Math 22
Exam 2 -A
October 20, 2010

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1. Identify the vertex; find all x- and y-intercepts, if any.

$$f(x) = -(x - \tfrac{1}{2})^2 + \tfrac{9}{4}$$

vertex: $\left(\frac{1}{2}, \frac{9}{4}\right)$

x-intercept(s): $f(x) = 0$

note: could expand then solve.

$-(x - \frac{1}{2})^2 + \frac{9}{4} = 0$

$-(x - 1/2)^2 = -9/4$

$(x - 1/2)^2 = 9/4$

$x - 1/2 = \pm 3/2 \rightarrow x = 1/2 \pm 3/2$

$x = -2, 1 \rightarrow (2,0), (-1,0)$

y-intercept:

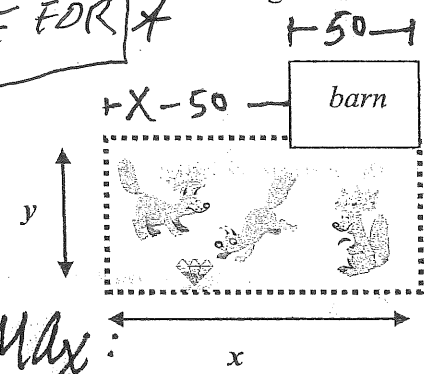
$f(0) = -(0 - \frac{1}{2})^2 + 9/4$

$= -(-1/2)^2 + 9/4$

$= -\frac{1}{4} + 9/4 = 2$

$(0,2)$

☆ 2. A breeder of prize pedigree show-squirrels wants to enclose a rectangular corral next to a 50 foot barn/suite, using the entire barn/suite as one side of the corral. The squirrel breeder has 250 feet of fencing available and will use of all it. Find the dimensions that enclose a maximum area, if no fencing is needed along the barn. *(not drawn to scale)* * NOT RESPONSABLE FOR TOPIC *



Constraint: $x + x - 50 + 2y = 250$

Primary: $A = xy$

Solution:

Constraint: $2x + 2y = 300$

$2y = 300 - 2x$

$y = 150 - x$

$A = x(150 - x)$

$A = 150x - x^2$

parabola opens down

find max:

find vertex:

$x = -b/2a$

$x = -150/2(-1) = 75$ ft

$y = 150 - 75 = 75$ ft

3. Use the Leading Coefficient Test to determine the left and right hand end behavior of the graphs of polynomial functions. Also, determine the ***maximum*** number of zeros or roots, and ***maximum*** number of extrema for each the polynomial functions.

$f(x) = -4x^5 - 2x^3 + 4x^2 - 5$

odd degree, leading coefficient <0
5th degree, $-4 < 0$

Left and right hand end behavior:

rises left, falls right

maximum number of zeros:

at most 5 real zeros

maximum number of extrema:

at most $5-1 = 4$ extrema

4. Find all ***real*** zeros of the function ***by factoring***: $f(x) = x^4 - 7x^2 - 8$

$f(x) = 0$

$x^4 - 7x^2 - 8 = 0$

$(x^2 - 8)(x^2 + 1) = 0$

$x^2 - 8 = 0$ $\quad$ $x^2 + 1 = 0$

no real zeros

$x^2 = 8$

$x = \pm\sqrt{8}$

$x = \pm 2\sqrt{2}$

5. Use ***the remainder theorem*** to evaluate the function, $f(x) = x^4 + 4x^3 - 4x + 1$ at $x = -4$.

remainder theorem: $f(k) = r$

$$\begin{array}{r|rrrrr} -4 & 1 & 4 & 0 & -4 & 1 \\ & & -4 & 0 & 0 & 16 \\ \hline & 1 & 0 & 0 & -4 & 17 \end{array}$$

so $f(-4) = 17$

6. Use the rational zero test to create a list of all the possible rational zeros of the function,

$f(x) = 2x^3 - 4x^2 - 5x + 12$. ***Do not attempt to find the real zeros.***

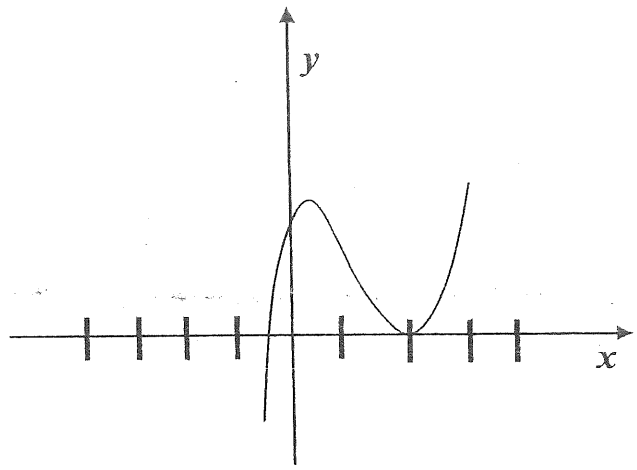
possible rational zeros:

$$\frac{\text{factors of constant}}{\text{factors of leading coefficient}}$$

: $\dfrac{\pm 1 \pm 2 \pm 3 \pm 4 \pm 6 \pm 12}{\pm 1 \pm 2}$

: $\pm \frac{1}{2}, \pm 1, \pm \frac{3}{2}, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12$

7. Find all **real** zeros of the function using the graph, synthetic division, factoring and/or the quadratic formula. $f(x) = 2x^3 - 7x^2 + 4x + 4$.



possible rational zeros:

$\frac{\pm 1 \pm 2 \pm 4}{\pm 1 \pm 2}$

: $\pm \frac{1}{2}, \pm 1, \pm 2, \pm 4$

2	2	-7	4	4
		4	-6	-4
	2	-3	-2	0

$x = 2$ is a zero

$2x^2 - 3x - 2 = 0$

$(2x+1)(x-2) = 0$

$2x+1 = 0 \quad x-2 = 0$

$x = -\frac{1}{2} \quad x = 2$

Real Zeros:

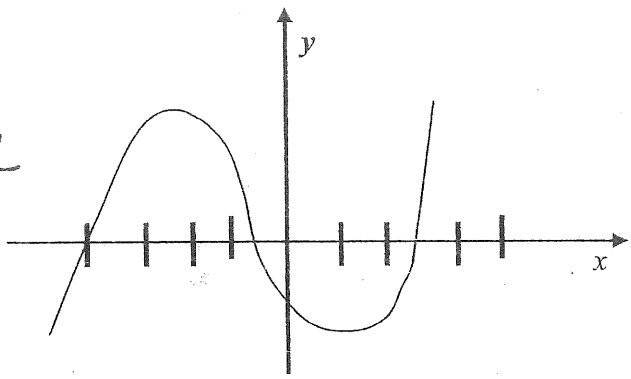
$x = -\frac{1}{2}, 2$ *

$x = 2$ even multiplicity

8. Find all **real** zeros of the function using the graph, synthetic division, factoring and/or the quadratic formula. $f(x) = x^3 + 2x^2 - 9x - 4$.

Possible rational zeros:

$$\frac{\pm 1 \quad \pm 2 \quad \pm 4}{\pm 1} = \pm 1, \pm 2, \pm 4$$



$$\begin{array}{r|rrrr} -4 & 1 & 2 & -9 & -4 \\ & & -4 & 8 & 4 \\ \hline & 1 & -2 & -1 & 0 \end{array}$$

$x = -4$ is a zero

$$x^2 - 2x - 1 = 0$$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-1)}}{2(1)}$$

$$x = \frac{2 \pm \sqrt{8}}{2} = \frac{2 \pm 2\sqrt{2}}{2} = \frac{\cancel{2}(1 \pm \sqrt{2})}{\cancel{2}} = 1 \pm \sqrt{2}$$

Real zeros:

$x = -4, 1 \pm \sqrt{2}$

9. Find the *equations* of the vertical asymptote(s), if any, and the equations horizontal asymptote(s), if any, of the graph of the function, $f(x)=\dfrac{2x^2-x-3}{x^2-4}$ *(no common factors)*

Equations of the
Vertical asymptote(s), if any.

$x^2-4=0$

$x^2=4$

$x=\pm 2$

Equations of the
Horizontal asymptote(s), if any.

$\dfrac{2^{nd}\text{ degree}}{2^{nd}\text{ degree}}$

$y=\dfrac{2}{1}$

$y=2$

10. Use long division to find the *equation* of the slant asymptote of the graph of the function,

$f(x)=\dfrac{2x^3-4x^2+3x-5}{x^2+1}$.

$$
\begin{array}{r}
2x-4 \\
x^2+1\,\overline{)\,2x^3-4x^2+3x-5} \\
-(2x^3 \qquad +2x) \\
\hline
-4x^2+x-5 \\
-(-4x^2 \qquad +4) \\
\hline
x-9
\end{array}
$$

Equation of the
Slant asymptote:

$y=2x-4$

$f(x)=2x-4+\dfrac{x-9}{x^2+1}$