Math 141
Quiz 6

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- Test the series for convergence or divergence.
- Name the test used, and support your conclusion.
- Whenever possible, find the sum, if convergent.

$\sum_{n=1}^{\infty} \frac{n}{n^2+1}$ LCT to $\sum_{1}^{\infty} \frac{1}{n}$ div. p-series $p=1 \leq 1$ Test/name: Support/Reasons: $\lim_{n\to\infty} \left(\frac{\frac{n}{n^2+1}}{\frac{1}{n}} \right)$ $\lim_{n\to\infty} \left(\frac{n^2}{n^2+1} \right) = 1$ finite and pos. both diverge Converge/Diverge: Sum, if possible: Note: Integral Test also possible	$\sum_{n=0}^{\infty} (-1)^n \frac{1}{n+1}$ AST (for convergence) Test/name: Support/Reasons: $a_n = \frac{1}{n+1}$ $a_{n+1} = \frac{1}{n+2}$ 1) $\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{1}{n+1} = 0$ 2) $a_{n+1} \leq a_n$ $\frac{1}{n+2} \leq \frac{1}{n+1}$ series converges Converge/Diverge: Sum, if possible:

$\sum_{n=1}^{\infty}(-1)^{n+1}\frac{2}{5^{n-1}}$

Test/name:

geometric

Support/Reasons:

$\sum_{1}^{\infty}(-1)^n(-1)\frac{2}{5^n 5^{-1}}$

$\sum_{1}^{\infty}(-10)\left(\frac{-1}{5}\right)^n$

$r=-1/5 \qquad 0<\left|\frac{-1}{5}\right|<1$

converges

$S=\frac{2}{1-(-1/5)}$

$=\frac{2}{6/5}=5/3$

Converge/Diverge:

Sum, if possible:

Note: AST also possible although "no sum"

$\sum_{n=1}^{\infty}\frac{2^n}{n!}$ Ratio

Test/name:

Support/Reasons:

$a_n=\frac{2^n}{n!} \quad a_{n+1}=\frac{2^{n+1}}{(n+1)!}$

$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right|$ <1 converges (abs)
>1 or ∞, div.
$=1$ test fails

$\lim_{n\to\infty}\left|\frac{2^{n+1}}{(n+1)!}\cdot\frac{n!}{2^n}\right|$

$\lim_{n\to\infty}\left|\frac{2^{n+1}}{2^n}\cdot\frac{n!}{(n+1)!}\right|$

$\lim_{n\to\infty}\left|\frac{2}{1}\cdot\frac{1}{n+1}\right|=0<1$

Converge/Diverge: Converges (abs.)

Sum, if possible:

1 2 3 4 5

$1+\frac{1}{4\sqrt{2}}+\frac{1}{9\sqrt{3}}+\frac{1}{32}+\frac{1}{25\sqrt{5}}+\ldots$

$$\sum_{n=1}^{\infty}\frac{1}{n^2\sqrt{n}} = \sum_{n=1}^{\infty}\frac{1}{n^{5/2}}$$

Test/name: p-series

Support/Reasons:

$p = 5/2 > 1$

Converges

Converge/Diverge:

Sum, if possible:

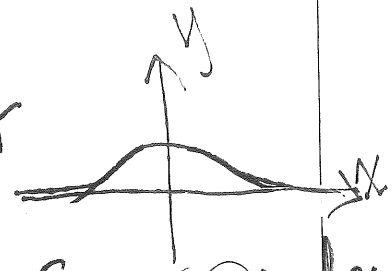
1 2 3 4 5 6

$\frac{1}{2}+\frac{1}{5}+\frac{1}{10}+\frac{1}{17}+\frac{1}{26}+\frac{1}{37}+\ldots$

$$\sum_{n=1}^{\infty}\frac{1}{n^2+1}$$

Test/name:

Support/Reasons: Integral test



$f(x) = \frac{1}{x^2+1}$, f cont., pos., dec. for $x \geq 1$

$\int_1^{\infty}\frac{1}{x^2+1}\,dx$ $u=x$ $a=1$ $du=dx$

$= \left[\arctan x\right]_1^{\infty}$

$= \arctan\infty - \arctan 1$

$= \pi/2 - \pi/4 = \pi/4$

not sum of series

improper int. converge

series converges

Converge/Diverge:

Sum, if possible:

Note: LCT to $\sum_1^{\infty}\frac{1}{n^2}$ conv. p-series $p=2>1$ also possible