Math 41 Chapter 1

Section 1: Lines in the Plane

The **slope** of a nonvertical line is the number of units the line rises or falls vertically for each unit of horizontal change from left to right.

Definition of the Slope of a Line

The **slope** m of the nonvertical line passing through the points (x_1, y_1) and (x_2, y_2) is

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\text{Change in } y}{\text{Change in } x} = \frac{\text{Rise}}{\text{Run}}$$

where $x_1 \neq x_2$. (See Figure 3.24.)

Slope of a Line

1. A line with positive slope $(m > 0)$ *rises* from left to right.
2. A line with negative slope $(m < 0)$ *falls* from left to right.
3. A line with zero slope $(m = 0)$ is *horizontal.*
4. A line with undefined slope is *vertical.*

Slope-Intercept Form of the Equation of a Line

The graph of the equation

$$y = mx + b$$

is a line whose slope is m and whose y-intercept is $(0, b)$. (See Figure 3.34.)

Parallel Lines

Two distinct nonvertical lines are parallel if and only if they have the same slope.

Perpendicular Lines

Consider two nonvertical lines whose slopes are m_1 and m_2. The two lines are perpendicular if and only if their slopes are *negative reciprocals* of each other. That is,

$$m_1 = -\frac{1}{m_2}, \quad \text{or equivalently,} \quad m_1 \cdot m_2 = -1.$$

In real-life problems, slope can describe a **constant rate of change** or an **average rate of change.** In such cases, units of measure are used, such as miles per hour.

Point-Slope Form of the Equation of a Line

The **point-slope form** of the equation of the line that passes through the point (x_1, y_1) and has a slope of m is

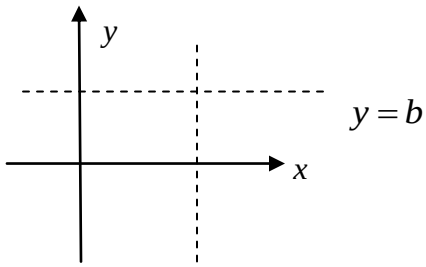
$$y - y_1 = m(x - x_1).$$

The **general form** of the equation of a line is $ax + by + c = 0$ General form

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1).$$ Two-point form

$y = b.$ Horizontal Line

$x = a$ Vertical Line



Summary of Equations of Lines

1. Slope of a line through (x_1, y_1) and (x_2, y_2): $m = \dfrac{y_2 - y_1}{x_2 - x_1}$
2. General form of equation of line: $ax + by + c = 0$
3. Equation of vertical line: $x = a$
4. Equation of horizontal line: $y = b$
5. Slope-intercept form of equation of line: $y = mx + b$
6. Point-slope form of equation of line: $y - y_1 = m(x - x_1)$
7. Parallel lines (equal slopes): $m_1 = m_2$
8. Perpendicular lines (negative reciprocal slopes): $m_1 = -\dfrac{1}{m_2}$

Section 2: Functions

Function: A **function** is a relationship between two variables such that to each value of the independent variable there corresponds exactly one value of the dependent variable.

Function Notation: $y = f(x)$

f is the **name** of the function.

y is the **dependent variable.**

x is the **independent variable.**

$f(x)$ is the **value of the function at x.**

Domain: The **domain** of a function is the set of all values (inputs) of the independent variable for which the function is defined. If x is in the domain of f, then f is said to be **defined** at x. If x is not in the domain of f,then f is said to be **undefined** at x.

Range: The **range** of a function is the set of all values (outputs) assumed by the dependent variable (that is, the set of all function values).

Implied Domain: If f is defined by an algebraic expression and the domain is not specified, the **implied domain** consists of all real numbers for which the expression is defined.

Function Notation

In the notation $f(x)$:

f is the **name** of the function.

x is the **domain** (or input) value.

$f(x)$ is the **range** (or output) value y for a given x.

The symbol $f(x)$ is read as *the value of f at x* or simply *f of x*.

Section 1.3 Graphs of Functions

Consider a function f whose domain and range are the set of real numbers. The **graph** of f is the set of ordered pairs

$(x, f(x))$, where x is in the domain of f.

x = x-coordinate of the ordered pair

$f(x)$ = y-coordinate of the ordered pair

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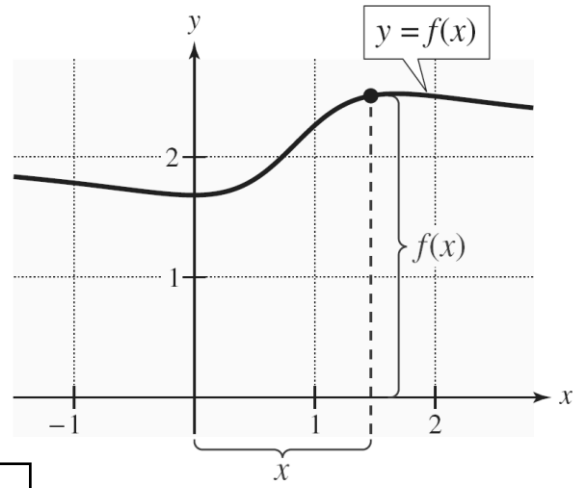
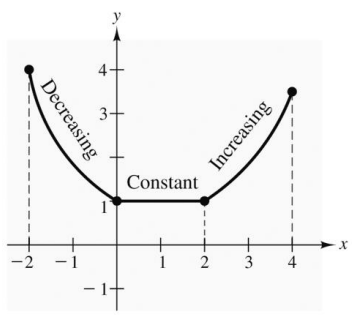


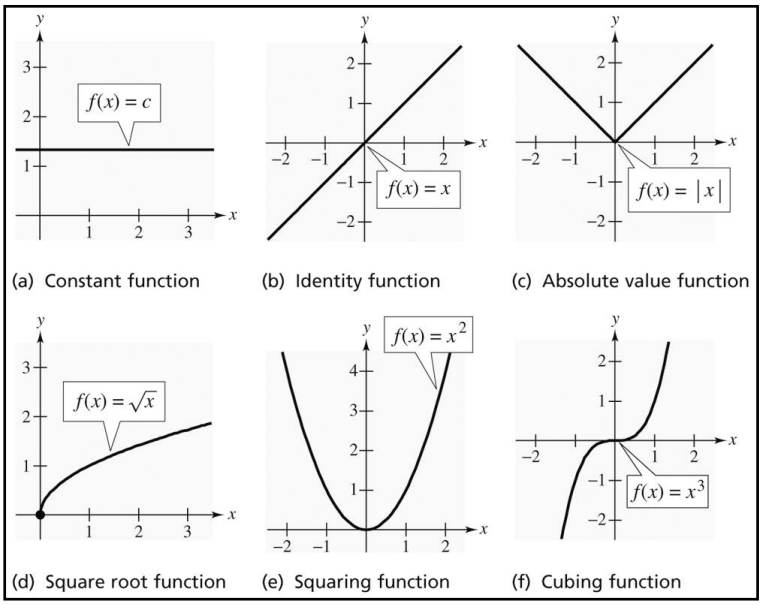
A function f is **increasing** on an interval if, for any x_1 and x_2 in the interval, $x_1 < x_2$ implies $f(x_1) < f(x_2)$.

A function f is **decreasing** on an interval if, for any x_1 and x_2 in the interval, $x_1 < x_2$ implies $f(x_1) > f(x_2)$.

A function f is **constant** on an interval if, for any x_1 and x_2 in the interval, $f(x_1) = f(x_2)$.



Six basic functions,



Section 1.4 Transformations of Functions and their graphs

Vertical and Horizontal Shifts

Let c be a positive real number. **Vertical** and **horizontal shifts** of the graph of the function $y = f(x)$ are represented as follows.

1. Vertical shift c units *upward:* $h(x) = f(x) + c$
2. Vertical shift c units *downward:* $h(x) = f(x) - c$
3. Horizontal shift c units to the *right:* $h(x) = f(x - c)$
4. Horizontal shift c units to the *left:* $h(x) = f(x + c)$

Reflections in the Coordinate Axes

Reflections of the graph of $y = f(x)$ are represented as follows.

1. **Reflection in the x-axis:** $h(x) = -f(x)$
2. **Reflection in the y-axis:** $h(x) = f(-x)$

Section 1.5 The Algebra of Functions: Combinations of Functions

Arithmetic Combinations of Functions

Let f and g be two functions with overlapping domains. Then, for all x common to both domains, the **sum, difference, product,** and **quotient** of f and g are defined as follows.

1. *Sum:* $(f + g)(x) = f(x) + g(x)$
2. *Difference:* $(f - g)(x) = f(x) - g(x)$
3. *Product:* $(fg)(x) = f(x) \cdot g(x)$
4. *Quotient:* $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}, \quad g(x) \neq 0$

Composition of Functions

Let f and g be functions of x. Then the **composite** functions, $(f \circ g)(x) = f(g(x))$, $(g \circ f)(x) = g(f(x))$, where the domain of $f \circ g$ is all x in the domain of g such that $g(x)$ is in the domain of f.

Section 1.6 Inverse Functions

A function f from a set A to a set B is one-to-one if each element y in the set B (range, outputs) is mapped from exactly one element x in the set A (domain, inputs).

Determine if a function is one-to-one using the Horizontal Line Test. If you find a horizontal line that crosses the graph in more than one point, the graph does not represent a one-to-one function.

The range of a function is the domain of its inverse. The domain of a function is the range of its inverse. Graphs of inverse function are reflections in the line $y = x$. Every point (a,b) of f corresponds to a point (b,a) of f^{-1}.Test for inverse functions,

$$f^{-1}(f(x)) = f(f^{-1},(x)) = x.$$

To find an inverse, , $f^{-1}(x)$, if it exists.

1. In the equation for $f(x)$, replace $f(x)$ by y.
2. Interchange the roles of x and y.
3. Solve the new equation for y. If the new equation does not represent y as a function of x, the function f does not have an inverse function. If the new equation does represent y as a function of x, continue to Step 4.
4. Replace y by $f^{-1}(x)$ in the new equation.
5. Verify that f and f^{-1} are inverse functions of each other by showing that the domain of f is equal to the range of f^{-1}, the range of f is equal to the domain of f^{-1}, and $f(f^{-1}(x)) = x = f^{-1}(f(x))$.