Math 41
Worksheet

1. $3(x-4)+2=9-2(x-3)$
$3x-12+2=9-2x+6$
$5x=25$
$x=5$

out of order ü

3. $\parallel$ to y-axis $\rightarrow$ vertical line through $\left(\frac{1}{2}, -\frac{5}{3}\right)$ with undefined slope

$x=\frac{1}{2}$

out of order ü

2. $\perp$ to $y=-\frac{1}{2}x-6 \rightarrow m_{\perp}=2$

through $(0,5)$

$y-y_1=m(x-x_1)$
$y-5=2(x-0)$
$y=2x+5$

ü out of order

4. $(t, P)$
$(0, 300)$ and $(10, 400)$

$m=\frac{\Delta P}{\Delta t}=\frac{400-300}{10-0}=\frac{100}{10}=10$

$(0, 300)$

$P-P_1=m(t-t_1)$ $\quad P-300=10(t-0)$
$P=10t+300$

5 (a) No. Dr. Doofenshmertz is assigned to more than one element.

(b) No. One value of $x$ is assigned to more than one $y$-value. For example

$$x = 0 \begin{cases} \nearrow y = 2 \\ \searrow y = -2 \end{cases}$$

(c) Yes. Each element $x$ is assigned to exactly one element $y$.

6. $f(x) = 2x - 5$

(a) $f(t) = 2t - 5$

(b) $f(3) = 2(3) - 5 = 1$

(c) $f(t+3) = 2(t+3) - 5$
$= 2t + 1$

(d) $f(t) + f(3) = (2t-5) + (1) = 2t - 4$

7. $g(x) = -(x+2)^2 - 1$ is a

horizontal shift 2 units to the left, a reflection about the x-axis and a vertical shift down one units

8. a. domain: $[-3, 5]$ or $-3 \le x \le 5$

b. range: $[-3, 6]$ or $-3 \le y \le 6$

c. $f(3) = 6$

d. $f(x) = 0$ at $x = -5/2$ and $x = 2$

9. Assuming no stretching or shrinking

$g(x) = -|x-3| + 2$

10. $\left(-3x^{-2}y^{3}\right)^{-2} \cdot \left(\frac{2x^{-3}}{5y^{4}}\right)^{0}$

$\frac{1}{(-3x^{-2}y^{3})^2} \cdot (1) = \frac{1}{9x^{-4}y^{6}} = \frac{x^4}{9y^6}$

11. $(3x^2-5)(3x^2+5) = 9x^4-25$

12. $(3x^2-5)^2 = 9x^4 - 30x^2 + 25$

13. $8x^3 - y^3 = (2x)^3 - (y)^3$

$= (2x-y)(4x^2+2xy+y^2)$

14 $6x^2 - 11x - 10 = (3x+2)(2x-5)$

15. $x^3 + 3x^2 - 4x - 12 = 0$

$x^2(x+3) - 4(x+3) = 0$

$(x+3)(x^2-4) = 0$

$x+3=0 \qquad x^2-4=0$

$x=-3 \qquad x^2=4$

$x=\pm 2$

16. $\dfrac{4}{x+2} + \dfrac{x}{(x-2)(x+2)} = \dfrac{4(x-2)+x}{(x-2)(x+2)}$

$= \dfrac{4x-8+x}{(x-2)(x+2)}$

$= \dfrac{5x-8}{(x-2)(x+2)}$

17. $\dfrac{x^2-1}{2x-6} \div \dfrac{x^2+x}{x^2-x-6}$

$= \dfrac{\cancel{(x+1)}(x-1)}{2\cancel{(x-3)}} \cdot \dfrac{\cancel{(x-3)}(x+2)}{x\cancel{(x+1)}}$

$= \dfrac{(x-1)(x+2)}{2x}, \; x \neq 3, -2, -1$

18. $\dfrac{2x}{x-2} = 3 + \dfrac{x}{x-2}$

$(x-2)\left(\dfrac{2x}{x-2}\right) = \left(3 + \dfrac{x}{x-2}\right)(x-2)$

$2x = 3(x-2) + x$

$2x = 3x - 6 + x$

$-2x = -6$

$x = 3$

19. a $x^2\sqrt{x} = x^2 x^{1/2}$

$= x^{5/2}$

b. $\left(\frac{1}{8}\right)^{-\frac{2}{3}} = 8^{2/3} = \left(\sqrt[3]{8}\right)^2 = 2^2 = 4$

20. $5\sqrt{2x} - x\sqrt{20x} + 3\sqrt{5x^3}$

$5\sqrt{2x} - 2x\sqrt{5x} + 3x\sqrt{5x}$

$5\sqrt{2}x + x\sqrt{5x}$

21. $(4\sqrt{x} - y)(3\sqrt{x} + 5y)$

$= 12x + 20y\sqrt{x} - 3y\sqrt{x} - 5y^2$

$= 12x + 17y\sqrt{x} - 5y^2$

22. $\sqrt{x+3} - 1 = x$

$\sqrt{x+3} = x+1$

$(\sqrt{x+3})^2 = (x+1)^2$

$x+3 = x^2 + 2x + 1$

$x^2 + x - 2 = 0$

$(x+2)(x-1) = 0$

$x+2=0 \quad x-1=0$

$x=-2 \quad x=1$

check $x=-2$

$\sqrt{-2+3} - 1 = -2$

$\sqrt{1} - 1 \neq 2$ extraneous

$x=1$

$\sqrt{1+3} - 1 = 1$

$\sqrt{4} - 1 = 1$

$2 - 1 = 1$ ✓

23. $x^2 - 2x - 1 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-1)}}{2(1)}$$

$$x = \frac{2 \pm \sqrt{4+4}}{2} = \frac{2 \pm \sqrt{8}}{2} = \frac{2 \pm 2\sqrt{2}}{2}$$

$$= \frac{2(1 \pm \sqrt{2})}{2}$$

$$= 1 \pm \sqrt{2}$$

24. $(2x-1)^2 = 8$

$2x - 1 = \pm\sqrt{8}$

$2x - 1 = \pm 2\sqrt{2}$

$2x = 1 \pm 2\sqrt{2}$

$$x = \frac{1 \pm 2\sqrt{2}}{2}$$

25. $x(2x-3) = 1$

$2x^2 - 3x + 1 = 0$

$(2x-1)(x-1) = 0$

$2x - 1 = 0 \quad x - 1 = 0$

$2x = 1$

$x = \frac{1}{2} \quad x = 1$