

PINK

name

Denise R. Shaking

*no calculators

*show all relevant work to receive full credit

*any evidence of academic dishonesty = 0 grade

*only pencils, pens = - 5 from grade

*late for exam = - 5 from grade

1. Find the slope intercept equation of the line through the point $(-2, 5)$ and(a) parallel to the line $2x - y + 4 = 0$

$$y = 2x + 4 \quad m = 2$$

use same slope

$$y - 5 = 2(x - (-2))$$

$$y - 5 = 2(x + 2)$$

$$y - 5 = 2x + 4$$

$$y = 2x + 9$$

(b) perpendicular to the line $x = 3$.

$$x = 3 \text{ vertical}$$

so our line horizontal

$$y = 5$$

2. The population of a small town in central Pennsylvania was 8000 in the year 2010, and had decreased to 7500 by the year 2015. Find a linear model giving the population P in terms of the year t , where $t = 0$ corresponds to the year 2010.

points (t, P) $(0, 8000)$
 $(5, 7500)$

$$m = \frac{\Delta P}{\Delta t}$$

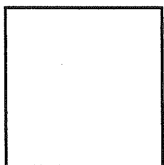
$$= \frac{8000 - 7500}{0 - 5}$$

$$= \frac{500}{-5} = -100$$

$$P - P_1 = m(t - t_1)$$

$$P - 8000 = -100(t - 0)$$

$$P = -100t + 8000$$



3. Determine if the equation represents a function of y in terms of x . If it is not a function, explain your reasoning.

a. $y^2 - x^2 = 16$

yes or no, if no, explain.

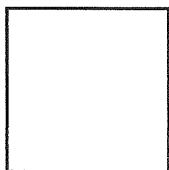
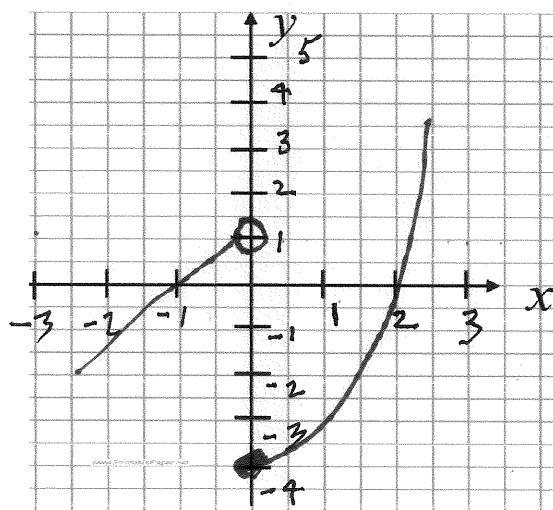
no, y^2 term

b. $y = 4x^2 - 1$

yes or no, if no, explain

yes,

4. Sketch a graph of the function. $f(x) = \begin{cases} x+1, & x < 0 \\ x^2 - 4, & x \geq 0 \end{cases}$



5. Determine if the statement is true **T** or false **F**. If false, explain why it is false or provide a counter-example.

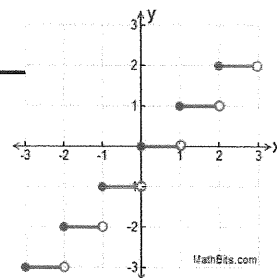
a. The graph of an odd function is symmetrical to the x-axis. F

If false, explain.

odd function symmetrical to the origin
no functions are symmetrical to x-axis

b. The graph shown is that of the greatest integer function, $f(x) = \llbracket x \rrbracket$ T

If false, explain.



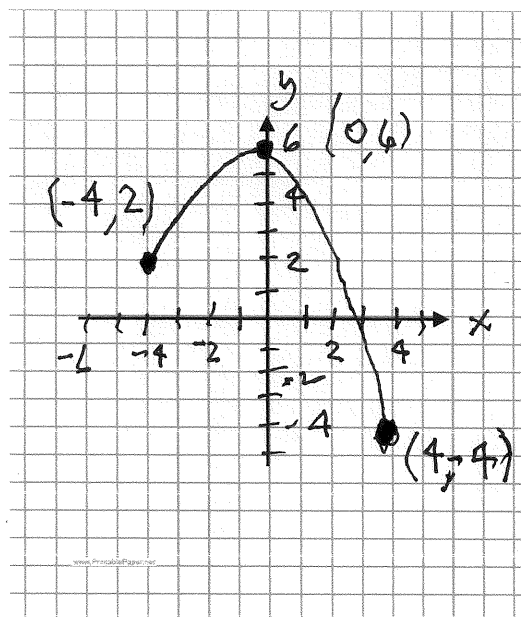
6. Use the graph of $y = f(x)$ to find the following:

(a) domain of f

$[-4, 4]$

(b) range of f

$[-4, 6]$



7. For the function $f(x) = x^2 - 3x + 5$, find and simplify the difference quotient.

$$\frac{f(x+h) - f(x)}{h} = \frac{(x+h)^2 - 3(x+h) + 5 - (x^2 - 3x + 5)}{h}$$

$$= \frac{\cancel{x^2} + 2\cancel{x}h + \cancel{h^2} - 3\cancel{x} - 3h + 5 - \cancel{x^2} + 3\cancel{x} - 5}{h}$$

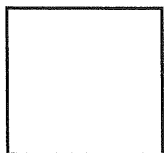
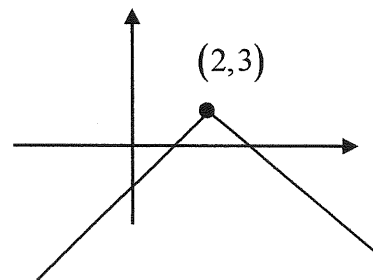
$$= \frac{h(2x + h - 3)}{h}$$

$$= 2x + h - 3$$

8. Use the graph of $f(x) = |x|$ to write the equation for the function g whose graph is shown.

Assume no vertical stretch or shrink.

$$g(x) = -|x - 2| + 3$$



9. Describe the shape and the rigid transformations that gives the graph of

$g(x) = -\frac{1}{4}(x-3)^2 - 2$ using the graph of $f(x) = x^2$. Not looking for a graph, but a written description.

A parabola shifted 3 units right,
two units down, reflected about x-axis
and a vertical shrink.

10. Find each of the following for the given functions, $f(x) = x^2 + 1$
 $g(x) = 2x - 3$

a. $(f \cdot g)(x) = f(x)g(x)$

$$= (x^2 + 1)(2x - 3)$$
$$= 2x^3 - 3x^2 + 2x - 3$$

b. $(f - g)(x) = f(x) - g(x)$

$$= (x^2 + 1) - (2x - 3)$$
$$= x^2 - 2x + 4$$



11. For the composite function, $f(g(x))$ find two functions f and g , if $f(g(x)) = \sqrt{4x-5}$

$$f(x) = \sqrt{x}$$

$$g(x) = 4x-5$$

12. Find $f^{-1}(x)$, if it exists, for $f(x) = 27x^3 - 1$. If it does not exist, explain the reason(s).

$$y = 27x^3 - 1$$

$$x = \sqrt[3]{\frac{y+1}{27}}$$

$$x+1 = 27y^3$$

$$\frac{x+1}{27} = y^3$$

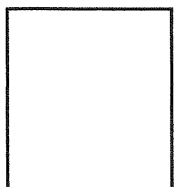
$$y = \sqrt[3]{\frac{x+1}{27}}$$

or

$$y = \frac{1}{3} \sqrt[3]{x+1}$$

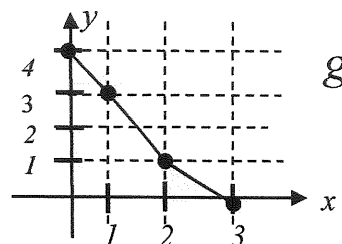
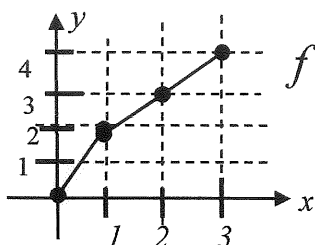
$$f^{-1}(x) = \frac{1}{3} \sqrt[3]{x+1}$$

13. (4 points): 4 points: Any plans for Valentine's Day?



Answer varies.

Extra: Use the graph of f and g to find each of the following:



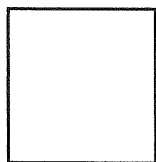
a. $g(f(1)) =$

b. $(f + g^{-1})(3) =$

c. $(g \circ f^{-1})(3) =$

d. $g^{-1}(g(1)) =$

Happy Valentine's Day



1. Find the slope intercept equation of the line through the point $(-2, 5)$ and

(a) parallel to the line $2x + y - 4 = 0$

$$y = -2x + 4 \quad m = -2$$

use same slope

$$y - 5 = -2(x - (-2))$$

$$y - 5 = -2(x + 2)$$

$$y - 5 = -2x - 4$$

$$y = -2x + 1$$

(b) perpendicular to the line $x = 4$.

given line vertical
so line horizontal $m = 0$

$$y = 5$$

2. The population of a small town in central Pennsylvania was 800 in the year 2010, and had decreased to 750 by the year 2015. Find a linear model giving the population P in terms of the year t , where $t = 0$ corresponds to the year 2010.

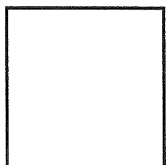
points (t, P) $(0, 800)$ $(5, 750)$

$$m = \frac{\Delta P}{\Delta t} = \frac{800 - 750}{0 - 5} = \frac{50}{-5} = -10$$

$$P - P_1 = m(t - t_1)$$

$$P - 800 = -10(t - 0)$$

$$P = -10t + 800$$



3. Determine if the equation represents a function of y in terms of x . If it is not a function, explain your reasoning.

a. $y - x^2 = 16$

yes or no, if no, explain.

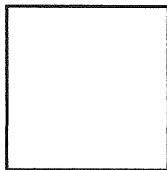
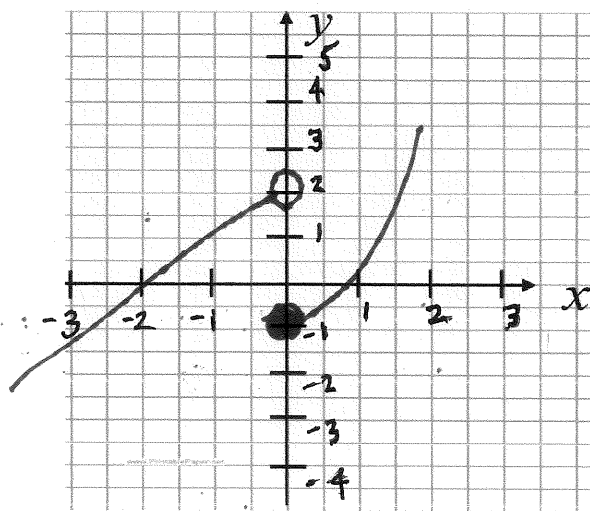
yes

b. $y^2 + 4x = 12$

yes or no, if no, explain

no, y^2 term

4. Sketch a graph of the function. $f(x) = \begin{cases} x+2, & x < 0 \\ x^2-1, & x \geq 0 \end{cases}$



5. Determine if the statement is true **T** or false **F**. If false, explain why it is false or provide a counter-example.

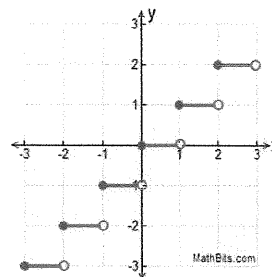
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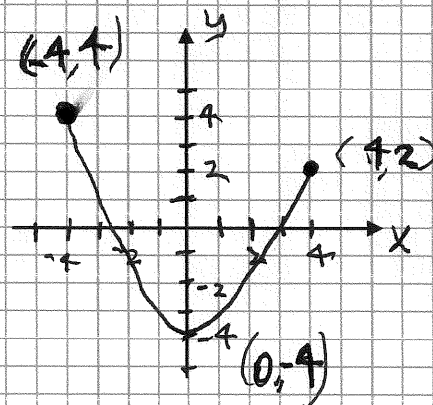
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$$\frac{f(x+h) - f(x)}{h} = \frac{(x+h)^2 - 3(x+h) + 5 - (x^2 - 3x + 5)}{h}$$

$$= \frac{x^2 + 2xh + h^2 - 3x - 3h + 5 - x^2 + 3x - 5}{h}$$

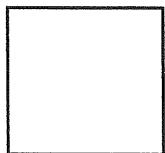
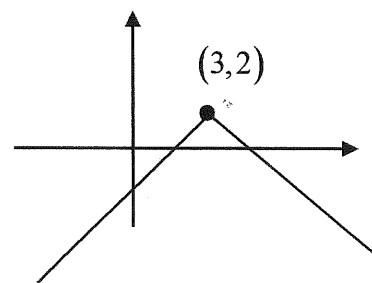
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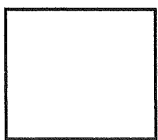
10. Find each of the following for the given functions,
 $f(x) = x^2 + 1$
 $g(x) = 2x - 3$

a. $(f \cdot g)(x) = f(x)g(x)$

$$= (x^2 + 1)(2x - 3)$$
$$= 2x^3 - 3x + 2x - 3$$

b. $(f - g)(x) = f(x) - g(x)$

$$= (x^2 + 1) - (2x - 3)$$
$$= x^2 - 2x + 4$$



11. For the composite function, $f(g(x))$ find two functions f and g , if $f(g(x)) = \sqrt{4x-5}$

$$f(x) = \sqrt{x}$$

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12. Find $f^{-1}(x)$, if it exists, for $f(x) = 27x^3 - 1$. If it does not exist, explain the reason(s).

$$y = 27x^3 - 1$$

$$x = 27y^{-1}$$

$$x+1 = 27y^3$$

$$\frac{x+1}{27} = y^3$$

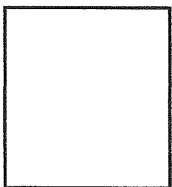
$$y = \sqrt[3]{\frac{x+1}{27}}$$

or

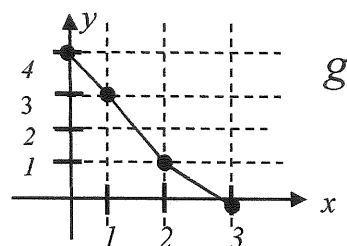
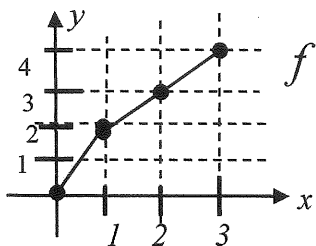
$$y = \frac{1}{3} \sqrt[3]{x+1}$$

$$f^{-1}(x) = \frac{1}{3} \sqrt[3]{x+1}$$

13. (4 points): 4 points: Any plans for Valentine's Day?



Extra: Use the graph of f and g to find each of the following:



a. $g(f(2)) =$

b. $(f^{-1} + g)(2) =$

c. $(g \circ f^{-1})(2) =$

d. $g^{-1}(g(2)) =$

Happy Valentine's Day

