Math 141
Exam 3 4/20/16
#1-8:12 pts each. #9: 4 pts.

Purple

Tamara Knight

show all relevant work for full credit

No calculators.
No pens: -10 pts.
No questions regarding content: -10 pts.
No late exams accepted. -100 pts.
No smart phones anywhere: -100 pts.

1. Test the series for convergence or divergence. Name the test utilized. Support your conclusion. If convergent, find the sum whenever possible.

a. $\sum_{n=0}^{\infty} \frac{2n^2}{n^2+1}$

Test : n^{th} term test for divergence

Apply test / reasoning :

$\lim_{n\to\infty} \frac{2n^2}{n^2+1} = 2 \neq 0$

series diverges

Converge / diverge :

Sum, if possible :

b. $\sum_{n=0}^{\infty} (-1)^n \frac{4}{3^{n-1}} = \sum_{n=0}^{\infty} 12\left(-\frac{1}{3}\right)^n$

Test : geometric

Apply test / reasoning :

$r = -\frac{1}{3} \quad | \quad 0 < \left|-\frac{1}{3}\right| < 1$

converges

$S = \frac{12}{1-(-\frac{1}{3})} = \frac{12}{4/3}$

$= 9$

Converge / diverge :

Sum, if possible :

P

2. Find the **center**, **radius**, and the (open) **interval of convergence** for the power series. You need ***NOT*** test endpoints.

$$f(x)=\sum_{n=1}^{\infty}\frac{(-1)^n(x-5)^n}{3^n\cdot n}$$

$$u_n=\frac{(x-5)^n}{3^n\cdot n} \qquad u_{n+1}=\frac{(x-5)^{n+1}}{3^{n+1}(n+1)}$$

Center: $c=5$

$$\lim_{n\to\infty}\left|\frac{(x-5)^{n+1}}{3^{n+1}(n+1)}\cdot\frac{3^n\cdot n}{(x-5)^n}\right|$$

$$\lim_{n\to\infty}\left|\frac{(x-5)^{n+1}}{(x-5)^n}\cdot\frac{3^n}{3^{n+1}}\cdot\frac{n}{n+1}\right|$$

$$\left|\frac{x-5}{3}\right|\lim_{n\to\infty}\left|\frac{n}{n+1}\right|=\left|\frac{x-5}{3}\right|(1)$$

for convergence $\left|\frac{x-5}{3}\right|<1$

$$|x-5|<3$$

Radius: $R=3$

(open) Interval of Convergence $(2,8)$

P

3. Find the **center**, **radius** , and the (open) **interval of convergence** for the power series,

$f(x)=\sum_{n=0}^{\infty}\frac{(-1)^{n+1}x^{n+1}}{(n+1)!}$. You need ***NOT*** test endpoints.

Center: $c=0$

$u_n = \frac{x^{n+1}}{(n+1)!}$ $u_{n+1} = \frac{x^{n+2}}{(n+2)!}$

$$\lim_{n\to\infty}\left|\frac{x^{n+2}}{(n+2)!}\cdot\frac{(n+1)!}{x^{n+1}}\right|$$

$$\lim_{n\to\infty}\left|\frac{x^{n+2}}{x^{n+1}}\cdot\frac{(n+1)!}{(n+2)!}\right|$$

$$\lim_{n\to\infty}\left|\frac{x}{1}\cdot\frac{1}{n+2}\right| = |x|\lim_{n\to\infty}\left|\frac{1}{n+2}\right|$$

$$= |x|(0)$$

for convergence

$|x|(0)<1$ true for

all x $R=\infty$

Radius: $R=\infty$

(open) Interval of Convergence $(-\infty,\infty)$

P

4. Given the power series, $f(x)=\sum_{n=1}^{\infty}\frac{(-1)^n(x-2)^n}{3^n\cdot n}$, and the fact that the radius of convergence is $R=3$ to find the interval of convergence, ***including*** testing endpoints.

Open interval of convergence prior to testing endpoints: $\boxed{(-1,5)}$

TEST LEFT ENDPOINT:

$x=-1$

$$f(-1)=\sum_{n=1}^{\infty}\frac{(-1)^n(-1-2)^n}{3^n\cdot n}$$

$$=\sum_{n=1}^{\infty}\frac{(-1)^n(-3)^n}{3^n\cdot n}$$

$(-1)^n 3^n$

$$=\sum_{n=1}^{\infty}\frac{(-1)^{2n}}{n}$$

$$=\sum_{n=1}^{\infty}\frac{1}{n}$$

div. p-series $p=1\le 1$

TEST RIGHT ENDPOINT:

$x=5$

$$f(5)=\sum_{n=1}^{\infty}\frac{(-1)^n(5-2)^n}{3^n\cdot n}$$

$$=\sum_{n=1}^{\infty}\frac{(-1)^n 3^n}{3^n\cdot n}$$

$$=\sum_{n=1}^{\infty}\frac{(-1)^n}{n}$$

$a_n=\frac{1}{n}$ $a_{n+1}=\frac{1}{n+1}$

1) $\lim_{n\to\infty}\frac{1}{n}=0$

2) $\frac{1}{n+1}\le\frac{1}{n}$

converges AST

FINAL interval of Convergence Radius: (after testing endpoints) $\boxed{(-1,5]}$

P

5. For the power series, $f(x)=\sum_{n=0}^{\infty}\frac{(-1)^n(x-1)^{n+1}}{2^n\cdot(n+1)!}$ find ***AND*** simplify or rewrite n^{th} term for each of the following. You need ***NOT*** find interval of convergence, nor test endpoints.

$$f(x)=\sum_{n=0}^{\infty}\frac{(-1)^n(x-1)^{n+1}}{2^n\cdot(n+1)!}=(x-1)-\frac{1}{2\cdot 2!}(x-1)^2+\frac{1}{2^2\cdot 3!}(x-1)^3-\frac{1}{2^3\cdot 4!}(x-1)^4+\frac{1}{2^4\cdot 5!}(x-1)^5-\ldots$$

a. $f'(x)=\sum_{n=}^{\infty}\frac{(-1)^n(n+1)(x-1)^n}{2^n(n+1)!}$

$$=\sum_{n=0}^{\infty}\frac{(-1)^n(x-1)^n}{2^n\cdot n!}$$

$$f(x)=\sum_{n=0}^{\infty}\frac{(-1)^n(x-1)^{n+1}}{2^n\cdot(n+1)!}=(x-1)-\frac{1}{2\cdot 2!}(x-1)^2+\frac{1}{2^2\cdot 3!}(x-1)^3-\frac{1}{2^3\cdot 4!}(x-1)^4+\frac{1}{2^4\cdot 5!}(x-1)^5-\ldots$$

b. $\int f(x)dx=C+\sum_{n=}^{\infty}\frac{(-1)^n(x-1)^{n+2}}{2^n(n+2)(n+1)!}$

$$=C+\sum_{n=0}^{\infty}\frac{(-1)^n(x-1)^{n+2}}{2^n\cdot(n+2)!}$$

P

6. Use the definition of the Taylor Polynomial and Taylor Series to find the 5th Taylor Polynomial for $f(x) = \sin 2x$ centered at $c = \pi$. Then use your polynomial to write the Taylor Series for $f(x) = \sin 2x$ centered at $c = \pi$.

$$P_5(x) = a_0 + a_1(x-\pi) + a_2(x-\pi)^2 + a_3(x-\pi)^3 + a_4(x-\pi)^4 + a_5(x-\pi)^5$$

$n=0$	$f(x) = \sin 2x$	$f(\pi) = 0$	$a_0 = 0/0! = 0$
$n=1$	$f'(x) = 2\cos 2x$	$f'(\pi) = 2$	$a_1 = 2/1! = 2$
$n=2$	$f''(x) = -4\sin 2x$	$f''(\pi) = 0$	$a_2 = 0/2! = 0$
$n=3$	$f'''(x) = -8\cos 2x$	$f'''(\pi) = -8$	$a_3 = -8/3! = -4/3$
$n=4$	$f^{(4)}(x) = 16\sin 2x$	$f^{(4)}(\pi) = 0$	$a_4 = 0/4! = 0$
$n=5$	$f^{(5)}(x) = 32\cos 2x$	$f^{(5)}(\pi) = 32$	$a_5 = 32/5! = 4/15$

5th Taylor polynomial: $P_5(x) = \frac{2}{1!}(x-\pi) - \frac{8}{3!}(x-\pi)^3 + \frac{32}{5!}(x-\pi)^5$

Taylor Series: $P(x) = \sum_{n=0}^{\infty} \frac{(-1)^n 2^{2n+1}(x-\pi)^{2n+1}}{(2n+1)!}$

or

$$= \sum_{n=1}^{\infty} (-1)^{n+1} \frac{2^{2n-1}(x-\pi)^{2n-1}}{(2n-1)!}$$

P

7. Use the Maclaurin Series for $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$ to find **AND** simplify the series for

a. $f(x) = e^{-2x^4}$

$$e^{-2x^4} = \sum_{n=0}^{\infty} \frac{(-2x^4)^n}{n!}$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n 2^n x^{4n}}{n!}$$

b. $g(x) = x^3 e^x$

$$x^3 e^x = x^3 \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

$$= \sum_{n=0}^{\infty} \frac{x^n x^3}{n!}$$

$$= \sum_{n=0}^{\infty} \frac{x^{n+3}}{n!}$$

8. Suppose the interval of convergence for a particular power series $f(x)$ is $(-2,1]$. Answer the following, **TRUE OR FALSE**.

a. It is possible that the interval of convergence of $f'(x)$ is $(-2,1)$. True.

b. It is possible that the interval of convergence of $f'(x)$ is $[-2,1]$. False.

c. It is possible that the interval of convergence of $\int f(x)\,dx$ is $(-2,1)$. False.

d. It is possible that the interval of convergence of $\int f(x)\,dx$ is $[-2,1]$. True.

9. **(4 points):** In your opinion the greatest song (or recording) of all time (in any musical genre).

Answers vary.

There is no wrong answer.

Math 141
Exam 3 4/20/16
#1-8:12 pts each. #9: 4 pts.

GREEN

Tamara Knight

show all relevant work for full credit

No calculators.
No pens: -10 pts.
No questions regarding content: -10 pts.
No late exams accepted. -100 pts.
No smart phones anywhere: -100 pts.

1. Test the series for convergence or divergence. Name the test utilized. Support your conclusion. If convergent, find the sum whenever possible.

a. $\sum_{n=0}^{\infty} \frac{2n^2}{n^2+1}$

Test : n^{th} term test for divergence

Apply test / reasoning :

$\lim_{n \to \infty} \frac{2n^2}{n^2+1} = 2 \neq 0$

series diverges

Converge / diverge :

Sum, if possible :

b. $\sum_{n=0}^{\infty} (-1)^n \frac{4}{2^{n-1}} = \sum_{n=0}^{\infty} 8\left(-\frac{1}{2}\right)^n$

Test : geometric

Apply test / reasoning :

$r = -\frac{1}{2}$

$0 < \left|-\frac{1}{2}\right| < 1$

Converges

$S = \frac{8}{1-(-\frac{1}{2})} = \frac{8}{\frac{3}{2}}$

$= \frac{16}{3}$

Converge / diverge :

Sum, if possible :

G

2. Find the **center**, **radius**, and the (open) **interval of convergence** for the power series. You need ***NOT*** test endpoints.

$$f(x)=\sum_{n=1}^{\infty}\frac{(-1)^n(x-3)^n}{2^n\cdot n}$$

$$u_n=\frac{(x-3)^n}{2^n\cdot n} \qquad u_{n+1}=\frac{(x-3)^{n+1}}{2^{n+1}(n+1)}$$

Center: $c=3$

$$\lim_{n\to\infty}\left|\frac{(x-3)^{n+1}}{2^{n+1}(n+1)}\cdot\frac{2^n\cdot n}{(x-3)^n}\right|$$

$$\lim_{n\to\infty}\left|\frac{(x-3)^{n+1}}{(x-3)^n}\cdot\frac{2^n}{2^{n+1}}\cdot\frac{n}{n+1}\right|$$

$$\left|\frac{x-3}{2}\right|\lim_{n\to\infty}\left|\frac{n}{n+1}\right|=\left|\frac{x-3}{2}\right|(1)=\left|\frac{x-3}{2}\right|$$

for convergence $\left|\frac{x-3}{2}\right|<1$

$$|x-3|<2$$

Radius: $R=2$

(open) Interval of Convergence $(1,5)$

G

3. Find the **center**, **radius** , and the (open) **interval of convergence** for the power series,

$f(x)=\sum_{n=0}^{\infty}\frac{(-1)^{n+1}x^{n+1}}{(n+1)!}$. You need ***NOT*** test endpoints.

Center: $c=0$

$u_n=\frac{x^{n+1}}{(n+1)!}$ $u_{n+1}=\frac{x^{n+2}}{(n+2)!}$

$$\lim_{n\to\infty}\left|\frac{x^{n+2}}{(n+2)!}\cdot\frac{(n+1)!}{x^{n+1}}\right|$$

$$\lim_{n\to\infty}\left|\frac{x^{n+2}}{x^{n+1}}\cdot\frac{(n+1)!}{(n+2)!}\right|$$

$$|x|\lim_{n\to\infty}\left|\frac{1}{n+2}\right|=|x|(0)$$

for convergence $|x|(0)<1$

true for all x

$R=\infty$

Radius: $R=\infty$

(open) Interval of Convergence $(-\infty,\infty)$

G

4. Given the power series, $f(x)=\sum_{n=1}^{\infty}\frac{(-1)^n(x-2)^n}{3^n\cdot n}$, and the fact that the radius of convergence is $R=3$ to find the interval of convergence, ***including*** testing endpoints.

Open interval of convergence, prior to testing endpoints: $\boxed{(-1,5)}$

TEST LEFT ENDPOINT:

$x=-1$

$$f(-1)=\sum_{n=1}^{\infty}\frac{(-1)^n(-1-2)^n}{3^n\cdot n}$$

$$=\sum_{n=1}^{\infty}\frac{(-1)^n(-3)^n}{3^n\cdot n}$$

$$=\sum_{n=1}^{\infty}\frac{(-1)^{2n}}{n}$$

$$=\sum_{n=1}^{\infty}\frac{1}{n}$$

div. p-series $p=1\le 1$

TEST RIGHT ENDPOINT:

$x=5$

$$f(5)=\sum_{n=1}^{\infty}\frac{(-1)^n(5-2)^n}{3^n\cdot n}$$

$$=\sum_{n=1}^{\infty}\frac{(-1)^n 3^n}{3^n\cdot n}$$

$$=\sum_{n=1}^{\infty}\frac{(-1)^n}{n}$$

$a_n=\frac{1}{n}$ $a_{n+1}=\frac{1}{n+1}$

$\lim_{n\to\infty}\frac{1}{n}=0$

$\frac{1}{n+1}\le\frac{1}{n}$ converges AST

FINAL interval of Convergence Radius: (after testing endpoints) $\boxed{(-1,5]}$

G

5. For the power series, $f(x)=\sum_{n=0}^{\infty}\frac{(-1)^n(x-1)^{n+1}}{2^n\cdot(n+1)!}$ find ***AND*** simplify or rewrite n^{th} term for each of the following. You need ***NOT*** find interval of convergence, nor test endpoints.

$$f(x)=\sum_{n=0}^{\infty}\frac{(-1)^n(x-1)^{n+1}}{2^n\cdot(n+1)!}=(x-1)-\frac{1}{2\cdot 2!}(x-1)^2+\frac{1}{2^2\cdot 3!}(x-1)^3-\frac{1}{2^3\cdot 4!}(x-1)^4+\frac{1}{2^4\cdot 5!}(x-1)^5-\ldots$$

a. $f'(x)=\sum_{n=}^{\infty}\frac{(-1)^n(n+1)(x-1)^n}{2^n\cdot(n+1)!}$

$$=\sum_{n=0}^{\infty}\frac{(-1)^n(x-1)^n}{2^n\,n!}$$

$$f(x)=\sum_{n=0}^{\infty}\frac{(-1)^n(x-1)^{n+1}}{2^n\cdot(n+1)!}=(x-1)-\frac{1}{2\cdot 2!}(x-1)^2+\frac{1}{2^2\cdot 3!}(x-1)^3-\frac{1}{2^3\cdot 4!}(x-1)^4+\frac{1}{2^4\cdot 5!}(x-1)^5-\ldots$$

b. $\int f(x)dx=C+\sum_{n=}^{\infty}\frac{(-1)^n(x-1)^{n+2}}{2^n(n+2)(n+1)!}$

$$=C+\sum_{n=0}^{\infty}\frac{(-1)^n(x-1)^{n+2}}{2^n\cdot(n+2)!}$$

G

6. Use the definition of the Taylor Polynomial and Taylor Series to find the 5th Taylor Polynomial for $f(x) = \sin 2x$ centered at $c = \pi$. Then use your polynomial to write the Taylor Series for $f(x) = \sin 2x$ centered at $c = \pi$.

$$P_5(x) = a_0 + a_1(x-\pi) + a_2(x-\pi)^2 + a_3(x-\pi)^3 + a_4(x-\pi)^4 + a_5(x-\pi)^5$$

$n=0$	$f(x) = \sin 2x$	$f(\pi) = 0$	$a_0 = 0/0! = 0$
$n=1$	$f'(x) = 2\cos 2x$	$f'(\pi) = 2$	$a_1 = 2/1! = 2$
$n=2$	$f''(x) = -4\sin 2x$	$f''(\pi) = 0$	$a_2 = 0/2! = 0$
$n=3$	$f'''(x) = -8\cos 2x$	$f'''(\pi) = -8$	$a_3 = -8/3! = -4/3$
$n=4$	$f^{(4)}(x) = 16\sin 2x$	$f^{(4)}(\pi) = 0$	$a_4 = 0/4! = 0$
$n=5$	$f^{(5)}(x) = 32\cos 2x$	$f^{(5)}(\pi) = 32$	$a_5 = 32/5! = 4/15$

5th Taylor polynomial: $P_5(x) = 2(x-\pi) - \frac{8}{3!}(x-\pi)^3 + \frac{32}{5!}(x-\pi)^5$

Taylor Series: $P(x) = \sum_{n=0}^{\infty} \frac{(-1)^n 2^{2n+1} (x-\pi)^{2n+1}}{(2n+1)!}$

or

$$\sum_{n=1}^{\infty} (-1)^{n+1} \frac{2^{2n-1} (x-\pi)^{2n-1}}{(2n-1)!}$$

G

7. Use the Maclaurin Series for $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$ to find **AND** simplify the series for

a. $f(x) = e^{-4x^2}$

$$e^{-4x^2} = \sum_{n=0}^{\infty} \frac{(-4x^2)^n}{n!}$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n 4^n x^{2n}}{n!}$$

b. $g(x) = x^2 e^x$

$$x^2 e^x = x^2 \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

$$= \sum_{n=0}^{\infty} \frac{x^2 x^n}{n!}$$

$$= \sum_{n=0}^{\infty} \frac{x^{n+2}}{n!}$$

8. Suppose the interval of convergence for a particular power series $f(x)$ is $(-2,1]$. Answer the following, **TRUE OR FALSE**.

a. It is possible that the interval of convergence of $f'(x)$ is $(-2,1)$. True.

b. It is possible that the interval of convergence of $f'(x)$ is $[-2,1]$. False.

c. It is possible that the interval of convergence of $\int f(x)\,dx$ is $(-2,1)$. False.

d. It is possible that the interval of convergence of $\int f(x)\,dx$ is $[-2,1]$. True.

9. **(4 points):** In your opinion the greatest song (or recording) of all time (in any musical genre).

Answers vary.

There is no wrong answer.