Block-based conditional entropy coding for medical image compression

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ABSTRACT

In this paper, we propose a block-based conditional entropy coding scheme for medical image compression using the 2-D integer Haar wavelet transform. The main motivation to pursue conditional entropy coding is that the first-order conditional entropy is always theoretically lesser than the first and second-order entropies. We propose a sub-optimal scan order and an optimum block size to perform conditional entropy coding for various modalities. We also propose that a similar scheme can be used to obtain a sub-optimal scan order and an optimum block size for other wavelets. The proposed approach is motivated by a desire to perform better than JPEG2000 in terms of compression ratio. We hint towards developing a block-based conditional entropy coder, which has the potential to perform better than JPEG2000. Though we don't indicate a method to achieve the first-order conditional entropy coder, the use of conditional adaptive arithmetic coder would achieve arbitrarily close to the theoretical conditional entropy. All the results in this paper are based on the medical image data set of various bit-depths and various modalities.

Keywords: PACS storage systems, teleradiology, JPEG2000, lossless image compression, entropy coding, block-based coding, wavelet transform, scan order, block size, error resilience.

1. INTRODUCTION

The trend in health-care information technology has been increasingly multimedia-oriented.[1] With the advent of modern digital imaging capabilities, there has been a voluminous increase in the amount of medical data being generated every day in various health-care enterprizes and hospitals. The Picture Archiving And Communications Systems (PACS) community envisions an all-digital environment in hospitals for the acquisition, storage, communication and display of large volumes of images of various modalities. This directly translates to the need for higher and higher compression ratios so that the storage costs are kept at a minimum and speeding up of transmission across low band-width channels (eg. for teleradiology applications) is achieved. JPEG2000,[2] the new state-of-the-art image compression standard is designed for broad range of applications, including the compression and transmission of medical images. In spite of its superior performance, there has been active research in this field for compression schemes with better compression ratios.

Medical images seem to exhibit high pixel-to-pixel correlation[3] which would imply that their conditional entropy is quite low as opposed to the first and second-order entropies. There has been several schemes such as CALIC,[4] LOCO-I[5] which exploit this by way of contexts generated from neighboring pixels.

In this paper, we pursue the goal of achieving higher compression ratios and introduce concepts which has the potential to achieve the same. We investigate the first-order conditional entropy coding to exploit the pixel-to-pixel correlation. Note that, however, in our case the context is simply the previous data sample as opposed to more complicated contexts used in other schemes. We propose a sub-optimal scan order and an optimum block size to perform conditional entropy coding to achieve higher compression ratios.

The paper is organized as follows. In Section 2, we provide the motivation to perform block-based conditional entropy coding for lossless image compression and the necessity to determine the scan order. Section 3 presents a method to determine the sub-optimal scan order to perform conditional entropy coding. We present our

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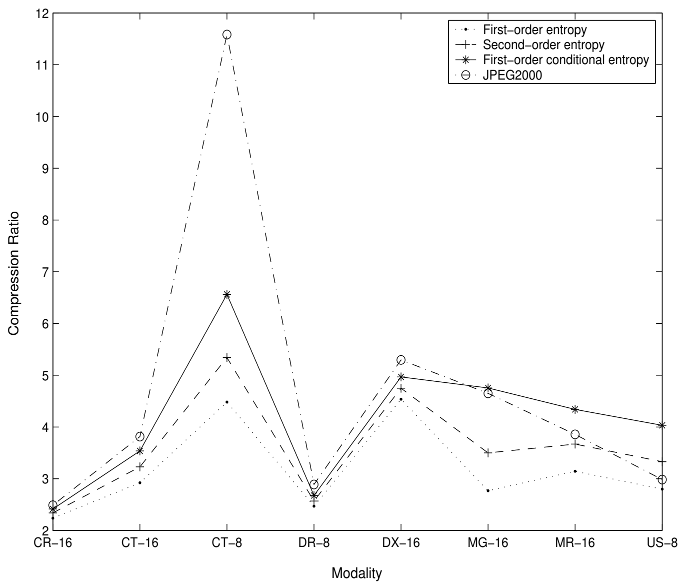


Figure 1. Comparison of compression ratios obtained with the first-order, second-order, first-order conditional entropy estimates and JPEG2000 for various modalities. CT-16 and CT-8 refer to 16-bit and 8-bit images from CT modality respectively. Similar is the case for other modalities.

proposed method of obtaining the optimal block size to achieve higher compression ratios in Section 4. All the results in this paper are based on the medical image data set of various bit-depths and various modalities. We conclude by hinting that a block-based conditional entropy coder is capable of performing possibly better than JPEG2000, thus aiding teleradiology applications and PACS storage systems.

2. MOTIVATION

The main motivation to pursue conditional entropy coding is that the first-order conditional entropy is always theoretically lesser than the first and second-order entropies, which is depicted in Figure 1. Figure 1 shows the comparison of compression ratios achieved with the first-order, second-order, first-order conditional entropy estimates and JPEG2000 for various modalities that are tabulated in Table 1. The compression ratio achieved using the first-order conditional entropy is greater than the one obtained by JPEG2000 for MR-16 and US-8 modalities and is closer for CR-16, CT-16, DR-8 and DX-16 modalities. For CT-8 modality, the compression ratio achieved using the first-order conditional entropy estimate is significantly less than the one achieved by JPEG2000. The conditional entropy is a function of the scan order, unlike the first-order entropy. Since, the first-order conditional distribution of a given pixel depends on the distribution of the preceding pixel, the order in which the data is arranged determines the first-order conditional entropy. It is therefore logical to determine a scan order such that the first-order conditional entropy is reduced resulting in high compression ratio. The conditional coding of the blocks would then be done with the prescribed scan order.

The motivation to adopt the block-based approach is that it provides a better estimate of the local statistic and thus providing lesser entropy than the non-block-based approach. The basic idea is to do determine the block size out of several different block sizes (64×64, 32×32, 16×16, 8×8, 4×4, 4×16, 2×16, 1×16), which minimizes the first-order conditional entropy or maximizes compression ratio computed from conditional entropy estimate. The optimum block size determination is based on the first-order conditional entropy estimate, which depends on the scan order. In order to reduce the complexity of the problem, we determine a universal scan order for all modalities which is sub-optimal and then determine an optimal block size.

Modality	Number of images
CR-16	15
CT-16	107
CT-8	52
DR-8	5
DX-16	21
MG-16	4
MR-16	370
US-8	24

Table 1. Medical image data set representing the modality and the number of images considered for experimentation in each modality. CT-16 and CT-8 refer to 16-bit and 8-bit images from CT modality respectively. Similar is the case for other modalities.

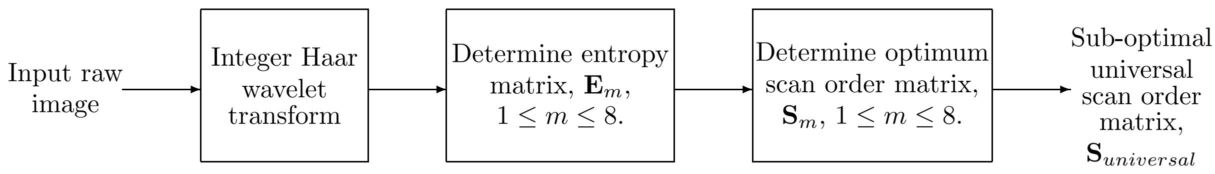


Figure 2. Block diagram to determine the sub-optimal universal scan order matrix.

3. DETERMINATION OF SUB-OPTIMAL SCAN ORDER

In this section, we will describe the method to determine the sub-optimal scan order. The block diagram explaining the method to determine the sub-optimal scan order is shown in Figure 2. The input raw image is transformed into wavelet domain by applying the 2-D integer Haar wavelet transform or S-transform.[6, 7] The 1-D integer Haar wavelet transform for a 1-D signal $x(n)$ is given by

$$l(n) = \left\lfloor \frac{x(2n+1) + x(2n)}{2} \right\rfloor \tag{1}$$

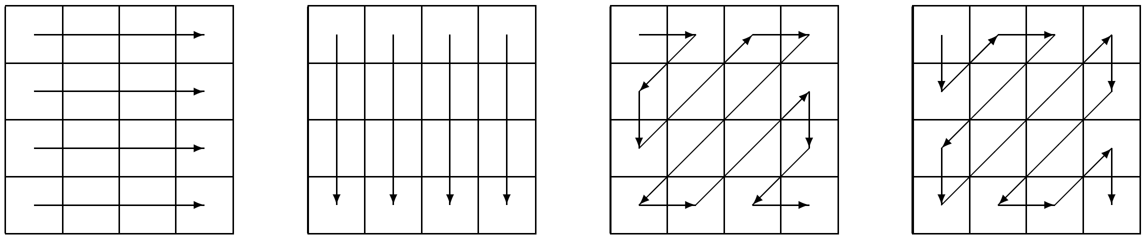
$$d(n) = x(2n+1) - x(2n) \tag{2}$$

where $l(n)$ and $d(n)$ are the 1-D sequences of approximation and detail coefficients respectively. $\lfloor k \rfloor$ represents the largest integer less than or equal to k. The 1-D integer Haar wavelet transform is applied separately first along the horizontal direction and then along the vertical direction. The experiment to determine the sub-optimal scan order is carried out on the wavelet coefficients obtained with one level of decomposition from the 2-D integer Haar wavelet transform. So, there will be four sub-bands which we term as LL, LH, HL and HH. The first letter represents the wavelet decomposition along the horizontal direction and the second letter represents the wavelet decomposition along the vertical direction as shown in Table 2. The idea is to determine the scan order for each sub-band in order to perform conditional coding instead of using a fixed scan order for all bands. This is particularly motivated by the fact that different sub-bands will have different statistics and a particular scan order will minimize the first-order conditional entropy for that sub-band. We performed only one level of the 2-D wavelet decomposition because we are interested in investigating the scan order for each of the four different sub-bands and the similar nature holds for sub-bands with higher levels of decomposition as the statistics are similar.

Four different scan orders namely horizontal raster (`HR`), vertical raster (`VR`), horizontal zig zag (`HZ`) and vertical zig zag (`VZ`), shown in Figure 3, are considered in the determination of sub-optimal scan order for each sub-band. All the four different scan orders are applied independently on the four sub-bands of a given image and the first-order conditional entropies are calculated. This procedure is carried out on all the images of a given modality and the mean conditional entropy estimates are obtained for different sub-bands with different

	L	H
L	LL	HL
H	LH	HH

Table 2. Wavelet sub-bands after one level of decomposition. The first letter in the sub-band notation represents the wavelet decomposition along the horizontal direction and the second letter represents the wavelet decomposition along the vertical direction.



(a) (b) (c) (d)

Figure 3. Different scan orders used in the determination of sub-optimal scan order. (a) Horizontal raster scan (`HR`), (b) Vertical raster scan (`VR`), (c) Horizontal zig zag scan (`HZ`) and (d) Vertical zig zag scan (`VZ`).

scan orders, each applied independently. Let e_{ij}^m be the mean conditional entropy estimate for modality m with i^{th} scan order for j^{th} sub-band. We define entropy matrix $\mathbf{E}_m$

$$\mathbf{E}_m = \begin{pmatrix} e_{11}^m & \cdots & e_{1q}^m \\ \vdots & \ddots & \vdots \\ e_{p1}^m & \cdots & e_{pq}^m \end{pmatrix} \tag{3}$$

The elements of $\mathbf{E}_m$ represent the mean entropy estimates for a given modality m, which can be any of the modalities shown in Table 1. p represents the number of scan orders and q represents the number of sub-bands under consideration. Here $p = 4$ and $q = 4$. So, $1 \leq i, j \leq 4$ where $i, j \in \mathbb{N}$. Since we considered 8 modalities considering bit-depths into account, $1 \leq m \leq 8$ where $m \in \mathbb{N}$. Hence, for different values of m, we have different entropy matrices. Let us define the sub-band vector, **sb** and scan order vector, **sc** as

$$\mathbf{sb} = \begin{pmatrix} \mathsf{LL} & \mathsf{HL} & \mathsf{LH} & \mathsf{HH} \end{pmatrix}^T \tag{4}$$

$$\mathbf{sc} = \begin{pmatrix} \mathtt{HR} & \mathtt{VR} & \mathtt{HZ} & \mathtt{VZ} \end{pmatrix}^T \tag{5}$$

where $\mathbf{sb}(i)$ and $\mathbf{sc}(i)$ represent the i^{th} element of the vectors **sb** and **sc** respectively and T represents matrix transposition. The optimal scan order matrix for a given modality m is given as $\mathbf{S}_m$

$$\mathbf{S}_m = \begin{pmatrix} s_1^m & s_2^m \\ s_3^m & s_4^m \end{pmatrix} \tag{6}$$

where s_i^m is the optimal scan order for the sub-band $\mathbf{sb}(i)$ of modality m and $1 \leq i \leq 4$. s_i^m is given as

$$s_i^m = \mathbf{sc}\left(\arg\min_{1 \leq j \leq p} \mathbf{e}_i^m\right), \; 1 \leq i \leq 4. \tag{7}$$

where $\mathbf{e}_i^m$ is the i^{th} column of entropy matrix $\mathbf{E}_m$, given as $\mathbf{e}_i^m = \begin{pmatrix} e_{1i}^m & e_{2i}^m & \cdots & e_{pi}^m \end{pmatrix}^T$ and e_{ji}^m is the j^{th} element of $\mathbf{e}_i^m$. Different optimal scan order matrices, $\mathbf{S}_1, \; \mathbf{S}_2, \cdots, \mathbf{S}_8$ are determined by carrying out the above mentioned procedure for different modalities. The percentage reduction in mean entropy for different modalities with the determined optimal scan order compared to `HR` in all the sub-bands is tabulated in Table 3. Since

Modality	Scan order matrix	LL	HL	LH	HH	Percentage reduction in mean first-order conditional entropy for $\mathbf{S}_m$-s with respect to HR in all the sub-bands
CR-16	$\mathbf{S}_1$	HR	VR	HR	VR	0.81
CT-16	$\mathbf{S}_2$	HZ	VR	HR	VR	0.99
CT-8	$\mathbf{S}_3$	HR	VR	HR	VR	6.87
DR-8	$\mathbf{S}_4$	VR	VR	HR	HR	1.26
DX-16	$\mathbf{S}_5$	VR	VR	HR	HR	0.73
MG-16	$\mathbf{S}_6$	VR	VR	VR	VR	2.09
MR-16	$\mathbf{S}_7$	HZ	VR	HR	VR	0.99
US-8	$\mathbf{S}_8$	HR	VR	HR	VR	0.43

Table 3. Percentage reduction in mean first-order conditional entropy for various modalities using the optimal scan orders as defined in (7) with respect to horizontal raster(HR) scan. LL, LH, HL and HH represent the bands after the 2-D wavelet decomposition. HR, VR, HZ, VZ are shown in Figure 3. CT-16 and CT-8 refer to 16-bit and 8-bit images from CT modality respectively. Similar is the case for other modalities.

many of the $\mathbf{S}_m$-s are different from each other, it would be desirable to have a universal scan order for all the modalities. The sub-optimal universal scan order, $\mathbf{S}_{universal}$ is given as

$$\mathbf{S}_{universal} = \begin{pmatrix} \texttt{HR} & \texttt{VR} \\ \texttt{HR} & \texttt{VR} \end{pmatrix} \quad (8)$$

The scan orders $\mathbf{S}_m$, $1 \leq m \leq 8$ represent the optimal scan order for each modality among the scan orders specified by **sc**. Since, the percentage reduction in entropy estimates tabulated in Table 3 did not decrease significantly with the universal scan order, we obtain $\mathbf{S}_{universal}$ which is sub-optimal. The sub-optimal scan order determines the conditional entropy estimates and hence compression ratio. This scan order is used in determining the conditional entropy when carrying out block-based conditional entropy coding.

4. DETERMINATION OF OPTIMAL BLOCK SIZE

In this section, we will describe the method to determine an optimum block size which provides higher compression ratio and error resilience. The block diagram explaining the method is shown in Figure 4. The first block in Figure 4 is same as that of Figure 2 except that we perform a multi-level wavelet decomposition unlike only one level as in the sub-optimal scan order determination. The number of levels of decomposition depends on the size of LL band. Eight different block sizes are considered for experimentation of which one of them is determined to be the optimum block size. Let r_k and c_k be the row and column dimensions of the blocks and $1 \leq k \leq l$, where l is the number of blocks and $k \in \mathbb{N}$. Here $l = 8$. Let us define row dimension vector, $\mathbf{r}$ and column dimension vector, $\mathbf{c}$ as $\mathbf{r} = \begin{pmatrix} 64 & 32 & 16 & 8 & 4 & 4 & 2 & 1 \end{pmatrix}^T$ and $\mathbf{c} = \begin{pmatrix} 64 & 32 & 16 & 8 & 4 & 16 & 16 & 16 \end{pmatrix}^T$ respectively. A block with row dimension r_k and column dimension c_k is defined as $r_k \times c_k$. Also, r_k and c_k are the k^{th} elements of $\mathbf{r}$ and $\mathbf{c}$ respectively.

The wavelet sub-bands are divided into blocks with block size, say $r_k \times c_k$. Using the sub-optimal scan order as specified by $\mathbf{S}_{universal}$, the first-order conditional entropies are estimated at sub-band level. The number of conditional entropy distributions depends on the number of levels of decompositions. d levels of wavelet decomposition result in $3d+1$ sub-bands and hence $3d+1$ conditional entropy distributions. The mean weighted entropies are estimated for the wavelet data by applying each of the blocks independently. Let $\mathbf{e}_b^m$ be the entropy vector with block-based approach for modality m which is defined as

$$\mathbf{e}_b^m = \begin{pmatrix} e_{b,1}^m & e_{b,2}^m & \cdots & e_{b,l}^m \end{pmatrix}^T \quad (9)$$

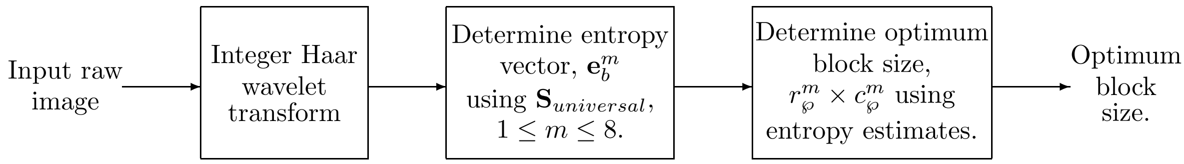


Figure 4. Block diagram to determine the optimum block size.

where $e_{b,k}^m$ represents the entropy estimate for modality m with block $r_k \times c_k$. The optimum block which maximizes the compression ratio or minimizes the entropy estimate for modality m is given as $r_\wp^m \times c_\wp^m$, where

$$\wp = \arg \min_{1 \leq k \leq l} \mathbf{e}_b^m \tag{10}$$

The above procedure is carried out for all modalities, i.e. $1 \leq m \leq 8$ and $r_\wp^m \times c_\wp^m$ is determined for all modalities. Figure 5 shows the percentage improvement in compression ratio for various block sizes with respect to 4×16 block for different modalities. It is clear from Figure 5 that 64×64 is the optimum block size for all modalities which provides high compression ratio with respect to other blocks. But 64×64 block has the least error-resilience capability compared to other blocks. This is because, in the event of error in any of the blocks, the number of affected pixels would be $r_k c_k$, which is maximum for 64×64 block. Since the maximum percentage reduction in compression ratio of 4×16 block with respect to 64×64 block for all modalities is less than 2.5% and the error resilience capability is 64 times better than 64×64, we choose 4×16 to be the optimum block size.

5. CONCLUSION

We have proposed a block-based conditional entropy coding scheme for medical image compression using the 2-D integer Haar wavelet transform. We proposed a sub-optimal scan order and an optimum block size to perform conditional entropy coding for various modalities. We deduce that a similar scheme can be used to obtain a sub-optimal scan order and an optimum block size for other wavelets also. The sub-optimal scan order along with 4×16 block provides better compression ratio and error resilience when compared with other block sizes. We conclude by saying that this paper attempts to build a case for block-based conditional entropy coder capable of performing possibly better than JPEG2000 and hence worth pursuing for future research. A lossless image compression scheme incorporating the proposed concepts would result in higher compression ratio thus aiding teleradiology applications and PACS storage systems.

APPENDIX A. DEFINITION OF ENTROPY

An ensemble 'X' is a random variable x with a set of possible *outcomes*, $\mathcal{A}_X = \{a_1, a_2, \cdots, a_I\}$, having probabilities $\{p_1, p_2, \cdots, p_I\}$, with $P(x = a_i) = p_i$, $p_i \geq 0$ and $\sum_{x \in \mathcal{A}_X} P(x) = 1$. The first-order entropy of discrete random variable X is defined by[8]

$$H_1(X) = \sum_{1 \leq i \leq I} P(x = a_i) \log \frac{1}{P(x = a_i)} \tag{11}$$

with the convention for $P(x) = 0$ that $0 \times \log \frac{1}{0} \equiv 0$, since $\lim_{\theta \to 0+} \theta \log \frac{1}{\theta} = 0$. The entropy measures the information content or 'uncertainty' of x.

The second-order entropy of X is defined as

$$H_2(X) = \frac{1}{2} \sum_{1 \leq i,j \leq I} P(x = a_i, x = a_j) \log \frac{1}{P(x = a_i, x = a_j)} \tag{12}$$

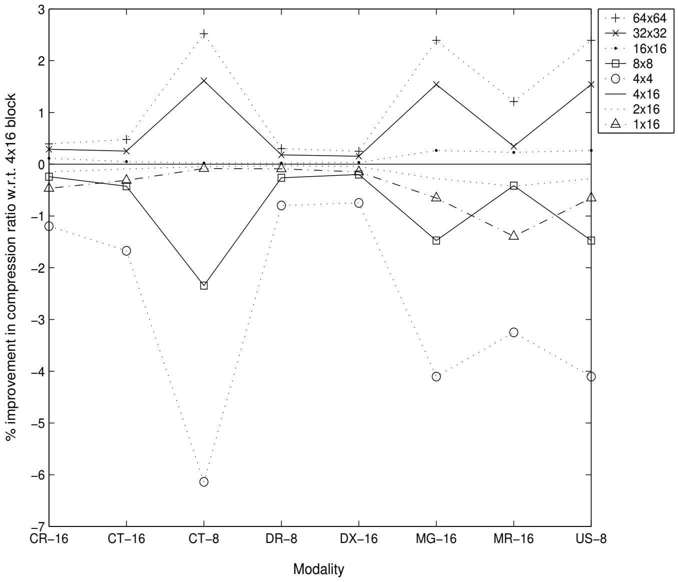


Figure 5. Percentage improvement in compression ratio for various block sizes with respect to 4×16 block for various modalities using the scan order specified by $\mathbf{S}_{universal}$ as defined in (8). CT-16 and CT-8 refer to 16-bit and 8-bit images from CT modality respectively. Similar is the case for other modalities.

The first-order conditional entropy of X is defined as

$$H_c(X) = \sum_{1 \leq i,j \leq I} P(x = a_i, x = a_j) \log \frac{1}{P(x = a_i / x = a_j)} \tag{13}$$

It can be shown that $H_c(X) = 2H_2(X) - H_1(X)$ and since $H_1(X) \geq H_2(X)$, we have $H_c(X) \leq H_2(X) \leq H_1(X)$.

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