Numerical Simulations Comparing Nonlinear Acoustic Effects in Straight and Tapered Tubes

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This talk is an UPDATE on our efforts to model streaming in tapered resonators.

Outline

I. Project description

II. Model equations

III. Computational grid

IV. Interim calculation without dissipation

V. Conclusions and future work

Project Objectives

- Numerically simulate the experiment performed by Olsen and Swift on the effect of a tapered pulse tube on streaming in a pulse tube refrigerator.

  J. Olson and G. Swift, “Acoustic streaming in pulse tube refrigerators: tapered pulse tubes” Cryogenics, 1997, **37**, 769-776

- Investigate two predictions from their paper:

  1. Importance of temperature dependent viscosity on streaming
  2. Optimal taper angle to suppress streaming

- Manipulate parameters of the computational model to evaluate the effects of additional second-order terms.

Implementation

- Finite difference algorithm in two dimensions using the two-step MacCormack method

  V. Sparrow and R. Raspet, "A numerical method for general finite amplitude wave propagation in two dimensions and its application to spark pulses" JASA, Nov. 1991, **90** (5), 2683-2691

  – Third dimension simulated by assuming axial symmetry
  – Grid clustered near the boundaries

Model Equations

1. Continuity

$$\frac{\partial \rho'}{\partial t} + \rho_o \nabla \cdot \mathbf{u} = \underbrace{-\rho' \nabla \cdot \mathbf{u} - \mathbf{u} \cdot \nabla \rho'}_{nonlinear}$$

2. Momentum

$$\rho_o \frac{\partial \mathbf{u}}{\partial t} + \nabla p = \underbrace{-\rho' \frac{\partial \mathbf{u}}{\partial t} - \rho_o \mathbf{u} \cdot \nabla \mathbf{u}}_{nonlinear} + \underbrace{\frac{1}{3}\mu \nabla (\nabla \cdot \mathbf{u}) + \mu \nabla^2 \mathbf{u}}_{dissipative} + (\rho_o + \rho') f$$

3. Entropy Balance

$$\rho_o T_o(\frac{\partial s}{\partial t} + \underbrace{\overbrace{\mathbf{u}\cdot\nabla s) + (\rho_o T' + \rho' T_o)\frac{\partial s}{\partial t}}^{2nd\ order\ near\ boundary}}_{nonlinear} = \underbrace{\kappa_o \nabla^2 T' + \nabla\cdot\kappa'\nabla T'}_{dissipative}$$

4. State Equation I

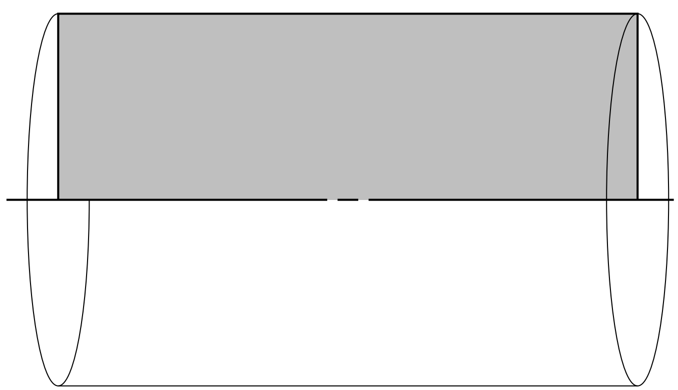
$$p - c_o^2\rho' = \underbrace{\frac{c_o^2}{\rho_o}\frac{B}{2A}\rho'^2}_{nonlinear} + \underbrace{c_o^2\left(\frac{\rho\beta T}{c_p}\right)_o s + \overbrace{\frac{1}{2}\left(\frac{\partial^2 P}{\partial s^2}\right)_{\rho,o} s^2}^{2nd\ order\ near\ boundary}}_{dissipative}$$

5. State Equation II

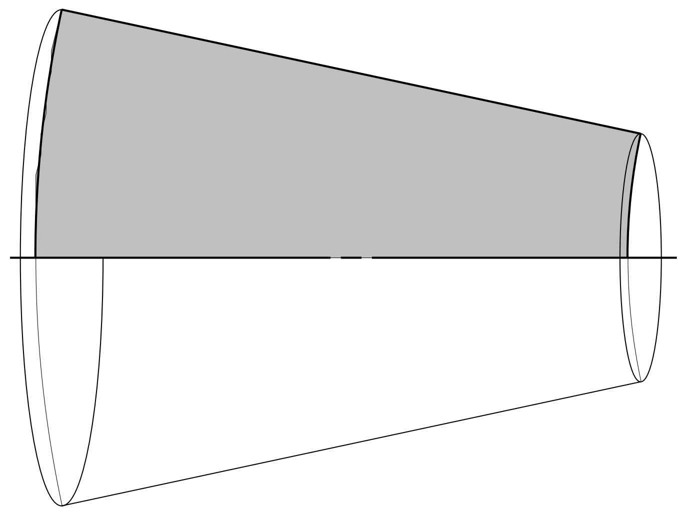
$$T' - \left(\frac{T\beta}{\rho c_p}\right)_o p = \underbrace{\left(\frac{T}{c_p}\right)_o s + \overbrace{\frac{1}{2}\left(\frac{\partial^2 T}{\partial P^2}\right)_{s,o} p^2 + \frac{1}{2}\left(\frac{\partial^2 T}{\partial s^2}\right)_{P,o} s^2}^{2nd\ order\ near\ boundary}}_{dissipative}$$

Orientation of computational domain

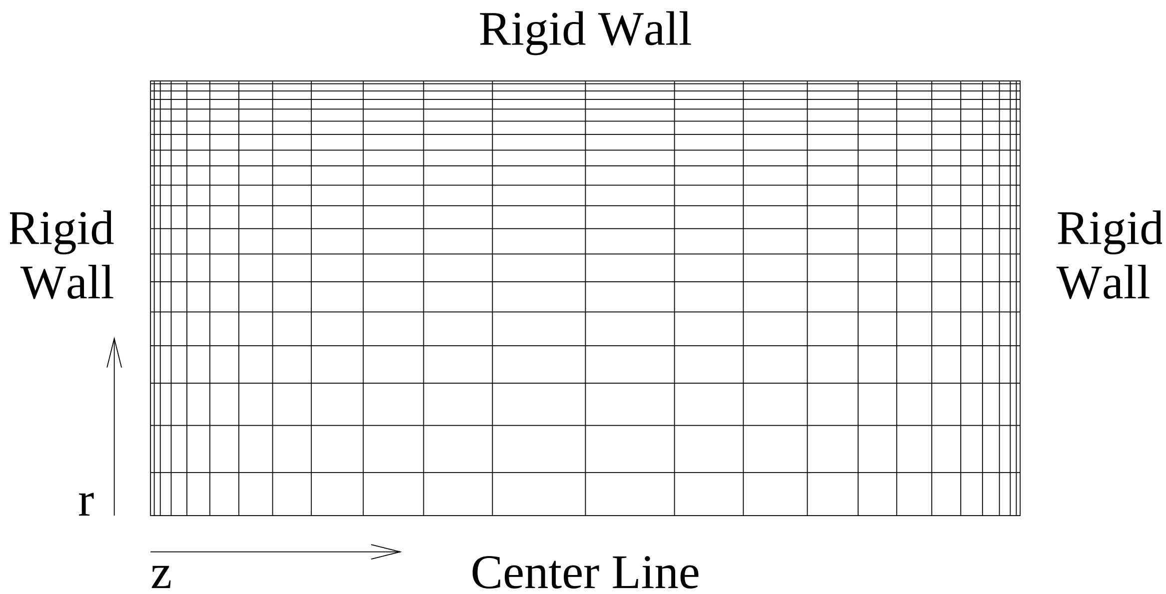
Cylindrical Coordinates



Spherical Coordinates



Clustered grid (representation)



Preliminary calculation without dissipation

- Observe the effects of nonlinear terms in the model equations.
- Leave out dissipative terms.
- Use uniform non-clustered grid for simplified calculation.

Model Equations without Dissipation

1. Continuity

$$\frac{\partial \rho'}{\partial t} + \rho_o \nabla \cdot \mathbf{u} = \underbrace{-\rho' \nabla \cdot \mathbf{u} - \mathbf{u} \cdot \nabla \rho'}_{nonlinear}$$

2. Momentum

$$\rho_o \frac{\partial \mathbf{u}}{\partial t} + \nabla p = \underbrace{-\rho' \frac{\partial \mathbf{u}}{\partial t} - \rho_o \mathbf{u} \cdot \nabla \mathbf{u}}_{nonlinear}$$

3. Entropy Balance

$$\rho_o T_o(\frac{\partial s}{\partial t} + \overbrace{\underbrace{\mathbf{u} \cdot \nabla s) + (\rho_o T' + \rho' T_o)\frac{\partial s}{\partial t}}_{nonlinear}}^{2nd\ order\ near\ boundary} = 0$$

4. State Equation I

$$p - c_o^2 \rho' = \underbrace{\frac{c_o^2}{\rho_o}\frac{B}{2A}{\rho'}^2}_{nonlinear}$$

5. State Equation II

$$T' - \left(\frac{T\beta}{\rho c_p}\right)_o p = 0$$

Animation

Conclusions

- We now have the full set of equations for modeling streaming in tapered tubes.

- We now know the equation transformations and grid stretching necessary to perform the full computation.

- Tests of nonlinearity in a cylindrical tube have been successful.

Future Work

- Model the tapered tube in spherical coordinates.
- Include dissipation terms.
  - Use refined grid.
  - Run in parallel using domain decomposition.
- Determine streaming at various different taper angles.
- Investigate the role of temperature dependent viscosity in streaming suppression.