Math 141
Quiz 4 A
February 16, 2016

Kent C. Strait

No Calculators (to protect the innocent)

If needed, $\sin 2u = 2\sin u \cos u$.

1. Find the indefinite integral: $\int \frac{1}{\sqrt{x^2-4}}\,dx$

$x = 2\sec\theta$

$dx = 2\sec\theta\tan\theta\,d\theta$

$\sqrt{x^2-4} = 2\tan\theta$

$$\int \frac{2\sec\theta\tan\theta\,d\theta}{2\tan\theta}$$

$$\int \sec\theta\,d\theta$$

$$= \ln|\sec\theta + \tan\theta| + C$$

$$= \ln\left|\frac{x}{2} + \frac{\sqrt{x^2-4}}{2}\right| + C$$

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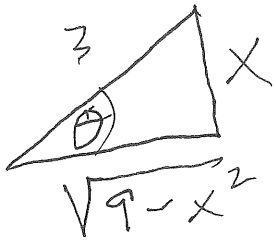
2. Find the indefinite integral: $\int \frac{1}{x\sqrt{9-x^2}}\,dx$

$x = 3\sin\theta$

$dx = 3\cos\theta\,d\theta$

$\sqrt{9-x^2} = 3\cos\theta$



$$\int \frac{3\cos\theta\,d\theta}{(3\sin\theta)(3\cos\theta)}$$

$$\frac{1}{3}\int \frac{1}{\sin\theta}\,d\theta$$

$$\frac{1}{3}\int \csc\theta\,d\theta$$

$$-\frac{1}{3}\ln|\csc\theta + \cot\theta| + C$$

$$= -\frac{1}{3}\ln\left|\frac{3}{x} + \frac{\sqrt{9-x^2}}{x}\right| + C$$

Math 141
Quiz 4 B
February 16, 2016

Kent G. Strait

No Calculators (to protect the innocent)

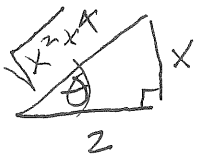
If needed, $\sin 2u = 2\sin u \cos u$.

1. Find the indefinite integral: $\int \frac{1}{\sqrt{x^2+4}}\,dx$

$x = 2\tan\theta$

$dx = 2\sec^2\theta\, d\theta$

$\sqrt{x^2+4} = 2\sec\theta$



$$= \int \frac{2\sec^2\theta\, d\theta}{2\sec\theta}$$

$$= \int \sec\theta\, d\theta$$

$$= \ln|\sec\theta + \tan\theta| + C$$

$$= \ln\left|\frac{\sqrt{x^2+4}}{2} + \frac{x}{2}\right| + C$$

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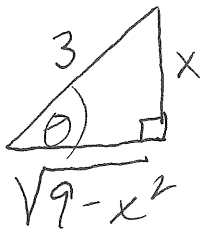
2. Find the indefinite integral: $\int \frac{1}{x^2\sqrt{9-x^2}}\,dx$

$x = 3\sin\theta$

$dx = 3\cos\theta\, d\theta$

$\sqrt{9-x^2} = 3\cos\theta$



$$\int \frac{3\cos\theta\, d\theta}{(3\sin\theta)^2(3\cos\theta)}$$

$$\frac{1}{9}\int \frac{1}{\sin^2\theta}\, d\theta$$

$$\frac{1}{9}\int \csc^2\theta\, d\theta = -\frac{1}{9}\cot\theta + C$$

$$= -\frac{1}{9}\left(\frac{\sqrt{9-x^2}}{x}\right) + C$$

Math 141
Quiz 4 C
February 17, 2016

Kent C. Strait

No Calculators (to protect the innocent)

If needed, $\sin 2u = 2\sin u \cos u$.

1. Find the indefinite integral: $\int \frac{1}{x^2\sqrt{x^2-4}}\,dx$

$x = 2\sec\theta$

$dx = 2\sec\theta\tan\theta\,d\theta$

$\sqrt{x^2-4} = 2\tan\theta$

$$\int \frac{\cancel{2}\sec\theta\cancel{\tan\theta}\,d\theta}{(2\sec\theta)^{\cancel{2}}(\cancel{2}\cancel{\tan\theta})}$$

$$= \frac{1}{4}\int \frac{1}{\sec\theta}\,d\theta$$

$$= \frac{1}{4}\int \cos\theta\,d\theta$$

$$= \frac{1}{4}\sin\theta + C$$

$$= \frac{1}{4}\left(\frac{\sqrt{x^2-4}}{x}\right) + C$$

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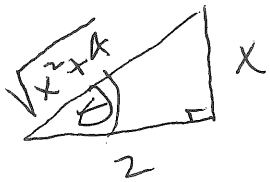
2. Find the indefinite integral: $\int \frac{1}{x\sqrt{x^2+4}}\,dx$

$x = 2\tan\theta$

$dx = 2\sec^2\theta\,d\theta$

$\sqrt{x^2+4} = 2\sec\theta$



$$\int \frac{\cancel{2}\sec^2\theta\,d\theta}{(2\tan\theta)(\cancel{2}\sec\theta)}$$

$$\frac{1}{2}\int \frac{\sec\theta}{\tan\theta}\,d\theta$$

$$\frac{1}{2}\int \frac{1}{\cos\theta}\cdot\frac{\cos\theta}{\sin\theta}\,d\theta$$

$$= \frac{1}{2}\int \frac{1}{\sin\theta}\,d\theta$$

$$= \frac{1}{2}\int \csc\theta\,d\theta$$

$$= -\frac{1}{2}\ln\left|\csc\theta + \cot\theta\right| + C$$

$$= -\frac{1}{2}\ln\left|\frac{\sqrt{x^2+4}}{x} + \frac{2}{x}\right| + C$$

Math 141
Quiz 4 D
February 17, 2016

Kent C. Strait

No Calculators (to protect the innocent)

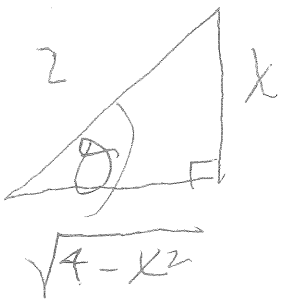
If needed, $\sin 2u = 2\sin u \cos u$.

1. Find the indefinite integral: $\int \frac{1}{x\sqrt{4-x^2}}\,dx$

$x = 2\sin\theta$

$dx = 2\cos\theta\, d\theta$

$\sqrt{4-x^2} = 2\cos\theta$



$$\int \frac{2\cos\theta\, d\theta}{(2\sin\theta)(2\cos\theta)}$$

$$\frac{1}{2}\int \frac{1}{\sin\theta}\, d\theta$$

$$\frac{1}{2}\int \csc\theta\, d\theta = -\frac{1}{2}\ln\left|\csc\theta + \cot\theta\right| + C$$

$$= -\frac{1}{2}\ln\left|\frac{2}{x} + \frac{\sqrt{4-x^2}}{x}\right| + C$$

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2. Find the indefinite integral: $\int \frac{1}{x\sqrt{4+x^2}}\,dx$

See Quiz 4C #2