Pattern Generation in Simple Inhibition-Dominated Networks

Katherine Morrison[1,2], Anda Degeratu[3], Vladimir Itskov[2], Carina Curto[2]

[1]University of Northern Colorado, [2]Pennsylvania State University, [3]University of Freiburg

katherine.morrison@unco.edu

Internally Generated Activity

Many networks exhibit ***internally generated*** patterns of activity – that is, ***emergent dynamics*** that are shaped by ***intrinsic properties of the network*** rather than inherited from an external fluctuating input.

- Spontaneous sequences in cortex
- Ripple sequences in hippocampus
- Patterned motor activity in central pattern generators (CPGs)

A common feature of all these networks is an ***abundance of inhibition***, and so we restrict to an ***inhibition-dominated regime***.

Possible explanations for pattern generation range from pacemaker or other intrinsically rhythmic neurons to network-level properties such as synaptic plasticity and the ***structure of connectivity***.

In order to focus on the ***role of connectivity***, we consider a model with simple ***perceptron-like neurons***, ***binary synapses***, and ***constant external input***, so that differences in the observed dynamics can be attributed to network connectivity alone.

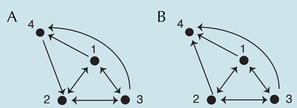
Key Mathematical Results

Theorem 1 gives conditions under which the network is guaranteed to never stabilize in a steady state.

Theorem 1: If G is an ***oriented graph*** (i.e., G has no bidirectional connections and no self-loops), and each neuron has ***out-degree at least 1***, then the network has ***no fixed point attractors***. The dynamics are thus ***guaranteed to be oscillatory or chaotic***.

Theorem 2 characterizes fixed point attractors that correspond to ***cliques*** (subsets of neurons that are all-to-all bidirectionally connected) in G.

Theorem 2: For any graph G, a clique σ is the ***support of a stable fixed point*** if and only if σ is a ***target-free clique***.



A vertex k is a ***target*** of a clique σ if $k \notin \sigma$ and $i \to k$ for each $i \in \sigma$. If σ has no targets, it is ***target-free***.

In (A), the clique $\sigma = \{1, 2, 3\}$ is target-free, while in (B) it is not, since vertex 4 is a target.

References: These results build on the theory of threshold-linear networks as developed in

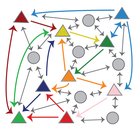
C. Curto and K. Morrison. Pattern completion in threshold-linear networks. Available at http://arxiv.org/abs/1512.00897.

C. Curto, A. Degeratu, V. Itskov. Encoding binary neural codes in networks of threshold-linear neurons. *Neural Comput*. 2013.

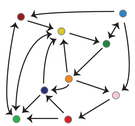
R.H. Hahnloser, H.S. Seung, J.J. Slotine. Permitted and forbidden sets in symmetric threshold-linear networks. *Neural Comput*. 2003.

Combinatorial Threshold-Linear Network (CTLN) Model Description

neural network



graph G



dynamics

$$\frac{dx_i}{dt} = -x_i + [Wx + \theta]_+$$

network model

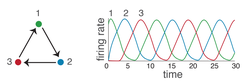
$$W_{ij} = \begin{cases} 0 & \text{if } i = j, \\ -1+\varepsilon & \text{if } i \leftarrow j \text{ in } G, \\ -1-\delta & \text{if } i \not\leftarrow j \text{ in } G. \end{cases}$$

$$\delta > 0, \qquad 0 < \varepsilon < \frac{\delta}{\delta+1}, \qquad \theta > 0$$

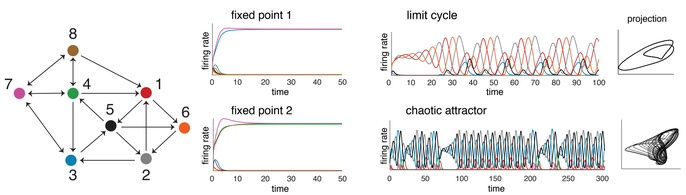
Unless otherwise noted, $\varepsilon = 0.25,\ \delta = 0.5,\ \theta = 1$ in all simulations. Thus, ***differences in dynamics*** are due only to ***differences in the graph G.***

Features of Dynamics

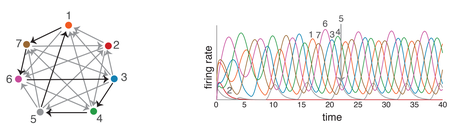
- Network activity ***follows the arrows*** in the graph.
  Note: arrows indicate ***less inhibition***, not net excitation.



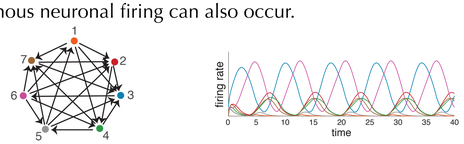
- All types of dynamics – ***multistability, limit cycles, chaos, and quasiperiodicity*** – can occur. Furthermore, multiple types of attractors can ***coexist*** in a single network.



- When activity falls into a limit cycle, the ***selected cycle*** in the graph is ***not always predictable*** from local graph statistics like the degree sequence.



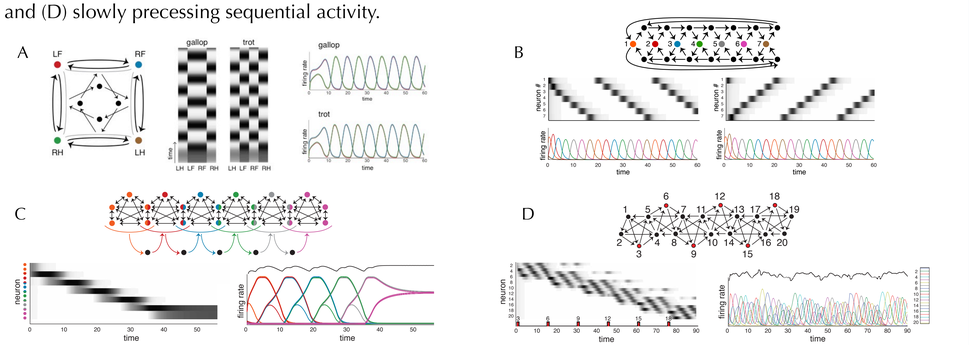
- When a sequence emerges, it is ***not always a simple sequence***; synchronous and/or quasi-synchronous neuronal firing can also occur.



Diversity of Dynamics

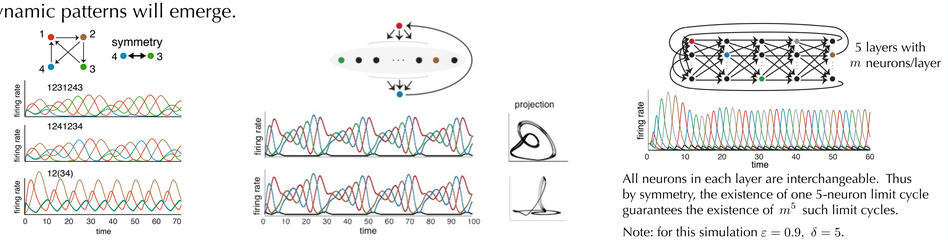
Engineered Networks

Theorem 2 enables one to engineer networks with prescribed patterns of activity, such as (A) a CPG for quadruped motion, (B) a network yielding forward and backward sequences, (C) a sequence of cell assemblies, and (D) slowly precessing sequential activity.



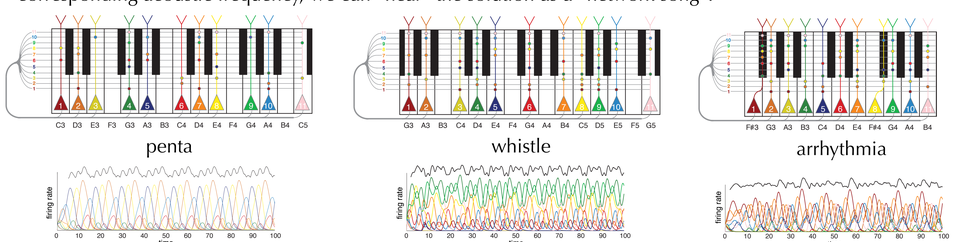
Networks with Symmetry

Graph symmetries (automorphisms) act on the space of attractors, and so can prove useful in predicting what dynamic patterns will emerge.



All neurons in each layer are interchangeable. Thus by symmetry, the existence of one 5-neuron limit cycle guarantees the existence of m^5 such limit cycles.

Note: for this simulation $\varepsilon = 0.9,\ \delta = 5$.

Musical Rhythms

By associating a piano key to each neuron, and using the firing rate to modulate the amplitude of the corresponding acoustic frequency, we can "hear" the solution as a "network song".