Math 141
Exam 2
March 2, 2016

Name Noah Riddle Moore

*no calculators or smart phones anywhere!
*show all relevant work to receive full credit
*any evidence of academic dishonesty = 0 grade
*only pencils, pens = - 5 from grade
*late for exam= - 5 from grade

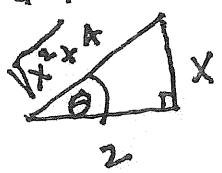
If needed $\sin 2u = 2\sin u \cos u$

1. Find the indefinite integral: $\int \frac{1}{\sqrt{x^2+4}}\,dx$

$x = 2\tan\theta\, d\theta$

$dx = 2\sec^2\theta\, d\theta$

$\sqrt{x^2+4} = 2\sec\theta$



$$\int \frac{2\sec^2\theta\, d\theta}{2\sec\theta}$$

$$\int \sec\theta\, d\theta$$

$$= \ln|\sec\theta + \tan\theta| + C$$

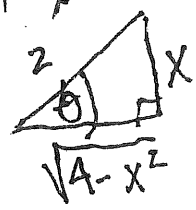
$$= \ln\left|\frac{\sqrt{x^2+4}}{2} + \frac{x}{2}\right| + C$$

2. Find the indefinite integral: $\int \frac{x^2}{\sqrt{4-x^2}}\,dx$

$x = 2\sin\theta$

$dx = 2\cos\theta\, d\theta$

$\sqrt{4-x^2} = 2\cos\theta$



$$\int \frac{(2\sin\theta)^2(2\cos\theta)\,d\theta}{2\cos\theta}$$

$$4\int \sin^2\theta\, d\theta$$

$$= 4\int\left(\frac{1-\cos 2\theta}{2}\right)d\theta$$

$$= 2\int(1-\cos 2\theta)\,d\theta$$

$$= 2\left[\theta - \frac{1}{2}\sin 2\theta\right] + C$$

$$= 2\left[\theta - \frac{1}{2}(2\sin\theta\cos\theta)\right] + C$$

$$= 2\left[\arcsin x/2 - \left(\frac{x}{2}\cdot\frac{\sqrt{4-x^2}}{2}\right)\right] + C$$

3. Find the partial fraction decomposition: $\frac{x^2+x+2}{x^2(x-2)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-2}$

$x^2+x+2 = Ax(x-2) + B(x-2) + Cx^2$

$x=0 \quad 2 = -2B$

$-1 = B$

$x=2 \quad 8 = 4C$

$2 = C$

$x=1 \quad 4 = -A-B+C$

$4 = -A+1+2$

$3 = -A$

$-3 = A$

p.f.d. $-\frac{3}{x} - \frac{1}{x^2} + \frac{2}{x-2}$

4. Find the partial fraction decomposition: $\frac{4x^2-5x+5}{(x-1)(x^2+1)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+1}$

$4x^2-5x+5 = A(x^2+1) + (Bx+C)(x-1)$

if $x=1 \quad 4 = 2A$

$2 = A$

* if expand $4x^2-5x+5 = Ax^2+A+Bx^2+Cx-Bx-C$

$= (A+B)x^2 + (C-B)x + (A-C)$

$A+B=4$ if $A=2$ $B=2$

$C-B=-5$ if $A=2$ $-C=-3$

$A-C=5$ $C=3$

p.f.d. $\frac{2}{x-1} + \frac{2x-3}{x^2+1}$

* or let $x=0$ and other value

5. Find the limit, if it exists.

$$\lim_{x\to 0}\left(\frac{e^{2x}-\cos x-2x}{x^2}\right) = \frac{e^0-\cos 0-0}{0} = \frac{0}{0}$$

L Hopital's

$$\lim_{x\to 0}\frac{2e^{2x}+\sin x-2}{2x} = \frac{2e^0+\sin 0-2}{0} = \frac{0}{0}$$

L Hopital's

$$\lim_{x\to 0}\frac{4e^{2x}+\cos x}{2} = \frac{4e^0+\cos 0}{2} = 5/2$$

6. Find the limit, if it exists.

$$\lim_{x\to 0^+}(x\ln x) = 0\cdot(-\infty)$$

$$\lim_{x\to 0^+}\frac{\ln x}{1/x} = -\frac{\infty}{\infty}$$

L Hopital's

$$\lim_{x\to 0^+}\frac{1/x}{-1/x^2} = \lim_{x\to 0^+}\left(\frac{1}{x}\right)\left(-\frac{x^2}{1}\right)$$

$$= \lim_{x\to 0^+}(-x) = 0$$

7. Determine if the improper integral converges or diverges. If convergent find its value.

$$\int_2^4 \frac{1}{\sqrt{16-x^2}}\,dx = \lim_{b\to 4^-} \int_2^b \frac{1}{\sqrt{16-x^2}}\,dx$$

$u = x \quad a = 4$

$du = dx$

$$= \lim_{b\to 4^-} \left[\left(\arcsin x/4\right)\right]_2^b$$

$$= \lim_{b\to 4^-} \arcsin b/4 - \arcsin 1/2$$

$$= \arcsin 1 - \arcsin 1/2 = \pi/2 - \pi/6 = \pi/3$$

converges

8. Determine if the improper integral converges or diverges. If convergent, find its value.

$$\int_{-\infty}^0 \frac{e^x}{1+e^x}\,dx = \lim_{a\to -\infty} \int_a^0 \frac{e^x}{1+e^x}\,dx$$

$u = 1+e^x$

$du = e^x\,dx$

$$= \lim_{a\to -\infty} \left[\ln|1+e^x|\right]_a^0$$

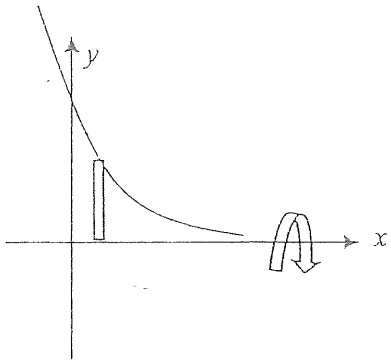
$$= \lim_{a\to -\infty} \ln(1+e^0) - \ln(1+e^a)$$

$$= \ln 2 - \ln(1+e^{-\infty})$$

$$= \ln 2 - \ln 1 = \ln 2$$

converges

Bonus. If finite, find the volume of the solid generated by rotating the region bounded by the graphs of $y = e^{-2x}$, and the x-axis for $x \geq 0$ about the x-axis If the volume is not finite, support your conclusion.



DISK

$$V = \pi \int_a^b R^2 \, dx$$

$$V = \pi \int_0^\infty \left(e^{-2x}\right)^2 dx$$

$$= \pi \int_0^\infty e^{-4x} \, dx$$

$$= \pi \lim_{b \to \infty} \int_0^b e^{-4x} \, dx$$

$$= \pi \lim_{b \to \infty} \left[-\frac{1}{4} e^{-4x} \right]_0^b$$

$$= \pi \lim_{b \to \infty} -\frac{1}{4} e^{-4b} + \frac{1}{4} e^0$$

$$= \pi \lim_{b \to \infty} -\frac{1}{4} e^{-\infty} + \frac{1}{4}$$

$$= \pi \left(0 + \frac{1}{4}\right) = \pi/4 \text{ units}^3$$

converges

finite volume

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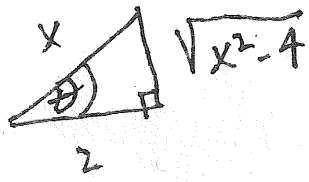
If needed $\sin 2u = 2\sin u \cos u$

1. Find the indefinite integral: $\int \frac{1}{x^2\sqrt{x^2-4}}\,dx$

$x = 2\sec\theta$

$dx = 2\sec\theta\tan\theta\,d\theta$

$\sqrt{x^2-4} = 2\tan\theta$



$$\int \frac{2\sec\theta\tan\theta\,d\theta}{(2\sec\theta)^2(2\tan\theta)}$$

$$\frac{1}{4}\int \frac{1}{\sec\theta}\,d\theta$$

$$\frac{1}{4}\int \cos\theta\,d\theta = \frac{1}{4}\sin\theta + C$$

$$= \frac{1}{4}\left(\frac{\sqrt{x^2-4}}{x}\right) + C$$

2. Find the indefinite integral: $\int \frac{x^2}{\sqrt{4-x^2}}\,dx$

$x = 2\sin\theta$

$dx = 2\cos\theta\,d\theta$

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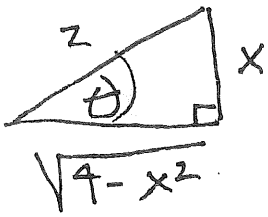

$$\int \frac{(2\sin\theta)^2(2\cos\theta)\,d\theta}{2\cos\theta}$$

$$4\int \sin^2\theta\,d\theta$$

$$4\int\left(\frac{1-\cos 2\theta}{2}\right)d\theta$$

$$2\int(1-\cos 2\theta)\,d\theta$$

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3. Find the partial fraction decomposition: $\frac{x^2-3x-2}{x^2(x-2)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-2}$

$x^2-3x-2 = Ax(x-2) + B(x-2) + Cx^2$

if $x=0$ $\quad -2=-2B$ $\quad$ if $x=2$ $\quad -4=4C$

$1=B$ $\quad -1=C$

$x=1$ $\quad -4=-A-B+C$

$-4=-A-1-1$

$-2=-A$

$2=A$

p.f.d.

$\frac{2}{x} + \frac{1}{x^2} - \frac{1}{x-2}$

4. Find the partial fraction decomposition: $\frac{4x^2-3x+3}{(x-1)(x^2+1)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+1}$

$4x^2-3x+3 = A(x^2+1) + (Bx+C)(x-1)$

if $x=1$ $\quad 4=2A$

$2=A$

*if expand $\quad 4x^2-3x+3 = Ax^2+A+Bx^2+Cx-Bx-C$

$= (A+B)x^2 + (C-B)x + (A-C)$

$A+B=4$ $\quad$ if $A=2$ $\quad B=2$

$C-B=-3$ $\quad$ if $A=2$ $\quad -C=1$

$A-C=3$ $\quad C=-1$

p.f.d. $\quad \frac{2}{x-1} + \frac{2x-1}{x^2+1}$

5. Find the limit, if it exists.

$$\lim_{x\to 0}\left(\frac{e^{2x}-\cos x-2x}{x^2}\right) = \frac{e^0-\cos 0-0}{0} = \frac{0}{0}$$

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$$\lim_{x\to 0^+}(x\ln x) = 0(-\infty)$$

$$\lim_{x\to 0^+}\frac{\ln x}{1/x} = \frac{-\infty}{\infty}$$

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$$\lim_{x\to 0^+}\frac{1/x}{-1/x^2}$$

$$\lim_{x\to 0^+}(1/x)(-x^2/1)$$

$$\lim_{x\to 0^+}(-x) = 0$$

7. Determine if the improper integral converges or diverges. If convergent find its value.

$$\int_1^2 \frac{1}{\sqrt{4-x^2}}\,dx = \lim_{b\to 2^-} \int_1^b \frac{1}{\sqrt{4-x^2}}\,dx$$

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converges

8. Determine if the improper integral converges or diverges. If convergent, find its value.

$$\int_0^\infty \frac{e^{-x}}{1+e^{-x}}\,dx$$

$u = 1+e^{-x}$

$du = -e^{-x}\,dx$

$-du = e^{-x}\,dx$

$$\lim_{b\to\infty} \int_0^b \frac{e^{-x}}{1+e^{-x}}\,dx$$

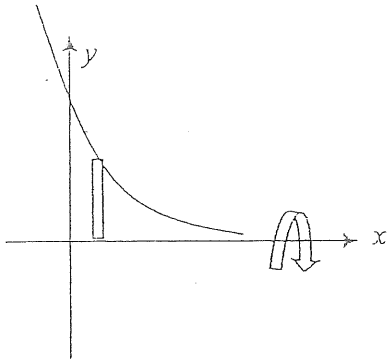
$$\lim_{b\to\infty} \left[-\ln|1+e^{-x}|\right]_0^b$$

$$\lim_{b\to\infty} -\ln|1+e^{-b}| + \ln|1+e^{0}|$$

$$= -\ln|1+e^{-\infty}| + \ln 2$$

$$= -\ln 1 + \ln 2 = \ln 2$$ converges

Bonus. If finite, find the volume of the solid generated by rotating the region bounded by the graphs of $y = e^{-2x}$, and the x-axis for $x \geq 0$ about the x-axis If the volume is not finite, support your conclusion.



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