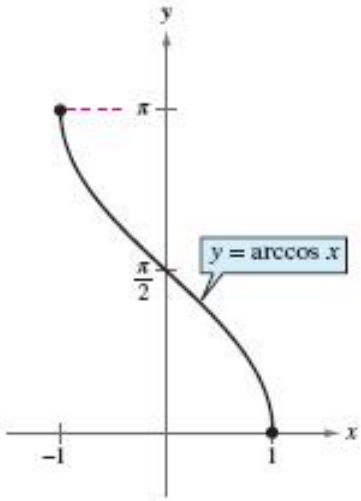
Symmetry: *y-axis*

Figure 4.61

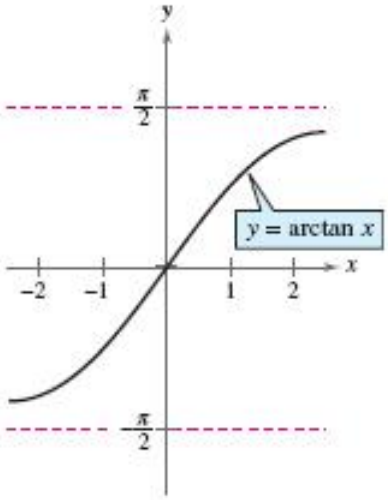
Graphs of Inverse Trig Functions



Domain: $[-1, 1]$; *Range:* $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$



Domain: $[-1, 1]$; *Range:* $[0, \pi]$

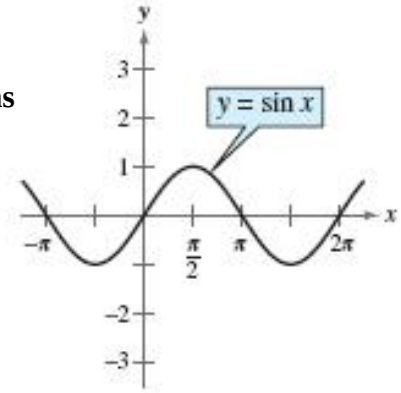


Domain: $(-\infty, \infty)$; *Range:* $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

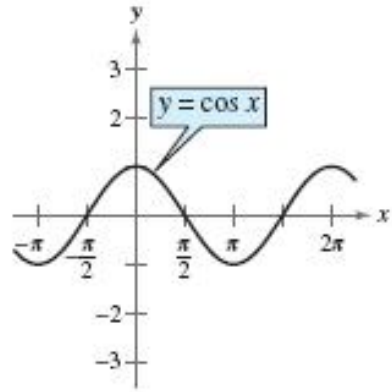
Figure 4.71

Graphs of

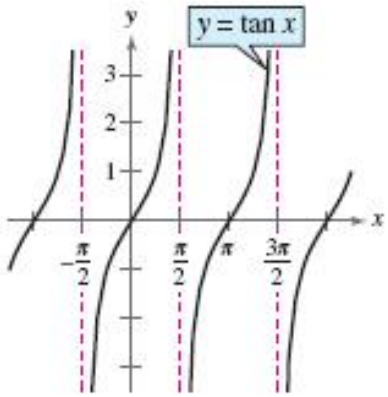
Basic Trigonometric Functions



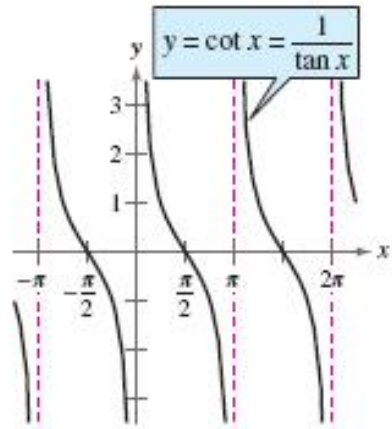
Domain: *all real numbers x*
Range: $[-1, 1]$
Period: 2π



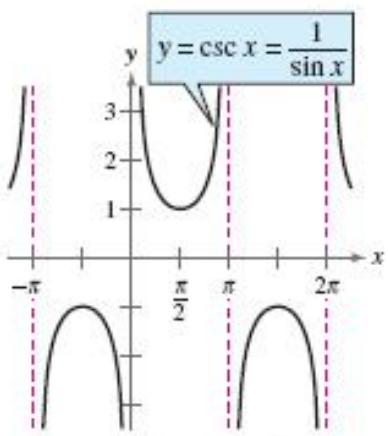
Domain: *all real numbers x*
Range: $[-1, 1]$
Period: 2π



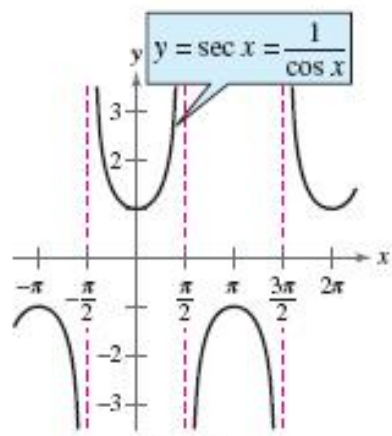
Domain: *all real numbers x,*
except $x = \frac{\pi}{2} + n\pi$
Range: $(-\infty, \infty)$
Period: π



Domain: *all real numbers x,*
except $x = n\pi$
Range: $(-\infty, \infty)$
Period: π



Domain: *all real numbers x,*
except $x = n\pi$
Range: $(-\infty, -1] \cup [1, \infty)$
Period: 2π



Domain: *all real numbers x,*
except $x = \frac{\pi}{2} + n\pi$
Range: $(-\infty, -1] \cup [1, \infty)$
Period: 2π

Figure 4.67