Math 41
Exam 3
April 22, 2016

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calculators ok! permitted

1. The value of a new SUV can be modeled by the equation $V = 40{,}000(0.75)^t$ and is depreciating in value where t is the number of years ***since*** purchase.

a. Find the value of the SUV when purchased.

$V(0) = 40{,}000(0.75)^0$

$= \$40{,}000\,(1)$

$= \$40{,}000$

b. Find the value after 4 years.

$V(4) = 40{,}000(0.75)^4$

$= \$12{,}656.25$

2. Use the appropriate formula in Problem #2. $A = P(1+\frac{r}{n})^{nt}$ and $A = Pe^{rt}$

Suppose very rich and very eccentric great Aunt Sofia Maria deposited \$10,000 into an account the day that you were born. Find the amount in that account on your 21st birthday, if the interest rate was 2% and the interest was compounded,

a. **Monthly.** $(12)(21)$

$A = 10{,}000\left(1 + \frac{0.02}{12}\right)$

$\approx \$15{,}214.30$

b. **Continuously.** $(0.02)(21)$

$A = 10{,}000e$

$\approx \$15{,}219.62$

3. Evaluate the logarithm without a calculator:

a. $\log_2 8 = 3$

because $2^3 = 8$

b. $\log_\pi 1 = 0$

because $\pi^0 = 1$

4. Use the change of base formula to approximate the logarithm, and round to three decimal places.

a. $\log_3 20 \approx 2.727$

$\frac{\ln 20}{\ln 3} \approx$

or

$\frac{\log 20}{\log 3} \approx$

b. $\log_{\frac{1}{2}} 5 \approx -2.322$

$\frac{\ln 5}{\ln(1/2)} \approx$

$\frac{\log 5}{\log(1/2)} \approx$

5. Use the properties of logarithms to write *(expand)* the expression as a sum, difference and/or multiple of logarithms.

$$\ln\left(\frac{\sqrt{x^2+1}}{x^2}\right) = \ln\sqrt{x^2+1} - \ln x^2$$

$$= \ln(x^2+1)^{1/2} - \ln x^2$$

$$= \tfrac{1}{2}\ln(x^2+1) - 2\ln x$$

6. Use the properties of logarithms to write *(condense)* the expression as the logarithm of a single quantity.

$$2\ln(x) + \tfrac{1}{3}\ln(x+1) = \ln x^2 + \ln(x+1)^{1/3}$$

$$= \ln\left[x^2(x+1)^{1/3}\right]$$

or

$$= \ln\left(x^2\sqrt[3]{x+1}\right)$$

7. Solve the exponential equation for x:
Leave your answer in terms of logarithms.

$$2e^{x-4} - 1 = 0$$

$$2e^{x-4} = 1$$

$$e^{x-4} = 1/2$$

$$\ln e^{x-4} = \ln(1/2)$$

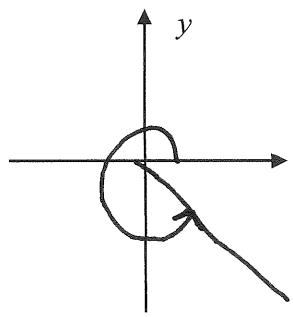
$$x - 4 = \ln(1/2) \rightarrow x = 4 + \ln(1/2)$$

8. a. Draw the angle is standard position and convert the angle to degree measure.

$$\theta = \frac{5\pi}{3}$$

$60°$

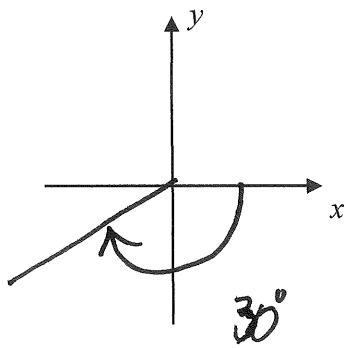
$\frac{5\pi}{3} \cdot \frac{180°}{\pi}$



$$\boxed{\theta = 300°}$$

b. Draw the angle in standard position and convert the angle to degree measure.

$$\theta = -\frac{5\pi}{6}$$



$30°$

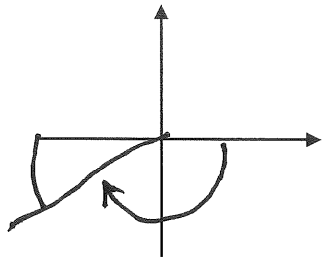
$-\frac{5\pi}{6} \cdot \frac{180°}{\pi}$

Degrees

$$\theta = -150°$$

9. a. Draw the angle in standard position and convert the angle to radian measure

$\theta = -150^\circ$

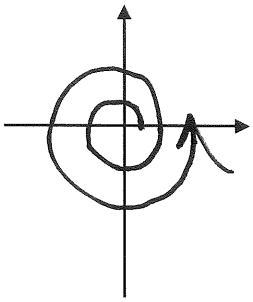


$\boxed{\theta = -5\pi/6}$

$-150^\circ \cdot \frac{\pi}{180^\circ}$

b. Draw the angle in standard position and convert the angle to radian measure

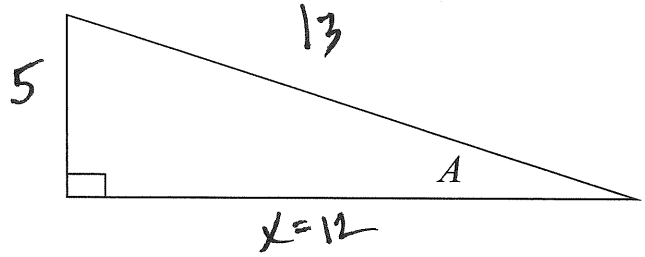
$\theta = 720^\circ$



$\boxed{\theta = 4\pi}$

$720 \cdot \frac{\pi}{180^\circ}$

10. Label the right triangle to find $\tan A$, given $\sin A = \frac{5}{13}$.



$x^2 + 5^2 = 13^2$

$x^2 + 25 = 169$

$x^2 = 144$

$x = 12$

$\tan A = 5/12$

$\sin\theta = o/h$

$\cos\theta = a/h$

$\tan\theta = o/a$

11. Find θ, $0^o < \theta < 90^o$, in degrees, for each equation. Do not use a calculator.

a. $\sin\theta = \frac{\sqrt{3}}{2}$

$\theta = 60^\circ$

b. $\csc\theta = \sqrt{2}$

$\sin\theta = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

$\theta = 45^\circ$

12. Find θ, $0 < \theta < \frac{\pi}{2}$, in radians, for each equation. Do not use a calculator.

a. $\tan\theta = \frac{\sqrt{3}}{3}$

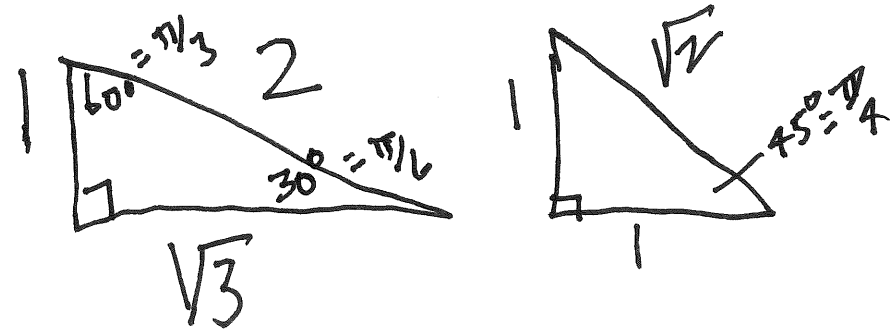
$\theta = \pi/6$

b. $\sec\theta = \sqrt{2}$

$\cos\theta = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

$\theta = \pi/4$

F.Y.I.



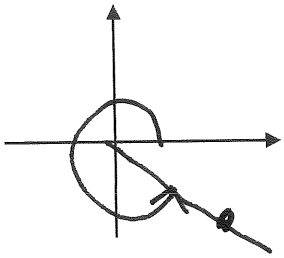
13. Find $\sin\theta$ of the angle θ whose terminal side passes through the point $(2,-1)$.

F.Y.I

$\sin\theta = y/r$

$\cos\theta = x/r$

$\tan\theta = y/x$



$(2,-1)$

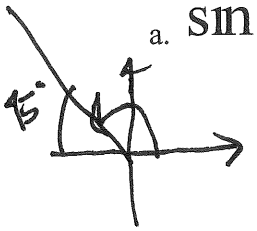
$r = \sqrt{x^2+y^2}$

$r = \sqrt{2^2+(-1)^2} = \sqrt{5}$

$\sin\theta = -\frac{1}{\sqrt{5}} = -\frac{\sqrt{5}}{5}$

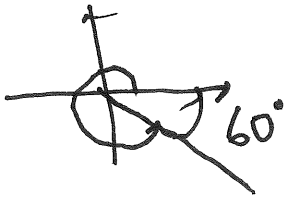
14. Evaluate the trigonometric function without using a calculator:

a. $\sin 135^\circ = +\sin 45^\circ$



$= \frac{\sqrt{2}}{2}$

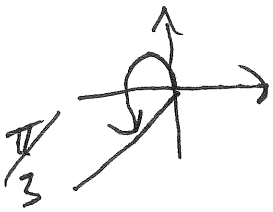
b. $\cos 300^\circ = +\cos 60^\circ$



$= \frac{1}{2}$

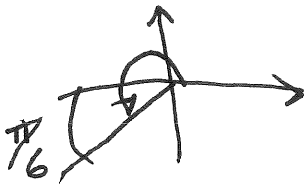
15. Evaluate the trigonometric function without using a calculator:

a. $\tan \frac{4\pi}{3} = + \tan \pi/3$



$= \sqrt{3}$

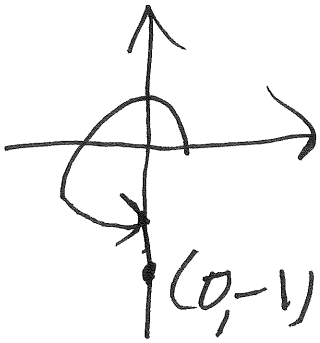
b. $\sin \frac{7\pi}{6} = - \sin \pi/6$



$= -1/2$

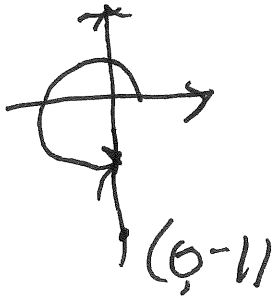
16. If possible, evaluate the trigonometric function of the quadrant angle.

a. $\tan \frac{3\pi}{2} =$ undefined



$r = 1$

b. $\cos 270^\circ = 0$

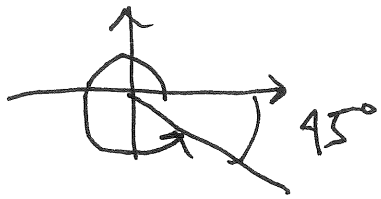


$r = 1$

17. Find two solutions in degrees $(0^\circ \le x < 360^\circ)$ of the equation: $\tan x = -1$

Q II, IV

$x' = 45^\circ$



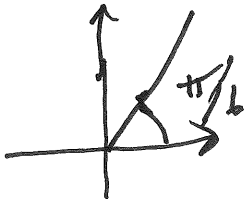
$x = 135^\circ$

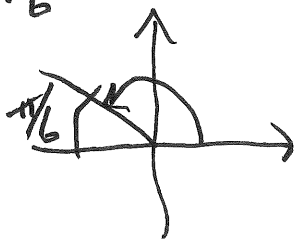
$x = 315^\circ$

18. Find two solutions in radians $(0 \le x < 2\pi)$ of the equation: $\sin x = \frac{1}{2}$

Q I, II

$x' = \pi/6$





$x = \pi/6$

$x = 5\pi/6$