

1. The value of a new SUV was \$35,000 in 2010 and is depreciating according to the exponential model, $V = 35,000(0.875)^t$ where V is the value and t is the number of years since purchase.

a. Find the value after 1 year.

$$V = 35,000(0.875)^1$$

$$\approx \$30,625$$

b. Find the value after 5 years.

$$V = 35,000(0.875)^5$$

$$\approx \$17,951.81$$

2. Use the appropriate formula in Problem #2.

$$A = P\left(1 + \frac{r}{n}\right)^{nt} \text{ and } A = Pe^{rt}$$

Suppose very rich and very eccentric great Aunt Sofia Maria deposited \$100,000 into an account the day that you were born. Find the amount in that account on your 21st birthday, if the interest rate was 3% and the interest was compounded,

a. Monthly.

$$A = 100,000\left(1 + \frac{0.03}{12}\right)^{(12)(21)}$$

$$\approx \$187,613.50$$

b. Continuously.

$$A = 100,000e^{(0.03)(21)}$$

$$\approx \$187,761.06$$

3. Write the logarithmic form of the exponential equation.

a. $2^{-5} = \frac{1}{32}$

$$\log_2 \frac{1}{32} = -5$$

b. $\sqrt{e} \approx 1.649 \rightarrow e^{1/2} \approx 1.649$

$$\ln 1.649 \approx \frac{1}{2}$$

4. Write the exponential form of the logarithmic equation

a. $\ln 12 \approx 2.485$

$$e^{2.485} \approx 12$$

b. $\log_2 16 = 4$

$$2^4 = 16$$

5. Evaluate the logarithm without a calculator:

a. $\log_4 16 = 2$

because $4^2 = 16$

b. $\log_{2/3} 1 = 0$

because $(2/3)^0 = 1$

6. Use the change of base formula to approximate the logarithm, and round to three decimal places.

a. $\log_3 30 \approx 3.096$

$$\log_3 30 = \frac{\ln 30}{\ln 3}$$

b. $\log_{1/2} 12 \approx -3.585$

$$\log_{1/2} 12 = \frac{\ln 12}{\ln(1/2)}$$

7. Use the properties of logarithms to write (*expand*) the expression as a sum, difference and/or multiple of logarithms.

$$\begin{aligned}\ln\left(x^3\sqrt{x^2+1}\right) &= \ln x^3 + \ln (x^2+1)^{1/2} \\ &= 3\ln x + \frac{1}{2}\ln(x^2+1)\end{aligned}$$

8. Use the properties of logarithms to write (*condense*) the expression as the logarithm of a single quantity.

$$\begin{aligned}\frac{1}{2}\ln(x+2) - 4\ln(x) &= \ln(x+2)^{1/2} - \ln x^4 \\ &= \ln\left(\frac{\sqrt{x+2}}{x^4}\right)\end{aligned}$$

9. Solve the logarithmic equation for x : $\ln(x+1) = 2$

Leave your answer in terms of exponentials. Then round to three decimal places.

$$\begin{aligned}\ln(x+1) &= 2 \\ e^{\ln(x+1)} &= e^2 \\ x+1 &= e^2 \\ x &= -1 + e^2 \approx 6.389\end{aligned}$$

10. The enrollment of Mrs. Puff's Driving School, a branch campus of Bikini Bottom University, has been increasing exponentially ever since SpongeBob passed his driving exam. Given that the enrollment was 2500 in the year 2000, and had increased to 5000 by the year 2010. Find the exponential growth model, $N = a \cdot e^{kt}$ giving the enrollment, N in terms of the year t , where $t = 0$ corresponds to the year 2000.



N in terms of $t \rightarrow (t, N)$

points: $(0, 2500)$ and $(10, 5000)$

$N = ae^{kt}$ using first point

$$2500 = ae^{kt} \quad k/9$$

$$2500 = ae^0$$

$$2500 = a$$

So $N = 2500e^{kt}$ using second pt.

$$5000 = 2500e^{10k}$$

$$2 = e^{10k} \rightarrow \ln 2 = \ln e^{10k} \rightarrow \frac{1}{10} \ln 2 = k$$

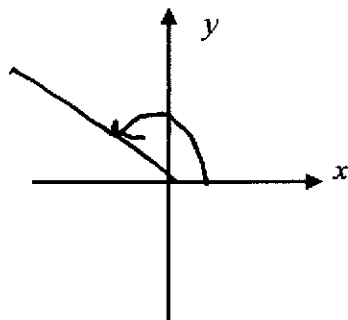
$$k \approx 0.0693$$

$$N = 2500e^{0.0693t}$$



11. a. Draw the angle in standard position and convert the angle to degree measure.

$$\theta = \frac{2\pi}{3}$$



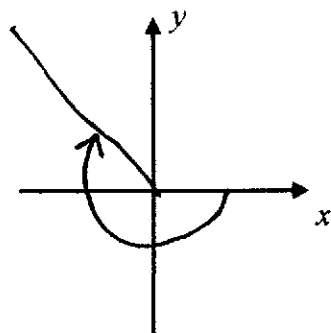
$$\frac{2\pi}{3} \cdot \frac{180^\circ}{\pi}$$

Degrees

$$\theta = 120^\circ$$

b. Draw the angle in standard position and convert the angle to degree measure.

$$\theta = -\frac{7\pi}{6}$$



$$-\frac{7\pi}{6} \cdot \frac{180^\circ}{\pi}$$

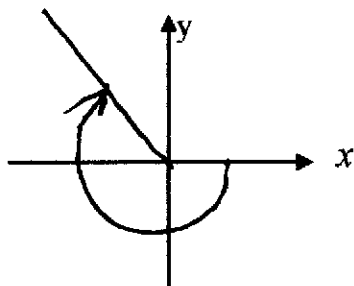
Degrees

$$\theta = -210^\circ$$

12. a. Draw the angle in standard position and convert the angle to radian measure

$$\theta = -225^\circ$$

$$-225^\circ \times \frac{\pi}{180^\circ}$$



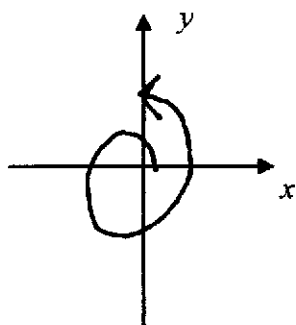
Radians

$$\theta = -\frac{5\pi}{4}$$

b. Draw the angle in standard position and convert the angle to radian measure

$$\theta = 450^\circ$$

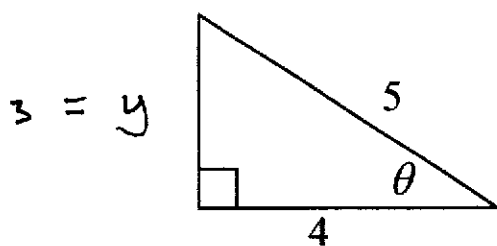
$$450^\circ \times \frac{\pi}{180^\circ}$$



Radians

$$\theta = \frac{5\pi}{2}$$

13. Use the right triangle to find each ratio of the angle, θ .



$$\begin{aligned} 4^2 + y^2 &= 5^2 \\ 16 + y^2 &= 25 \\ y^2 &= 9 \quad y = 3 \end{aligned}$$

$$\sin \theta = \frac{3}{5}$$

$$\csc \theta = \frac{5}{3}$$

$$\cos \theta = \frac{4}{5}$$

$$\sec \theta = \frac{5}{4}$$

$$\tan \theta = \frac{3}{4}$$

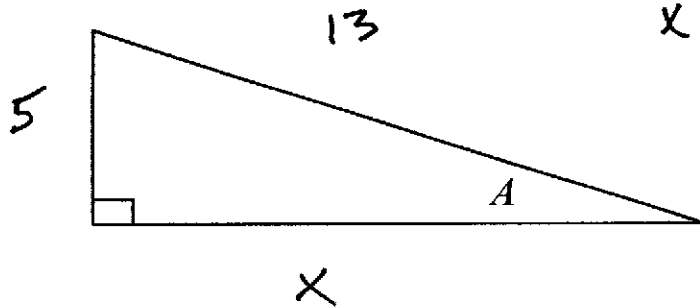
$$\cot \theta = \frac{4}{3}$$

14. Find the exact ratio of the special angle without a calculator:

a. $\sin \frac{\pi}{4} = \frac{\sqrt{2}}{2}$

b. $\sec 60^\circ = \frac{1}{\cos 60^\circ}$
 $= \frac{1}{\frac{1}{2}} = 2$

15. Label the right triangle to find $\tan A$, given $\sin A = \frac{5}{13}$.



$$\begin{aligned} x^2 + 5^2 &= 13^2 \\ x^2 + 25 &= 169 \\ x^2 &= 144 \\ x &= 12 \end{aligned}$$

$$\tan A = \frac{5}{12}$$

16. Find θ , $0^\circ < \theta < 90^\circ$, in degrees, for each equation. Do not use a calculator.

a. $\sin \theta = \frac{1}{2}$

$$\theta = 30^\circ$$

b. $\sec \theta = \sqrt{2}$

$$\cos \theta = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\theta = 45^\circ$$

17. Find θ , $0 < \theta < \frac{\pi}{2}$, in radians, for each equation. Do not use a calculator.

a. $\tan \theta = 1$

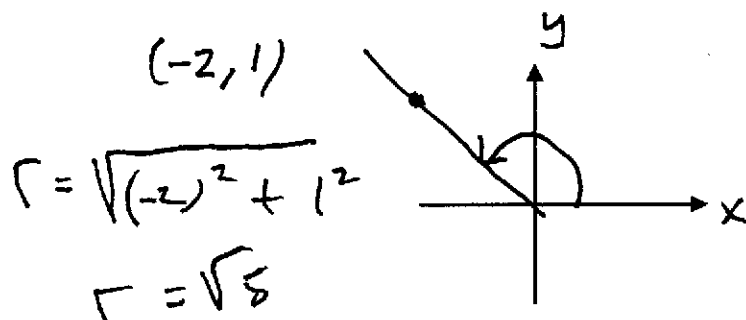
$$\theta = \pi/4$$

b. $\sec \theta = 2$

$$\cos \theta = 1/2$$

$$\theta = \pi/3$$

18. Find $\sin \theta$ of the angle θ whose terminal side passes through the point $(-2, 1)$.



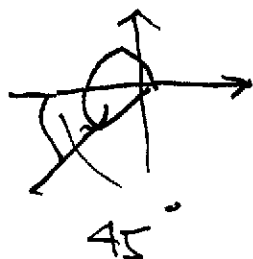
$$\sin \theta = y/r$$

$$\sin \theta = \frac{1}{\sqrt{5}} = \frac{\sqrt{5}}{5}$$

19. Evaluate the trigonometric function without using a calculator:

a. $\sin 225^\circ = -\sin 45^\circ$

$$= -\frac{\sqrt{2}}{2}$$



b. $\cos 330^\circ = +\cos 30^\circ$

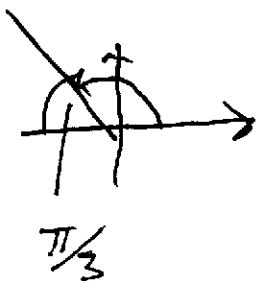
$$= \frac{\sqrt{3}}{2}$$



20. Evaluate the trigonometric function without using a calculator:

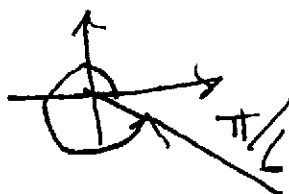
a. $\tan \frac{2\pi}{3} = -\tan \frac{\pi}{3}$

$= -\sqrt{3}$



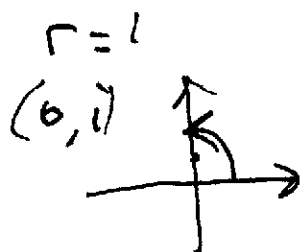
b. $\sin \frac{11\pi}{6} = -\sin \frac{\pi}{6}$

$= -\frac{1}{2}$

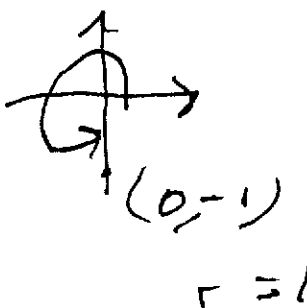


21. If possible, evaluate the trigonometric function of the quadrant angle.

a. $\tan \frac{\pi}{2} = \text{undefined}$



b. $\sin 270^\circ = -1$



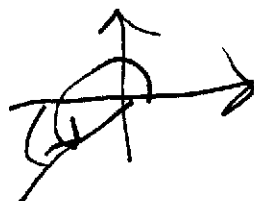
22. Find two solutions in degrees ($0^\circ \leq x < 360^\circ$) of the equation: $\tan x = 1$

$$\tan \theta > 0 \quad \text{Q I, III}$$

reference angle: 45°



$$x = 45^\circ$$

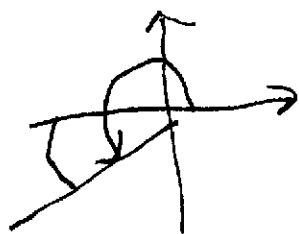


$$x = 225^\circ$$

23. Find two solutions in radians ($0 \leq x < 2\pi$) of the equation: $\sin x = -\frac{1}{2}$

$$\sin \theta < 0 \quad \text{Q III, IV}$$

reference angle: $\frac{\pi}{6}$



$$x = \frac{7\pi}{6}$$



$$x = \frac{11\pi}{6}$$