**Math 41**
**Exam 3 A**
**March 23, 2016**
**#1-8: 12 points each, #9: 4 points**

**name** Darryl Rhea A

**No calculators**

1. Find all intercept(s). Write the quadratic function in standard form. Identify the vertex

$f(x) = 4x^2 + 8x + 3$

*y*-intercept: $f(0) = 3$ or $(0, 3)$

---

*x*-intercept(s), *if any*:

$f(x) = 0$

$4x^2 + 8x + 3 = 0$

$(2x + 1)(2x + 3) = 0$

$2x + 1 = 0 \quad 2x + 3 = 0$

$x = -1/2 \quad x = -3/2$

$(-1/2, 0) \quad (-3/2, 0)$

---

standard equation and vertex:

$f(x) = 4x^2 + 8x + 3$

$f(x) = 4(x^2 + 2x) + 3$

$f(x) = 4(x^2 + 2x + 1 - 1) + 3$

$f(x) = 4(x^2 + 2x + 1) - 4 + 3$

$f(x) = 4(x + 1)^2 - 1$

vertex $(-1, -1)$

A

2. Use the function $f(x) = 2x^3 - 6x^2 + 4x - 5$ to determine the following:

a) the left and right hand behavior of the function

odd degree leading coefficient $< 0$
falls left, rises right

---

b) the number of real zeros

at most 3 real zeros

---

c) the number of extrema:

at most 2 extrema

---

3. Find all real zeros of the function <u>by factoring</u>: $f(x) = x^4 - 10x^2 + 16$

$f(x) = 0$ $\quad x^4 - 10x^2 + 16 = 0$

$(x^2 - 8)(x^2 - 2) = 0$

$x^2 - 8 = 0 \quad x^2 - 2 = 0$

$x^2 = 8 \quad x^2 = 2$

$x = \pm\sqrt{8} \quad x = \pm\sqrt{2}$

$x = \pm 2\sqrt{2}$

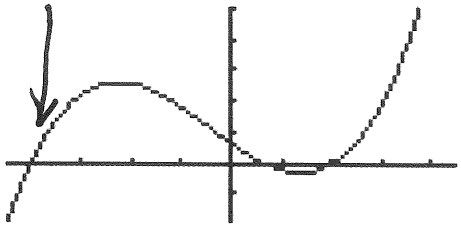
4. Use the remainder theorem to evaluate the function, $f(x) = 3x^5 - 5x^4 - 4x^3 - 12x^2 - 4x + 3$ at $x = -2$.

$$\begin{array}{r|rrrrrr} -2 & 3 & -5 & -4 & -12 & -4 & 3 \\ & & -6 & 22 & -36 & 96 & -184 \\ \hline & 3 & -11 & 18 & -48 & 92 & -181 \end{array}$$

$F(-2) = -181$

$x = -4?$

5. Find all real zeros of the function using the graph, synthetic division, factoring and/or the quadratic formula: $f(x) = 3x^3 + 4x^2 - 28x + 16$



poss rational zeros: $\dfrac{\pm 1 \ \pm 2 \ \pm 4 \ \pm 8 \ \pm 16}{\pm 1 \ \pm 3}$

$= 1, 2, 4, 8, 16, 1/3, 2/3, 4/3, 8/3, 16/3$

$$\begin{array}{r|rrrr} -4 & 3 & 4 & -28 & 16 \\ & & -12 & 32 & -16 \\ \hline & 3 & -8 & 4 & 0 \end{array}$$

$3x^2 - 8x + 4 = 0$

$(3x - 2)(x - 2) = 0$

$3x - 2 = 0 \quad x - 2 = 0$

$x = 2/3 \quad x = 2$

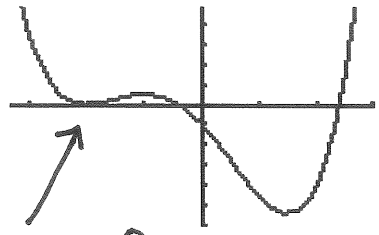
Real zeros:

$x = -4, 2/3, 2$

6. Find all real zeros of the function using the graph, synthetic division, factoring and/or the quadratic formula: $f(x) = x^4 + 2x^3 - 5x^2 - 12x - 4$

A

poss rational zeros: $\frac{\pm 1 \pm 2 \pm 4}{\pm 1}$



$x = -2$? = twice

$$\begin{array}{r|rrrrr} -2 & 1 & 2 & -5 & -12 & -4 \\ & & -2 & 0 & 10 & 4 \\ \hline & 1 & 0 & -5 & -2 & 0 \end{array}$$

$$\begin{array}{r|rrrr} -2 & 1 & 0 & -5 & -2 \\ & & -2 & 4 & 2 \\ \hline & 1 & -2 & -1 & 0 \end{array}$$

$x^2 - 2x - 1 = 0$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-1)}}{2(1)}$$

$$x = \frac{2 \pm \sqrt{8}}{2} = \frac{2 \pm 2\sqrt{2}}{2} = 1 \pm \sqrt{2}$$

Real zeros:

$x = -2, -2, 1 \pm \sqrt{2}$

7. Find the equations of all vertical and horizontal asymptote(s), if any, of the graph of the function:
*there are no common factors*

A

$$f(x) = \frac{x^2 - 4x + 4}{x^2 - 16}$$

Vertical Asymptote(s):

$x^2 - 16 = 0$

$x^2 = 16$

$x = \pm 4$

Horizontal Asymptote(s):

$y = \frac{1}{1}$

$y = 1$

equal powers

A

**8**. Find the slant asymptote(s) of the graph of the function:

$$f(x) = \frac{2x^3 - x + 5}{x^2 + 3}$$

Slant Asymptote(s):

slant

$$f(x) = 2x$$

$$x^2+3 \overline{\big) 2x^3 - x + 5}$$

$$-(2x^3 + 6x)$$

$$-7x + 5$$

$$f(x) = 2x + \frac{-7x+5}{x^2+3}$$

so $y = 2x$

9. Write an equation of a polynomial function, name it $f(x)$ with the following characteristics: $f(x)$ has a real zero at $x = 1$ of multiplicity of two and a real zero at $x = -2$ of multiplicity of one. *YOU DO NOT NEED TO EXPAND YOUR FUNCTION'S EQUATION.*

$$f(x) = (x-1)^2(x-(-2))$$

$$= (x-1)^2(x+2)$$

*Bonus:* Find the ***vertical asymptote(s) and/or hole(s),*** if any, of the graph of the function:

$f(x) = \dfrac{x^2 - 3x - 10}{x^2 - 25}$. ***Briefly*** explain your answer.

$$f(x) = \frac{(x-5)(x+2)}{(x-5)(x+5)}$$

$$= \frac{x+2}{x+5}, \quad x \neq 5$$

Vertical Asymptote(s):

at

$x = -5$

f undefined at $x = -5$

Hole(s):

at

$x = 5$

f undefined at $x = 5$

BUT like factors

**Math 41** name Darryl Rhea B
**Exam 3 B**
**March 23, 2016**
**#1-8: 12 points each, #9: 4 points** **No calculators**

1. Find all intercept(s). Write the quadratic function in standard form. Identify the vertex.

$f(x) = 4x^2 - 8x + 3$

*y*-intercept: $f(0) = 3$ or $(0, 3)$

---

*x*-intercept(s), *if any*: $f(x) = 0$

$4x^2 - 8x + 3 = 0$

$(2x-1)(2x-3) = 0$

$2x - 1 = 0 \quad 2x - 3 = 0$

$x = 1/2 \quad x = 3/2$ or $(1/2, 0)$ $(3/2, 0)$

---

standard equation and vertex:

$f(x) = 4x^2 - 8x + 3$

$= 4(x^2 - 2x) + 3$

$= 4(x^2 - 2x + 1 - 1) + 3$

$= 4(x^2 - 2x + 1) - 4 + 3$

$= 4(x-1)^2 - 1$

vertex $(1, -1)$

B

2. Use the function $f(x) = 2x^3 - 6x^2 + 4x - 5$ to determine the following:

a) the left and right hand behavior of the function

odd degree leading coefficient < 0
falls left, rises right

---

b) the number of real zeros

at most 3 real zeros

---

c) the number of extrema:

at most 2 extrema

---

3. Find all real zeros of the function <u>by factoring</u>: $f(x) = x^4 - 11x^2 + 24$

$f(x) = 0$

$x^4 - 11x^2 + 24 = 0$

$(x^2 - 8)(x^2 - 3) = 0$

$x^2 - 8 = 0 \quad x^2 - 3 = 0$

$x^2 = 8 \quad x^2 = 3$

$x = \pm\sqrt{8} \quad x = \pm\sqrt{3}$

$x = \pm 2\sqrt{2}$

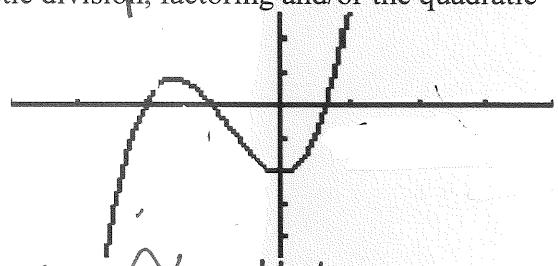
B

4. Use the remainder theorem to evaluate the function, $f(x) = 3x^5 - 5x^4 - 4x^3 - 12x^2 - 4x + 3$ at $x = -2$.

$$\begin{array}{r|rrrrrr} -2 & 3 & -5 & -4 & -12 & -4 & 3 \\ & & -6 & 22 & -36 & 96 & -184 \\ \hline & 3 & -11 & 18 & -48 & 92 & -181 \end{array}$$

$f(-2) = -181$

5. Find all real zeros of the function using the graph, synthetic division, factoring and/or the quadratic formula: $f(x) = 3x^3 + 16x^2 + 12x - 16$



Poss rationals: $\dfrac{\pm 1 \pm 2 \pm 4 \pm 8 \pm 16}{\pm 1 \pm 3}$

$= \pm 1, 2, 4, 8, 16, 1/3, 2/3, 4/3, 8/3, 16/3$

$$\begin{array}{r|rrrr} -2 & 3 & 16 & 12 & -16 \\ & & -6 & -20 & 16 \\ \hline & 3 & 10 & -8 & 0 \end{array}$$

$3x^2 + 10x - 8 = 0$

$(3x - 2)(x + 4) = 0$

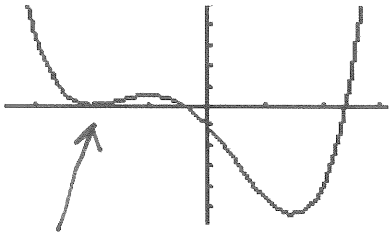
$3x - 2 = 0 \qquad x + 4 = 0$

$x = 2/3 \qquad x = -4$

Real zeros:

$x = -2, 2/3, -4$

6. Find all real zeros of the function using the graph, synthetic division, factoring and/or the quadratic formula: $f(x) = x^4 + 2x^3 - 5x^2 - 12x - 4$



$x = -2$ twice?

poss rationals: $\frac{\pm 1, \pm 2, \pm 4}{\pm 1}$

$$\begin{array}{r|rrrrr} -2 & 1 & 2 & -5 & -12 & -4 \\ & & -2 & 0 & 10 & 4 \\ \hline & 1 & 0 & -5 & -2 & 0 \end{array}$$

$$\begin{array}{r|rrrr} -2 & 1 & 0 & -5 & -2 \\ & & -2 & 4 & 2 \\ \hline & 1 & -2 & -1 & 0 \end{array}$$

$x^2 - 2x - 1 = 0$

$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-1)}}{2(1)}$

$x = \frac{2 \pm \sqrt{8}}{2}$

$x = \frac{2 \pm 2\sqrt{2}}{2}$

$x = 1 \pm \sqrt{2}$

Real zeros:

$x = -2, -2, 1 \pm \sqrt{2}$

7. Find the equations of all vertical and horizontal asymptote(s), if any, of the graph of the function:
*there are no common factors*

$$f(x) = \frac{x^2 - 8x + 16}{x^2 - 4}$$

Vertical Asymptote(s):

$x^2 = 4$

$x = \pm 2$

Horizontal Asymptote(s):

$y = \frac{1}{1}$

$y = 1$

equal powers

**8**. Find the slant asymptote(s) of the graph of the function:

$$f(x) = \frac{2x^3 - x + 5}{x^2 + 3}$$

Slant Asymptote(s):

$$\begin{array}{r} 2x \\ x^2+3 \overline{\big) 2x^3 \quad - x + 5} \\ -(2x^3 \quad + 6x) \\ \hline -7x + 5 \end{array}$$

$y = 2x$ $\qquad f(x) = 2x + \dfrac{-7x+5}{x^2+3}$

9. Write an equation of a polynomial function, name it $f(x)$ with the following characteristics: $f(x)$ has a real zero at $x = 1$ of multiplicity of two and a real zero at $x = -2$ of multiplicity of one. *YOU DO NOT NEED TO EXPAND YOUR FUNCTION'S EQUATION.*

$$f(x) = (x-1)^2(x+2)$$

$(x-(-2))$

*Bonus:* Find the ***vertical asymptote(s) and/or hole(s),*** if any, of the graph of the function:

$f(x) = \dfrac{x^2 - 3x - 10}{x^2 - 25}$. **Briefly** explain your answer.

$$f(x) = \frac{(x-5)(x+2)}{(x-5)(x+5)}$$

$$= \frac{x+2}{x+5}, \quad x \neq 5$$

Vertical Asymptote(s):

$x = -5$

division by zero after simplify.

Hole(s):

$x = 5$

like factor cancelled