HW 6 : Ch 7 problems 17, 18, 21, 25, 45, 56, 86, 94, 98, 100

7-17 The power output and thermal efficiency of a power plant are given. The rate of heat rejection is to be determined, and the result is to be compared to the actual case in practice.

**Assumptions** 1 The plant operates steadily. 2 Heat losses from the working fluid at the pipes and other components are negligible.

**Analysis** The rate of heat supply to the power plant is determined from the thermal efficiency relation,

\[
\dot{Q}_H = \dot{W}_{\text{net, out}} = \frac{600 \text{ MW}}{0.4} = 1500 \text{ MW}
\]

The rate of heat transfer to the river water is determined from the first law relation for a heat engine,

\[
\dot{Q}_L = \dot{Q}_H - \dot{W}_{\text{net, out}} = 1500 - 600 = 900 \text{ MW}
\]

In reality the amount of heat rejected to the river will be lower since part of the heat will be lost to the surrounding air from the working fluid as it passes through the pipes and other components.

7-18 The heat input and thermal efficiency of a heat engine are given. The work output of the heat engine is to be determined.

**Assumptions** 1 The plant operates steadily. 2 Heat losses from the working fluid at the pipes and other components are negligible.

**Analysis** Applying the definition of the thermal efficiency to the heat engine,

\[
W_{\text{net}} = \eta_{\text{th}} Q_H = (0.35)(1.3 \text{ kJ}) = 0.455 \text{ kJ}
\]

7-21 The heat rejection and thermal efficiency of a heat engine are given. The heat input to the engine is to be determined.

**Assumptions** 1 The plant operates steadily. 2 Heat losses from the working fluid at the pipes and other components are negligible.

**Analysis** According to the definition of the thermal efficiency as applied to the heat engine,

\[
W_{\text{net}} = \eta_{\text{th}} q_H
\]

\[
q_H - q_L = \eta_{\text{th}} q_H
\]

which when rearranged gives

\[
q_H = \frac{q_L}{1 - \eta_{\text{th}}} = \frac{1000 \text{ kJ/kg}}{1 - 0.4} = 1667 \text{ kJ/kg}
\]
7-25 The United States produces about 51 percent of its electricity from coal at a conversion efficiency of about 34 percent. The amount of heat rejected by the coal-fired power plants per year is to be determined.

**Analysis** Noting that the conversion efficiency is 34%, the amount of heat rejected by the coal plants per year is

\[
\eta_{th} = \frac{W_{\text{coal}}}{Q_{\text{in}}} = \frac{W_{\text{coal}}}{Q_{\text{out}} + W_{\text{coal}}}
\]

\[
Q_{\text{out}} = \frac{W_{\text{coal}}}{\eta_{th}} - W_{\text{coal}}
\]

\[
= \frac{1.878 \times 10^{12} \text{ kWh}}{0.34} - 1.878 \times 10^{12} \text{ kWh}
\]

\[
= 3.646 \times 10^{12} \text{ kWh}
\]

7-45 The COP and the work input of a heat pump are given. The heat transferred to and from this heat pump are to be determined.

**Assumptions** The heat pump operates steadily.

**Analysis** Applying the definition of the heat pump coefficient of performance,

\[
Q_H = \text{COP}_{\text{HP}} W_{\text{net,in}} = (1.7)(50 \text{ kJ}) = 85 \text{ kJ}
\]

Adapting the first law to this heat pump produces

\[
Q_L = Q_H - W_{\text{net,in}} = 85 \text{ kJ} - 50 \text{ kJ} = 35 \text{ kJ}
\]
A decision is to be made between a cheaper but inefficient air-conditioner and an expensive but efficient air-conditioner for a building. The better buy is to be determined.

**Assumptions** The two air conditioners are comparable in all aspects other than the initial cost and the efficiency.

**Analysis** The unit that will cost less during its lifetime is a better buy. The total cost of a system during its lifetime (the initial, operation, maintenance, etc.) can be determined by performing a life cycle cost analysis. A simpler alternative is to determine the simple payback period. The energy and cost savings of the more efficient air conditioner in this case is

\[
\text{Energy savings} = (\text{Annual energy usage of A}) - (\text{Annual energy usage of B})
\]

\[
= (\text{Annual cooling load})(1 / \text{COP}_A - 1 / \text{COP}_B)
\]

\[
= (120,000 \text{ kWh/year})(1/3.2 - 1/5.0)
\]

\[
= 13,500 \text{ kWh/year}
\]

\[
\text{Cost savings} = (\text{Energy savings})(\text{Unit cost of energy})
\]

\[
= (13,500 \text{ kWh/year})($0.10/\text{kWh}) = $1350/\text{year}
\]

The installation cost difference between the two air-conditioners is

\[
\text{Cost difference} = \text{Cost of B} - \text{cost of A} = 7000 - 5500 = $1500
\]

Therefore, the more efficient air-conditioner B will pay for the $1500 cost differential in this case in about 1 year.

**Discussion** A cost conscious consumer will have no difficulty in deciding that the more expensive but more efficient air-conditioner B is clearly the better buy in this case since air conditioners last at least 15 years. But the decision would not be so easy if the unit cost of electricity at that location was much less than $0.10/kWh, or if the annual air-conditioning load of the house was much less than 120,000 kWh.

An inventor claims to have developed a heat engine. The inventor reports temperature, heat transfer, and work output measurements. The claim is to be evaluated.

**Analysis** The highest thermal efficiency a heat engine operating between two specified temperature limits can have is the Carnot efficiency, which is determined from

\[
\eta_{\text{th,max}} = \eta_{\text{th,C}} = 1 - \frac{T_L}{T_H} = 1 - \frac{290 \text{ K}}{500 \text{ K}} = 0.42 \text{ or } 42\%
\]

The actual thermal efficiency of the heat engine in question is

\[
\eta_{\text{th}} = \frac{W_{\text{net}}}{Q_H} = \frac{300 \text{ kJ}}{700 \text{ kJ}} = 0.429 \text{ or } 42.9\%
\]

which is greater than the maximum possible thermal efficiency. Therefore, this heat engine is a PMM2 and the claim is false.

The minimum work per unit of heat transfer from the low-temperature source for a heat pump is to be determined.

**Assumptions** The heat pump operates steadily.

**Analysis** Application of the first law gives

\[
\frac{W_{\text{net, in}}}{Q_L} = \frac{Q_H - Q_L}{Q_L} = \frac{Q_H}{Q_L} - 1
\]

For the minimum work input, this heat pump would be completely reversible and the thermodynamic definition of temperature would reduce the preceding expression to
\[
\frac{W_{\text{net, in}}}{Q_L} = \frac{T_H}{T_L} - 1 = \frac{535 \text{ K}}{460 \text{ K}} - 1 = 0.163
\]
The refrigerated space and the environment temperatures for a refrigerator and the rate of heat removal from the refrigerated space are given. The minimum power input required is to be determined.

Assumptions  The refrigerator operates steadily.

Analysis  The power input to a refrigerator will be a minimum when the refrigerator operates in a reversible manner. The coefficient of performance of a reversible refrigerator depends on the temperature limits in the cycle only, and is determined from

$$COP_{R,rev} = \frac{1}{(T_H / T_L)^{\frac{1}{1}} - 1} = \frac{1}{(25 + 273 \text{ K})/(\text{-8 + 273} \text{ K}) - 1} = 8.03$$

The power input to this refrigerator is determined from the definition of the coefficient of performance of a refrigerator,

$$\dot{W}_{net,in, min} = \frac{Q_L}{COP_{R, max}} = \frac{300 \text{ kJ/min}}{8.03} = 37.36 \text{ kJ/min} = 0.623 \text{ kW}$$
An inventor claims to have developed a refrigerator. The inventor reports temperature and COP measurements. The claim is to be evaluated.

**Analysis** The highest coefficient of performance a refrigerator can have when removing heat from a cool medium at -12°C to a warmer medium at 25°C is

\[
\text{COP}_{R,\text{max}} = \frac{1}{(T_H / T_L) - 1} = \frac{1}{(25 + 273 \text{ K})/(-12 + 273 \text{ K}) - 1} = 7.1
\]

The COP claimed by the inventor is 6.5, which is below this maximum value, thus the claim is reasonable. However, it is not probable.

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The claim of a heat pump designer regarding the COP of the heat pump is to be evaluated.

**Assumptions** The heat pump operates steadily.

**Analysis** The maximum heat pump coefficient of performance would occur if the heat pump were completely reversible,

\[
\text{COP}_{\text{HP, max}} = \frac{T_H}{T_H - T_L} = \frac{300 \text{ K}}{300 \text{ K} - 260 \text{ K}} = 7.5
\]

Since the claimed COP is less than this maximum, this heat pump is possible.