Why Sellers Should Prefer Sequential Mechanisms

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Why Sellers Should Prefer Sequential Mechanisms

We analyze two mechanisms commonly used for selling an asset or a contract, in a setting in which bidders must incur an entry cost in order to learn how much the asset is worth to them: an English auction, and a sequential bidding process. A theory developed by Bulow and Klemperer (2009) predicts that sellers should prefer the auction, because it generates higher average revenues, while bidders should prefer the sequential mechanism, because it generates higher average bidder profits. We compare the two mechanisms in a controlled laboratory environment, varying the entry cost, and find that, contrary to the theoretical predictions, average seller revenues tend to be the same or higher under the sequential mechanism, while average bidder profits are approximately the same. We identify three behavioral causes that explain our result: (1) bidders do not enter the auction 100% of the time, and (2) in the sequential mechanism, bidders do not set pre-emptive bids according to the threshold strategy, and (3) the subsequent bidders tend to over-enter in response to pre-emptive bids by the first bidders. We develop a model of noisy bidder entry decisions, related to quantal response equilibrium, and show, using maximum-likelihood parameter estimation techniques, that our model does a good job in organizing the experimental data, both, in terms of point predictions, as well as in terms of qualitative comparisons of the two mechanisms.

Keywords: Auctions, Experimental Economics, Behavioral Mechanism Design.

1. Introduction

In this study we analyze a setting in which an asset or a contract is up for bid, and potential bidders must incur an entry cost prior to bidding, in order to learn their valuations. This setting is used in a number of contexts. For instance, in procurement activities, suppliers must commit significant resources to estimate the value of a contract up for bid. Similarly, in mergers and acquisitions, one firm must incur the due diligence cost to research the value of the other company. In any of these scenarios, the bid taker can choose from a number of mechanisms to award the asset to a bidder. Two of the mechanisms often used in practice, and the subject of our study, are: (1) an auction in which bidders make simultaneous entry decisions, and (2) a stylized sequential negotiation in which the bid taker approaches bidders one at a time, giving them opportunities to place pre-emptive bids, thus possibly deterring the entry by subsequent bidders.

Past empirical work suggests that bidders and sellers differ on their preference between auctions and sequential mechanisms. For example, in a recent poll of private equity firms,
80 percent said that, when acting as sellers, they prefer running auctions. However, 90 percent of those same companies said that when they act as bidders, they prefer to avoid auctions (Stephenson, Jones and Di Lapigio 2006). Similarly, Warren Buffet, when describing the Berkshire Hathaway acquisition criteria in his 2008 annual report writes “We don’t participate in auctions.” (Berkshire Hathaway (2009) p. 24).

Theoretical research supports this order of preference between auctions and sequential mechanisms by participants; auctions are generally revenue maximizing for sellers and sequential mechanisms are generally profit maximizing for bidders (see Fishman (1988), Bulow and Klemperer (2009), and references therein). Auctions force all bidders to incur entry costs and compete with each other simultaneously. This improves revenue for sellers, but creates inefficiencies because some bidders incur entry costs unnecessarily. Sequential mechanisms allow early bidders to circumvent the auction and set preemptive jump bids, which can deter future entry by competitors and allow the initial bidder to capture higher profits than they would in auctions. While both of these mechanisms have been studied in the theoretical literature, and were used in roughly 50% of public takeover activities in the 1990s, which represented over $1 trillion in deals (Boone and Mulherin 2007), they have not been compared in a controlled laboratory setting.

The objective of this study is to understand how auctions and sequential mechanisms compare and contrast in a controlled, laboratory environment. Specifically, we examine whether bidder behavior is similar to theoretical predictions of the Bulow and Klemperer (2009) model that result in the conclusions that an English auction generates higher seller revenues and the sequential mechanism generates higher bidder profits. There are a number of reasons why this theoretical result may not translate into practice. For instance, past experimental work has shown that bidders in auctions may not conform to standard game theoretic predictions (see Kagel (1995) for a survey of laboratory auction research). Similarly, the sequential mechanism model incorporates signaling behavior that assumes bidders are perfect optimizers who can make complex inferences and calculations related to entry decisions and preemptive bidding behavior. Issues of bounded rationality by bidders may affect their behavior and ultimately the normative predictions of revenue and profits for the two mechanisms (see Simon (1984) and Conlisk (1996) for summaries of bounded rationality models). By comparing the performance of auctions and sequential mechanisms in a controlled setting of a laboratory, with well-defined rules that match the Bulow and Klemperer (2009) model, we can test whether this model is a good predictor of actual behavior. We can
also begin to understand the causes of potential differences between the observed behavior and the theoretical predictions, and extend the model to capture some additional behavioral features.

There is a considerable amount of theoretical work comparing auctions and sequential mechanisms. Bulow and Klemperer (2009), who develop the model that we directly test here, compare a standard English auction to a sequential bidding mechanism. Fishman (1988) is an earlier paper with a similar, but a less general model. In both papers the authors prove that the sequential mechanism results in higher average seller revenue than the auction. Hirshleifer and Png (1989) also study a sequential bargaining process with two bidders, however, they assume that bidding itself is costly. In their setting, the sequential mechanism can generate higher revenue compared to an auction, even in theory. Bernhardt and Scoones (1993) present a specific application of a sequential mechanism to wage offers. Arnold and Lippman (1995) compare an auction to a sequential process with information asymmetries, discounting, and costly search by sellers.

To our knowledge, we are the first to directly compare auctions and sequential mechanisms in a setting in which the equilibrium structure is well understood. A number of field empirical studies have investigated auctions compared to negotiations (see, for example (Lusht 1996, Bajari, McMillan and Tadelis 2008) among others). However, our paper focuses directly on the model in Bulow and Klemperer (2009), in which a free-form negotiation is not an option.

Our main finding is that the preference of the two mechanisms for bidders and sellers is different from the theoretical prediction. We find that with moderate entry costs, the sequential mechanism results in similar average seller revenue and bidder profits, while with high entry costs the sequential mechanism actually results in higher seller revenues than does the auction, while average bidder profits continue to be similar. Therefore, our laboratory results indicate that it may well be that sellers should prefer sequential mechanisms over auctions, while bidders should be relatively indifferent between the two. We find that the differences between our data and theoretical predictions result primarily from two behavioral phenomena. First, in the auction, bidders do not enter 100% of the time, as the standard theory predicts, thus driving its revenue slightly below that of normative benchmarks. Second, in the sequential mechanism, the first bidders do set positive preemptive bids, but the second bidders nevertheless enter the auctions more often than they should. This second bidder over-entry causes the revenues in the sequential mechanism to be significantly higher.
than it should in theory, especially when entry costs are high.

We proceed to develop a new alternative model of bidder behavior that better organizes our data. Our model is related to the quantal response equilibria model developed in McKelvey and Palfrey (1995) and McKelvey and Palfrey (1998). We show that if individual bidders derive some (random) benefit or cost from entering auctions\(^1\), we can generate predictions that are largely consistent with what we observe in the laboratory. We fit this model to our data using maximum-likelihood estimation to illustrate that it predicts behavior better than the standard theory.

In the next section we describe our experimental design along with standard theoretical predictions for both the auction and sequential mechanism. In Section 3 we present the results of all the treatments in our experiment. Following this, in Section 4 we present an alternative model that builds on the standard theory but better describes our data. Lastly, in Section 5, we conclude our investigation with a summary and comment on future research.

2. Experimental Design

Each subject in every treatment acted as a first or second bidder of a single, indivisible object. In each round a first and second bidder were randomly matched together.\(^2\) In the first set of treatments, the auction, each round began with both bidders making their entry decisions privately and simultaneously. If both bidders entered the auction, they were then shown their own private values, and proceeded to compete for the item via an ascending clock auction in which the initial price was 0. The bidder who dropped out of the auction first lost the auction, and this drop-out price established the winning bid for the other bidder.

In the second set of treatments, the sequential mechanism (Seqmech), each round began with the first bidder (bidder 1) of each pair deciding whether or not to enter the auction, and, if she chose to enter the auction, setting an initial preemptive bid for the auction. After bidder 1 made these decisions, the second bidder (bidder 2) then made her entry decision after observing bidder 1’s preemptive bid. If both bidders entered the auction, then they

\(^1\)The benefit could come from a variety of sources such as the joy of competing, or to overestimating the probability of winning the auction to name a few possible sources. We do not specifically model the source of the cost or benefit. Rather our intent is to demonstrate that such factors might have a dramatic impact on both individual behavior and aggregate performance of the mechanism.

\(^2\)We called the two bidders “bidder A” and “bidder B” in the actual experiment to avoid any framing effects.
competed for the item in an ascending clock auction in which the initial price corresponded to bidder 1’s pre-emptive bid (please see the online appendix for sample instructions).

The private values for all bidders were uniformly distributed between 1 and 100, independent and identically distributed, in each round of all treatments. Each subject participated in a single treatment only, and each treatment included 30 rounds. To eliminate the possibility of losses, we provided each subject with an initial endowment of 20 laboratory dollars per round in all four treatments in our study.

In both the auction and sequential mechanism, we ran one set of treatments with an entry cost of 3 (c = 3), which we will refer to as Lowcost. In a second set of treatments we set the entry cost to 10 (c = 10), which we will refer to as Highcost. We varied the entry costs between treatments to help determine if any potential results were influenced by entry costs rather than the selling mechanism. Table 1 summarizes our design of the experiment and sample sizes. In each treatment, 36 subjects were placed in five to six cohorts, where we will use the cohort as the main unit of statistical analysis in the next section.

Table 1: Experimental design and number of participating subjects.

<table>
<thead>
<tr>
<th>Treatment</th>
<th>Auction</th>
<th>Seqmech</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lowcost</td>
<td>36</td>
<td>36</td>
<td>72</td>
</tr>
<tr>
<td>Highcost</td>
<td>36</td>
<td>36</td>
<td>72</td>
</tr>
<tr>
<td>Total</td>
<td>72</td>
<td>72</td>
<td>144</td>
</tr>
</tbody>
</table>

Following the completion of each round of each treatment, we provided the following information to the bidders: who entered the auction, the outcome of any auction (the winning bid was 0 if a single bidder entered the auction), who won the object, and the resulting private profits.

We conducted all sessions at a large northeast U.S. university in the spring of 2010. Subjects in all four treatments were students, mostly undergraduates, from a variety of majors. Before each session subjects were allowed a few minutes to read the instructions themselves. Following this, we read the instructions aloud and answered any questions. Each individual was recruited through an online recruitment system where cash was the only incentive offered. Subjects were paid a $5 show-up fee plus an additional amount that was based on their personal performance for all 30 rounds. Average compensation for the

\(^3\)Values were rounded to integer values.
participants, including the show-up fee, was $22. Each session lasted approximately 45 minutes and we programmed the software using the zTree system (Fischbacher 2007).

2.1 Predictions

Given our experimental parameters we begin by calculating the predicted seller revenue and bidder profits for both the auction and sequential mechanism. We refer the interested reader to Bulow and Klemperer (2009) for the details for the general theory with \( n \) bidders and an arbitrary value distribution.

Under both mechanisms, there are two potential bidders who must decide whether or not to pay a common cost \( c \) to learn their private valuations. Values are drawn independently from the continuous uniform distribution on 0 to 1.\(^4\)

The timing of decisions under the two selling mechanisms are different. Under the sequential mechanism, bidder 1 ‘arrives’ first and has the opportunity to pay the cost \( c \) to learn her value \((v_1)\) and then enter the auction. We will denote the possibly mixed strategy between entry or not by the first bidder with the probability of entry of \( \beta_1 \) and not entry \( 1 - \beta_1 \). Contingent upon entry, the first bidder learns her valuation and has the opportunity to place a preemptive bid that might depend upon her valuation and is denoted by \( p(v_1) \). Bidder 2 ‘arrives’ next and observes whether or not the first bidder entered and the preemptive bid. She then decides whether or not to enter and learn her valuation \((v_2)\). We denote the possibly mixed entry strategy of the second bidder by \( \beta_2(p) \), (enter) and \( 1 - \beta_2(p) \) (not enter).\(^5\) After the entry decision of the second bidder, the item is sold in an English auction with the starting price of either 0 (if the first bidder did not enter) or \( p \) if the first bidder entered. Assuming both bidders play the weakly dominant strategy of bidding up to their value in the auction, any auction with only one bidder will end at either 0 (in the event bidder 1 did not enter but bidder 2 did) or \( p \) (bidder 1 enters but bidder 2 does not). An auction with both bidders will proceed to the maximum of the second highest valuation of the two bidders \((\min\{v_1, v_2\})\) and the preemptive bid \( p \).

\(^4\)In the actual experiment, valuations were drawn uniformly on the integer valuations between 1 and 100. When comparing theoretical results with our experimental predictions, we simply divide the experimental results by 100. As is standard in experimental auction studies, we assume that the application of the continuous theory to a discrete implementation is sufficiently precise.

\(^5\)The entry strategy of the second bidder can depend upon the observed preemptive bid, the entry decision of the first bidder as well as the second bidder’s knowledge of the particular equilibrium being played (e.g. \( p(v) \) and \( \beta_1 \)). For notational clarity, we do not include the observed and equilibrium entry decisions of the first bidder. It is obvious that, contingent upon non-entry by the first bidder, the second bidder will have a dominant strategy to always enter for the parameter values of \( c \) in our experiment.
The auction mechanism is similar except that the pre-emptive bid opportunity is not available to the first bidder. Therefore, in effect, both bidders simultaneously decide whether or not to enter and, after learning their valuations, compete in an English auction. As before, the English auction will progress to a price of 0 (only one bidder entered) or to the second highest valuation of the two entering bidders.

We first examine the equilibrium in the auction mechanism since it is easily derived from well-known auction results. The following payoff table depicts each player’s (ex ante) expected profits from entry:

<table>
<thead>
<tr>
<th>Bidder 2</th>
<th>Enter</th>
<th>Not Enter</th>
</tr>
</thead>
<tbody>
<tr>
<td>Enter</td>
<td>$\frac{1}{6} - c$, $\frac{1}{6} - c$</td>
<td>$\frac{1}{2} - c$, 0</td>
</tr>
<tr>
<td>Not Enter</td>
<td>0, $\frac{1}{2} - c$</td>
<td>0, 0</td>
</tr>
</tbody>
</table>

As long as $c < \frac{1}{6}$, it is a dominant strategy for both bidders to enter resulting in expected bidder profits of $\frac{1}{6} - c$, and a seller expected revenue of $\frac{1}{3}$.

Now consider the equilibrium in the sequential mechanism. The preemptive bidding strategy of the first bidder is the crucial element of the sequential mechanism since it allows for the first bidder to transmit information about her valuation to the second bidder, which might induce the second bidder to not enter. Note that the auction mechanism outcome can always be replicated by the first bidder entering and following a ‘pooling’ preemptive bidding strategy of always bidding zero (e.g. $p(v_1) = 0$ for all $v_1 \in [0, 1]$). On the other hand, a completely revealing preemptive bid strategy (e.g. $p(v_1)$ is an increasing continuous function of $v_1$) is not tenable since low valuing first bidders would want to mimic high valuing bidders who can discourage competition from the second bidder; the second bidder would never enter if she knew the first bidder’s value was greater than $1 - \sqrt{2c}$. Therefore, the equilibrium preemptive bidding strategy is of a ‘partially pooling’ nature where low valuing bidders bid zero and all others bid a common preemptive bid. Bulow and Klemperer (2009) show that in equilibrium the first bidder sets the preemptive bid of 0 if her value is below the cut-off value $v_s$ (called the deterring value), and $p^*$ otherwise. The first bidder chooses $p^*$ in a way that makes the second bidder indifferent between not entering and paying $c$ to compete against a bidder whose value is above $v_s$. At the same time she selects $p^*$ that makes herself indifferent between competing in the auction against the second bidder whose value is uniformly distributed on $[0, 1]$ or winning the auction outright with the bid of $p^*$.
Formally, the equilibrium preemptive bid has the following form:

\[
p(v) = \begin{cases} 
0 & v < v_s \\
p^* & v \geq v_s
\end{cases}
\]  

(1)

where \( p^* \leq v_s \) to ensure individual rationality for the first bidder. Given this preemptive bid strategy, the second bidder can calculate her expected auction profits (denoted \( \pi_a^2 \)) for each preemptive bid:

\[
\pi_a^2(p) = \begin{cases} 
(v_s)^2 + \frac{1-v_s}{2} & p = 0 \\
\frac{(v_s)^2}{6} + \frac{1-v_s}{2} & p = p^*
\end{cases}
\]  

(2)

To make preemptive bidding worthwhile, the second bidder must be induced to not enter whenever \( p^* \) is observed or \( \beta_2(p^*) = 0 \). Also, note that the second bidder’s expected auction profits only depend upon the cutoff value \( v_s \) (since \( p^* \leq v_s \)). The first bidder’s (interim) auction profits contingent upon entry by the second bidder, however, does depend upon the chosen level of the preemptive bid and is given by:

\[
\pi_1^0(p, v_1, e_2) = \frac{v_1^2 - p^2}{2}.
\]  

(3)

The equilibrium is therefore found by selecting a \( v_s \) that makes the second bidder indifferent between entry and not, and a \( p^* \) that ensures that low valuing first bidders prefer a preemptive bid of 0 to \( p^* \). Since Equation 3 is an increasing function of \( v_1 \), this is found by finding the \( p^* \) such that \( \pi_1^0(0, p^*, e_2) = \pi_1^0(p^*, p^*, e_2) \). Given our parameterization, these values are given by:

\[
v_s = 1 - (6c)^{\frac{1}{2}}
\]  

(4)

\[
p^* = \frac{1}{2} - 3c
\]  

(5)

whenever \( c < \frac{1}{6} \). Given the enhanced profitability of this preemptive bidding strategy, the first bidder will always enter (\( \beta_1 = 1 \)).

Expected profits of both the bidders and the sellers can be calculated given the equilibrium and our parameterizations. Table 2 summarizes predicted seller revenue, bidder profits (net endowments), deterring values, and preemptive bids for our experimental parameters (since our bidders’ values were \( U[0, 100] \) theoretical predictions in the remainder of the paper are scaled by 100 relative to the analysis we presented above).

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\( ^6 \)More generally, the second bidder’s expected auction profits from competition against a bidder whose values lie uniformly in the sub-interval of the original distribution given by \( [v, \overline{v}] \) is \( \pi_a^2 = \frac{(v-\overline{v})^2}{6} + \frac{(1-v)(1-\overline{v})}{2} \).
Table 2: Experimental predictions.

<table>
<thead>
<tr>
<th>Treatment</th>
<th>Auction</th>
<th>Seqmech</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Seller Revenue</td>
<td>33.33</td>
</tr>
<tr>
<td></td>
<td>1st bidder Profit</td>
<td>13.67</td>
</tr>
<tr>
<td>Lowcost ((c = 3))</td>
<td>2nd bidder Profit</td>
<td>13.67</td>
</tr>
<tr>
<td></td>
<td>Deterring Value, (v_s)</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>Preemptive Bid, (p^*)</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>Seller Revenue</td>
<td>33.33</td>
</tr>
<tr>
<td></td>
<td>1st bidder Profit</td>
<td>6.67</td>
</tr>
<tr>
<td>Highcost ((c = 10))</td>
<td>2nd bidder Profit</td>
<td>6.67</td>
</tr>
<tr>
<td></td>
<td>Deterring Value, (v_s)</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>Preemptive Bid, (p^*)</td>
<td>-</td>
</tr>
</tbody>
</table>

Note that the Bulow and Klemperer (2009) theory predicts that the seller revenue is higher in the auction, and bidder expected profit (particularly bidder 1’s profit) is higher under the sequential mechanism. Furthermore, the difference in seller revenue between the two mechanisms should be increasing in \(c\); we intentionally selected the cost parameters such that the expected differences between the two mechanism was quite high in the Highcost treatment whereas the difference was smaller in the Lowcost treatment. Finally, the predicted deterring value and optimal preemptive bid are both decreasing in \(c\).

3. Results

Table 3 summarizes average seller revenue, bidder profits, preemptive bids, and entry rates in the experiment.

We can see from Table 3 that in both cost treatments, the sequential mechanism generates equal or higher revenue for the seller compared to the auction (in the Highcost treatment the difference is significant at 5%)\(^7\). This is counter to the predictions of the Bulow and Klemperer (2009) model, which predicts that the auction should generate higher seller revenues in both cost treatments.

Comparing Tables 2 and 3 we can also see that the auction, particularly in the Highcost

\(^{7}\)All statistical comparisons we report in this section are done using a Mann-Whitney test and using cohort averages as the unit of analysis.
Table 3: Summary of the data (standard errors in parenthesis).

<table>
<thead>
<tr>
<th>Treatment</th>
<th>Auction</th>
<th>Seqmech</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lowcost ($c = 3$)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Seller Revenue</td>
<td>32.12 (0.66)</td>
<td>34.64 (1.62)</td>
</tr>
<tr>
<td>1st bidder Profit</td>
<td>15.58 (1.54)</td>
<td>11.55 (1.64)</td>
</tr>
<tr>
<td>2nd bidder Profit</td>
<td>12.99 (1.34)</td>
<td>12.59 (2.05)</td>
</tr>
<tr>
<td>Preemptive Bid</td>
<td>7.26 (1.22)</td>
<td></td>
</tr>
<tr>
<td>1st bidder Entry Proportion</td>
<td>0.967 (0.015)</td>
<td>0.983 (0.011)</td>
</tr>
<tr>
<td>2nd bidder Entry Proportion</td>
<td>0.953 (0.023)</td>
<td>0.965 (0.012)</td>
</tr>
<tr>
<td>Highcost ($c = 10$)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Seller Revenue</td>
<td>27.13 (0.87)</td>
<td>31.37 (1.79)</td>
</tr>
<tr>
<td>1st bidder Profit</td>
<td>7.43 (1.98)</td>
<td>6.42 (1.91)</td>
</tr>
<tr>
<td>2nd bidder Profit</td>
<td>9.54 (1.37)</td>
<td>8.35 (1.36)</td>
</tr>
<tr>
<td>Preemptive Bid</td>
<td>9.70 (1.37)</td>
<td></td>
</tr>
<tr>
<td>1st bidder Entry Proportion</td>
<td>0.857 (0.031)</td>
<td>0.939 (0.022)</td>
</tr>
<tr>
<td>2nd bidder Entry Proportion</td>
<td>0.933 (0.029)</td>
<td>0.850 (0.051)</td>
</tr>
</tbody>
</table>

treatment, generates seller revenue that is slightly below theoretical predictions (the difference in the Highcost treatment is significant at 5%). In our data we see that bidders in the auction do play the dominant strategy of bidding up to their values, so lower than predicted auction revenues are due to entry behavior. Bidders enter the auction only 96.02% of the time in the Lowcost treatment, and 89.54% of the time in the Highcost treatment, and entry rates do not increase over time.\(^8\) Lower than 100% entry rates account for the auction’s revenues being slightly below the predicted values, and suggest that bidders respond to the magnitude of the entry cost when making their entry decisions. We will explore alternative models for this entry behavior in later sections.

Turning to the bidders’ profits, 1st bidders should fare better in the sequential mechanism than in the auction, as it provides them with the opportunity to set a preemptive jump bid, potentially deterring bidder 2 from entering the auction. On the other hand, if bidder 1 acts optimally, bidder 2 earns the same average profits under the two mechanisms.

In the auction, both 1st and 2nd bidder profits are largely in line with the predicted values (there is a slight increase in bidder profits in our data due to the entry decisions mentioned previously, however this does not cause any of these differences to be statistically significant).

\(^8\)We ran a logit regression (with random effects) of entry on decision period and find that entry rates for bidders did not change over time–subjects did not learn to enter more.
significant). In the sequential mechanism, 2nd bidder profits are also not statistically different from theoretical predictions. But 1st bidder profits in the sequential mechanism are far below theoretical predictions. This last finding also results in first bidders’ average profits being roughly the same between the auction and the sequential mechanism.

Thus far we have shown that the sequential mechanism sometimes results in higher seller profit than an auction, which is contrary to the Bulow and Klemperer (2009) model. This higher profit for the seller is achieved primarily at the expense of bidder 1. Next, we examine both bidders’ decisions in more detail to better understand the causes of these findings.

![Figure 1: Proportion of preemptive bids equalling zero in our data (left panel) and in theory (right panel).](image)

We begin by examining how 1st bidders set preemptive bids. In the Bulow and Klemperer (2009) model, 1st bidders follow a threshold strategy, shown in the right panel of Figure 1: 1st bidders should set the preemptive bid to 0 if their value is below $v_s$, and when their value is above $v_s$ they should set the preemptive bid to $p^*$. The left panel of Figure 1 shows the proportion of preemptive bids set to zero in our data, as a function of value.\(^9\) It is clear from Figure 1 that bidders do not follow the threshold strategy, but instead, their probability of setting a preemptive bid of zero decreases in value up to some point, and then levels off. The point up to which the probability of a zero preemptive bid decreases in value is clearly higher when $c = 3$ than when $c = 10$, which is the opposite to the theoretical predictions.

Next we examine the magnitude of preemptive bids. The preemptive bid should follow a threshold strategy shown on the right panel of Figure 2, and specifically, preemptive bids should be constant when $v \geq v_s$, and positive preemptive bids should be different for the two cost conditions. But in our data, summarized in the left panel of Figure 2, we see that

\(^9\)For all figures, we removed any observation where bidder 1 entered and set a preemptive bid equal to or above their cost, this occurred 19 out of all 1080 decision by first bidders.
the magnitude of positive preemptive bids increases in $v$ linearly, and moreover, there is no discernible difference in the two cost conditions.\footnote{We confirmed this formally with a random effect regression: the coefficient on $v$ is positive and significant, the coefficient on $HIGHCOST$ is not significant, and neither is the coefficient on the interaction variable $v \times HIGHCOST$.}

To summarize our conclusions about the behavior of bidder 1: 1st bidders enter the auctions less than 100\% of the time. When they do enter, they do not follow the threshold strategy in regards to either their decision to place a positive preemptive bid, or to the magnitude of the preemptive bid. For low $v$’s, 1st bidders’ probability of entering a positive preemptive bid increases in value, and for high $v$’s these probabilities reach a constant level, that is significantly below 100\%. So 1st bidders are more likely to place preemptive bids when their $v$’s are low, and are not likely enough to place positive preemptive bids when their $v$’s are high. The size of the preemptive bid itself increases in $v$ and does not depend on the entry cost. Overall, bidder 1’s behavior does not resemble the Bulow and Klemperer (2009) model.

Moving on to bidder 2’s behavior, as shown in the right panel of Figure 3, 2nd bidders should always enter as long as the preemptive bid is below $p^*$, and should never enter as long as the preemptive bid is above $p^*$. The critical difference between how our 2nd bidders enter and how the Bulow and Klemperer (2009) model says they should enter, is that they enter too much following high preemptive bids. Specifically, 2nd bidders in the $c = 10$ condition should never enter when preemptive bids are above 20. However, as we can see from the left panel of Figure 3, 2nd bidders still enter 50\% to 100\% of the time following preemptive bids of 60. Our 2nd bidders also sometimes fail to enter for low preemptive bids, but this effect

Figure 2: The magnitude of positive preemptive bids in our data (left panel) and in theory (right panel).
is not very large.

Figure 3: Entry proportion of 2nd bidder in response to the preemptive bid in our data (left panel) and in theory (right panel).

In sum, our data suggest that the sequential mechanism generates the same or higher revenue to sellers when compared to the auction, and roughly the same profits to bidders. These results stem from three primary behavioral findings: (1) in the auction, subjects do not enter quite enough, especially when entry costs are high, (2) in the sequential mechanisms, 1st bidders do not follow the threshold strategy in setting preemptive bids, and as a result they end up not setting preemptive bids frequently enough, and when they do set them, the size of the preemptive bid is positively correlated with bidder 1’s value; and (3) in the sequential mechanism, 2nd bidders enter even when 1st bidders set high preemptive bids. In the next section we calculate the relative effect of these three behavioral deviations from theory on the resulting seller’s profit.

3.1 Revenue Difference Analysis

Thus far our results suggest that both bidders’ behavior contributes to the revenue of the sequential mechanism exceeding that of the auction. However, which bidder’s behavior, the first or the second, accounts for most of the difference? To answer this question we take the observed revenue for the sequential mechanism for each decision, compare it to the theoretical prediction for that decision, and partition it between the two bidders.

The simplest way to understand this approach is by example. Suppose bidder 1’s value exceeds the deterring value. In theory, bidder 1 should set a preemptive bid equal to \( p^* \), and bidder 2 should be deterred from entering. As such, the revenue from this example, in theory, is \( p^* \). However, in our data 1st bidders often set the preemptive bid above \( p^* \), additionally, bidder 2 frequently entered the auction despite the preemptive bid being equal to or above \( p^* \). Let \( y \) be the final observed revenue for a given decision in our data. In this example, the actual revenue deviates from the theoretical prediction by \( (y - p^*) \). We can partition this deviation in revenue, \( (y - p^*) \), between the two bidders as follows; \( (p - p^*) \) gets attributed to bidder 1, since she set a preemptive bid \( p \) above \( p^* \), and the remaining \( (y - p) \) gets attributed to bidder 2, since her entry decision drove the revenue above \( p \).

We applied this approach to revenues in each round of both Seqmech treatments and identified which bidder’s behavior contributed most to the deviation in revenue in the sequential
mechanism. The results are illustrated in Table 4.

Table 4: 1st and 2nd bidder’s impact on the deviation in revenue for the sequential mechanism (the predicted revenue uses the actual bidder values drawn in the experiment).

<table>
<thead>
<tr>
<th></th>
<th>Lowcost ((c=3))</th>
<th>Highcost ((c=10))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Predicted Revenue</td>
<td>30.03</td>
<td>16.94</td>
</tr>
<tr>
<td>1st bidder Impact</td>
<td>-1.31</td>
<td>1.33</td>
</tr>
<tr>
<td>2nd bidder Impact</td>
<td>5.92</td>
<td>13.10</td>
</tr>
<tr>
<td>Observed Revenue</td>
<td>34.64</td>
<td>31.37</td>
</tr>
</tbody>
</table>

As one can see in Table 4, bidder 1’s behavior in both cost treatments did not contribute substantially to the revenue result in the Seqmech treatments (slightly negative in the Lowcost treatment, and slightly positive in the Highcost treatment, both close to zero). Therefore, when one considers that bidder 1’s decisions did not alter the revenue significantly, and that both bidder’s bidding decisions were close to optimal, it is clear that bidder 2’s entry behavior is what generates the dramatic increase in the final revenue for both cost conditions of the Seqmech treatments. Specifically, 2nd bidder’s behavior contributed to an increase of 5.92 in revenue, on average, in the Lowcost treatment, and 13.10, on average, in the Highcost treatment. In the next section we develop a model that captures the main feature of our data by allowing the second bidder’s entry decision to be noisy.

4. **A Theory of Noisy Bidder Entry**

In their theory, Bulow and Klemperer (2009) assume that bidders perfectly understand and calculate the payoffs associated with the particular perfect Bayesian signaling equilibrium being played. There are a number of reasons why these assumptions might not be realistic. First, there are actually many potential perfect Bayesian equilibria of which the utilized equilibrium is selected because it satisfies a particular (although intuitive) selection principle. Second, real bidders might be prone to errors in calculations regarding the expected payoffs from various entry and bidding decisions in the two mechanisms. For example, is it really reasonable to expect that a human bidder precisely calculates the expected profits associated with auction entry? Third, real bidders might also be prone to ‘behavioral’ biases whereby
they systematically perceive differences in payoffs from actions in the mechanisms that are not explained completely by the assumption of expected-utility maximizing behavior. For example, previous experimental work on bidding behavior in auctions demonstrated that behavioral biases such as joy of winning (Greenleaf 2004), anticipated regret (Engelbrecht-Wiggans and Katok 2008) fit the data well.

Since the seller expected revenue comparisons are generated given the assumption of a particular equilibrium being played. If, for whatever reason, real bidders fail to adhere to this equilibrium, it is possible that the revenue comparisons between the auction and the sequential mechanism will be impacted. To this end, we develop a model of noisy bidder entry decisions. In particular, we assume that in addition to paying a cost $c$ to enter and learn their values, each bidder ($i$) perceives an additional benefit/cost of $\epsilon_i$ for entry into the mechanism where $\epsilon_i$ is known by the bidder at the time of entry but is not known by the other bidder. We assume that $\epsilon_i$ is drawn independently from the normal distribution $N(\mu, \sigma)$.\textsuperscript{11} As is the case of the entry cost $c$, we assume the additional cost factor $\epsilon_i$ to be sunk at the time of entry decision so that it does not directly impact future decisions such as auction bidding strategies (for both bidders) or preemptive bidding strategies (for bidder 1).\textsuperscript{12}

There are a number of justifications for the inclusion of such a term. First, if bidders make calculation mistakes regarding the profitability of various auctions’ results, then it would be reasonable to assume that a mean zero error term ($\mu = 0$) might capture the impact of those errors on the equilibrium. Or, these errors might be generated as some type of idiosyncratic cost or benefit element. In the lab, for example, some subjects might view competition itself as mentally (and emotionally) taxing whereas other subjects might like the challenge and associate added benefits of competition. In practice, we feel it is reasonable that in high value auctions of the type where this model is probably most appropriate (such as mergers and acquisitions and procurement settings) that, in addition to the commonly known cost element, there might be idiosyncratic cost or benefit elements that are not known by the other participants. For example, a firm considering bidding on a procurement contracts might decide not to spend the considerable effort required to put together a cost estimate

\textsuperscript{11} Essentially equivalent results can be generated by assuming $\epsilon$ to be distributed logistically in which case choices probabilities are generated similarly to the familiar quantal response equilibrium probabilities of McKelvey and Palfrey (1995) and McKelvey and Palfrey (1998).

\textsuperscript{12} The addition of this noise term, however, does dramatically impact the admissibility of certain types of equilibria as we will show shortly.
(e.g. decided not to enter) because of internal issues within the firm. Lastly, modeling noise as a random cost or benefit element may serve as a useful approximation to capture other behavioral issues, such as regret or social preferences.\footnote{Indeed, exploratory attempts to more formally model these behaviors have yielded largely consistent results.}

Since the bidders in the sequential mechanism are asymmetric, it may well be reasonable for the noise terms of the two bidders to come from different distributions. One may argue that the first bidder’s entry decision is simpler than the second bidder’s, because he does not have to think about the pre-emptive bid, so it may well be that the first bidder’s term comes from a distribution with $\mu$ close to zero, and a small $\sigma$. In contrast, the second bidder must interpret the first bidder’s pre-emptive bid, and this complexity may well cause the second bidder’s $\sigma$ to be large. Knowing that failure to enter is sure to result in first bidder earning higher profit may trigger some inequality aversion, and $\mu > 0$ may be a reasonable approximation for modeling it.\footnote{Modeling social preferences is beyond the scope of this paper.}

4.1 The Model

To see the potential for such behavior to impact the revenue performance of the sequential mechanism, let us consider a model in which bidders 1 and 2 have (possibly) different noise parameters $\epsilon_1$ and $\epsilon_2$. First, let us examine the second bidder’s behavior. Since the payoff from non-entry is 0, bidder two will decide to enter (contingent upon an observed preemptive bid $p$ and entry by bidder 1) only if

$$\pi_2^a(p) - c + \epsilon_2 \geq 0.$$ 

This, of course, means that bidder two will only enter if

$$\epsilon_2 \geq c - \pi_2^a(p).$$

The (ex ante) entry probability for bidder two which is given by

$$\beta_2(p) = \Pr(\epsilon_2 \geq c - \pi_2^a(p))$$

$$= 1 - \Pr(\epsilon_2 \leq c - \pi_2^a(p))$$

$$= 1 - \Phi\left( \frac{[c - \pi_2^a(p)] - \mu}{\sigma} \right) \quad (6)$$

where $\Phi(\cdot)$ is the cdf of the standard normal distribution.
The first bidder faces a similarly drawn cost $\epsilon_1$ when making her entry decision. Having entered, the first bidder must then decide on an optimal preemptive bid function given that she knows that the second bidder will enter in a noisy fashion described by Equation 6. We also assume that, as in the original equilibrium notion, the second bidder is fully aware of the selected equilibrium and therefore makes the correct inferences regarding bidder one’s valuations from observed preemptive bids. While this may not be particularly realistic, we decided to allow for only one type of variation from equilibrium in order to understand the impact of such a model rather than potentially confounding the impact via the inclusion of another modeling assumption. We also do not consider the option of noisy bidding in the final auction stage or in the preemptive bidding decision by the first bidder for two reasons. First, as the experimental data indicates, bidding behavior in the final auction stage is largely consistent with theoretical predictions. Second, allowing for noisy behavior in the preemptive bid decision greatly complicates the analytical tractability of the theoretical model.

Let us first consider the impact of noisy behavior on the equilibrium entry decisions of both players in the auction. Because both players are now entering with less than probability one, each bidder must consider the fact that they may be the sole entrant into the auction and, therefore, obtain a greater profitability.

**Proposition 1** A symmetric equilibrium in the auction will be given by the entry probability $\beta^* = \beta_1 = \beta_2$ that is the solution to the following equation:

\[ 1 - \beta^* = \Phi \left( \frac{\frac{1}{3} \beta^* + c - \frac{1}{2} - \mu}{\sigma} \right) \]  

(7)

where each bidder enters if $\epsilon_i \geq \frac{1}{3} \beta^* + c - \frac{1}{2}$.

**Proof:** Given the entry probability of the other bidder $\beta_j$, bidder $i$’s expected payoff from entry into the auction is

\[ \pi_i(e_i) = \beta_j \frac{1}{6} + (1 - \beta_j) \frac{1}{2} - c + \epsilon_i \]  

(8)

\[ = \frac{1}{2} - \beta_j \frac{1}{3} - c + \epsilon_i \]  

(9)

where the first term in equation 8 is the expected (ex ante) profits to a bidder is a two-person auction and the second term is the expected profits in the event of non-entry by the other bidder so that the auction price is zero. Since the payoff from non-entry is zero, bidder $i$ will enter only if

\[ \frac{1}{2} - \beta_j \frac{1}{3} - c + \epsilon_i \geq 0 \]
or
\[ \epsilon_i \geq \beta_j \frac{1}{3} + c - \frac{1}{2} \]
which results in the following entry probability
\[ \beta_i = 1 - \Phi \left( \frac{\frac{1}{3} \beta_j + c - \frac{1}{2} - \mu}{\sigma} \right). \] (10)
Since equation 10 must hold for both bidder's in equilibrium, we have the equilibrium condition of the proposition.

Now, consider the sequential mechanism. The key strategic variable is now the preemptive bid. Since the choices of the second bidder depends upon the information inferred from the observed preemptive bid given a particular bid strategy, we have to consider the possibility of different types of preemptive bidding strategies and the information that they reveal to the second bidder. In particular, consider the following types of strategies:

1. **pooling**: Valuation type is not refined by observed preemptive bid. The only feasible (individually rational) bid of this type is \( p(v_1) = 0 \) for all \( v_1 \).

2. **partially pooling**: Preemptive bidding reveals sub-intervals of valuations as in the equilibrium under the standard theory (Equation 1).

3. **revealing**: The preemptive bid function is an increasing continuous function of valuation and therefore reveals (via inversion) bidder one’s valuation exactly.

4. **partially pooling revealing**: For low values, the bidders pool on a bid of 0 whereas for values above some cutoff the bid is revealing of bidder one’s valuation.

The following proposition shows that the only possible equilibrium must involve complete revelation of valuations.

**Proposition 2** Let \( \epsilon_i \sim N(\mu, \sigma) \) for \( i = 1, 2 \). If there is a pure strategy equilibrium, then it must involve a fully revealing preemptive bid function.

**Proof**: Suppose not. Then, if there exists an equilibrium it must be one of the three other types. We will rule out each type in turn.

---

\(^{15}\)There might in principle be many sub-intervals revealed in the strategy. The proposition below is sufficiently general to rule out such equilibria.
First, consider any partially pooling or pooling equilibrium. Let \( v_s \geq 0 \) be the cutoff for bidder’s to place a preemptive bid and \( p \leq v_s \) be the preemptive bid amount. Note that a fully pooling equilibrium is a simply a special case of the partially pooling equilibrium where \( v_s = p = 0 \).\(^{16}\) The expected profits of a first bidder with a value of \( v_1 = 1 \) is then given by

\[
\pi_1(p, 1) = \frac{1 - p^2}{2} + \Phi \left( \frac{c - \pi_2^a(p) - \mu}{\sigma} \right) \left( 1 - \frac{1 - p^2}{2} \right)
\]

where \( \pi_2^a(p) = \frac{(1 - v_s)^2}{6} > 0 \). Since this is assumed to be an equilibrium, a first bidder with a value of \( v_1 = 1 \) must prefer not to place a preemptive bid of \( p + \alpha \) where \( \alpha > 0 \) where this bid, assuming the equilibrium behavior of the other bidders, would reveal the first bidder’s value. This implies that, upon observing \( p + \alpha \) the second bidder would know that they are facing a first bidder with a value of 1, which implies \( \pi_2^a(p + \alpha) = 0 \). The first bidder’s expected profits from this deviation is then given by

\[
\pi_1(p + \alpha, 1) = \frac{1 - (p + \alpha)^2}{2} + \Phi \left( \frac{c - \mu}{\sigma} \right) \left( 1 - \frac{1 - (p + \alpha)^2}{2} \right). \quad (11)
\]

The assumption of equilibrium implies that \( \pi_1(p, 1) \geq \pi_1(p + \alpha, 1) \), however, letting \( \alpha \to 0 \) for equation 11 we arrive at

\[
\Phi \left( \frac{c - \pi_2^a(p) - \mu}{\sigma} \right) \geq \Phi \left( \frac{c - \mu}{\sigma} \right)
\]

which is a contradiction since \( \Phi(\cdot) \) is a distribution.

Second, consider a partially pooling revealing equilibrium. In this equilibrium there must exist some value \( \bar{v} \) for the first bidder such for all values below \( \bar{v} \) bidders prefer to pool by placing a zero bid and for all values greater than \( \bar{v} \) all bidders prefer to follow some preemptive bid function \( p(v) \) that perfectly reveals the bidder’s value and \( p(\bar{v}) = 0 \). Consider the first bidder with a valuation of \( v_1 = \bar{v} \). Then, in equilibrium, she must be indifferent between pooling with the low valuing bidders at a bid of zero and revealing with the high valuing bidders also with a bid of zero. The expected payoff from pooling on zero is given by:

\[
\pi_1(0, \bar{v}) = \frac{\bar{v}^2}{2} + \Phi \left( \frac{c - \pi_2^a(0) - \mu}{\sigma} \right) \left( \bar{v} - \frac{\bar{v}^2}{2} \right) \quad (12)
\]

where

\[
\pi_2^a(0) = \frac{\bar{v}^2}{6} + \left( \frac{1}{2} \right) (1 - \bar{v})
\]

\(^{16}\)This proof can also easily be extended to consider any form of general partial pooling equilibrium with many potential preemptive bid levels as long as the top valuing first bidder is pooled with lower valuing first bidders.
The expected payoff from revealing at zero is given by:

\[
\hat{\pi}_1(0, v) = \frac{v^2}{6} + \Phi \left( \frac{c - \hat{\pi}_2^a(0) - \mu}{\sigma} \right) \left( v - \frac{v^2}{2} \right)
\]  

(13)

where

\[
\hat{\pi}_2^a(0) = \left( 1 - \frac{v}{2} \right)(1 - v).
\]

Since in equilibrium it must be that \( \pi_2^a(0) = \hat{\pi}_1(0, v) \), it follows that the only \( v \) for which this is true is \( v = 0 \) which is a contradiction with the assumption that this equilibrium is partially revealing (e.g. it would be completely revealing).

Note that this is in contrast to the result of the standard theory where there exists many partially pooling equilibria and Bulow and Klemperer (2009) identify the equilibrium that maximizes bidder profits. The reason that such an equilibrium fails to exist in our setting is that increases in the preemptive bid by the first bidder will always have a measurable impact on the likelihood of entry by the second bidder. This provides sufficient incentive for high valuing first bidders to attempt to differentiate themselves by placing a higher preemptive bid. In contrast, under the standard theory, any increase of bid beyond the one specified in the equilibrium will only have a negative impact for first bidders since the second bidder is already not entering for sure so a higher preemptive bid only increases the price that the first bidder will pay.

The earlier proposition, however, does not prove that a revealing (pure strategy) equilibrium exists. It is possible, that an equilibrium must involve mixing of preemptive bids by the first bidder. Therefore, we proceed by characterizing the necessary conditions for a revealing equilibrium with noisy entry decisions.

Let us suppose there exists a revealing preemptive bid function \( p(v_1) \) with \( p(v_1) \leq v_1 \) for all \( v_1 \). Suppose that the bid function is differentiable and increasing everywhere so that \( p'(v_1) > 0 \). The boundary condition is that \( p(0) = 0 \). Let \( v^{-1}(p) \) be the inverse preemptive bid function. Then, bidder two’s expected profit from the auction is given by:

\[
\pi_2^a(p) = \frac{(1 - v^{-1}(p))^2}{2}.
\]

This is simply the expected value of \( v_2 - v_1 \) conditional on \( v_2 \geq v_1 \). The ex ante entry probability of the second bidder is given by equation 6 with this expected profit term substituted into the equation. Bidder one’s expected profit from the auction contingent upon entry by

\footnote{Proposition 4.1 rules out the possibility of a non-zero boundary value.}
the second bidder is still given by equation 3. The first bidder’s expected profits from a particular preemptive bid level is therefore given by:

\[
\pi_1(p, v_1) = \beta_2(p)\pi_1^a(p, v_1, e_2) + (1 - \beta_2(p))(v_1 - p)
\]

\[
= \pi_1^a(p, v_1, e_2) + \Phi \left( \frac{[c - \pi_2(p)] - \mu}{\sigma} \right) [v_1 - p - \pi_1^a(p, v_1, e_2)]
\] (15)

In order for the preemptive bid strategy to be an equilibrium it must be that the prescribed preemptive bid maximizing expected profits for a first bidder with that valuation or the necessary first order condition is given by:

\[
\frac{\partial \pi_1(p, v_1)}{\partial p} = 0
\] (16)

\[
\frac{\partial \pi_1^a(p, v_1, e_2)}{\partial p} - \phi(\gamma(p)) \frac{\partial \pi_2(p)}{\partial p} [v_1 - p - \pi_1^a(p, v_1, e_2)] - \Phi(\gamma(p)) \left( 1 + \frac{\partial \pi_1^a(p, v_1, e_2)}{\partial p} \right) = 0
\] (17)

where \(\gamma(p) = \frac{[c - \pi_2(p)] - \mu}{\sigma}\). Since,

\[
\frac{\partial \pi_2^a(p)}{\partial p} = -(1 - v^{-1}(p)) \frac{\partial v^{-1}(p)}{\partial p}
\]

and

\[
\frac{\partial \pi_1^a(p, v_1, e_2)}{\partial p} = -p
\]

equation 17 can be rewritten as follows:

\[
-p + \phi(\gamma(p)) \left( 1 - v^{-1}(p) \right) \frac{\partial v^{-1}(p)}{\partial p} [v_1 - p - \pi_1^a(p, v_1, e_2)] - \Phi(\gamma(p)) (1 - p) = 0.
\]

If this is in equilibrium, then is must be that \(v^{-1}(p) = v_1\) and utilizing the fact that \(\frac{\partial v^{-1}(p)}{\partial p} = \frac{1}{p'(v_1)}\), we have that

\[
-p + \phi(\gamma(p)) \frac{1}{p'(v_1)} \frac{1}{\sigma} (1 - v_1) [v_1 - p - \pi_1^a(p, v_1, e_2)] - \Phi(\gamma(p)) (1 - p) = 0.
\]

Solving for \(p'(v_1)\) we arrive at the following differential equation:

\[
p'(v_1) = \phi(\gamma(p(v_1))) \frac{1}{\sigma} (1 - v_1) \left[ (v_1 - p(v_1)) \left( 1 - \frac{v_1 + p(v_1)}{2} \right) \right].
\] (18)

\[18\]Note that the \(c\) and \(\epsilon_1\) terms are dropped from these equations since they are sunk at the time of preemptive bid decision making. This is primarily done for notational simplicity when considering the first bidder entry decision.
While this differential equation does not readily admit an analytic solution, it can be easily solved for numerically. Let $p^*(v_1)$ be the solution to the differential equation 18. Given this solution, we can now move to the earlier stage, where bidder one makes her entry decision. In order to treat both players symmetrically, we assume this decision to be noisy as well. Therefore, the first bidder’s expected payoff from entry ($e_1$) is given by

$$\pi_1(e_1) = \int_{-\infty}^{\infty} \pi_1(p^*(v_1), v_1) f(v_1) dv_1 - c + \epsilon_1$$

where $\pi_1(p, v)$ is given by equation 15. Using the fact that values are distributed uniformly on the $[0, 1]$ interval we have that

$$\pi_1(e_1) = \int_{0}^{1} \pi_1(p^*(v_1), v_1) dv_1 - c + \epsilon_1.$$  

Since the payoff from non-entry is 0, bidder one will decide to enter only if

$$\int_{0}^{1} \pi_1(p^*(v_1), v_1) dv_1 - c + \epsilon_1 \geq 0.$$  

This, of course, means that bidder one will only enter if

$$\epsilon_1 \geq c - \int_{0}^{1} \pi_1(p^*(v_1), v_1) dv_1.$$  

The (ex ante) entry probability for bidder one then is given by

$$\beta_1 = \Pr(\epsilon_1 \geq c - \int_{0}^{1} \pi_1(p^*(v_1), v_1) dv_1)$$

$$= 1 - \Pr(\epsilon_1 \leq c - \int_{0}^{1} \pi_1(p^*(v_1), v_1) dv_1)$$

$$= 1 - \Phi \left( \frac{[c - \int_{0}^{1} \pi_1(p^*(v_1), v_1) dv_1] - \mu}{\sigma} \right)$$  (19)

The entry probabilities of the two bidders $\beta_1$ (Equation 19) and $\beta_2$ (Equation 6) given the preemptive bid $p^*(v_1)$ characterize the equilibrium under noisy entry decision and can be easily utilized to calculate expected revenue and profit results for the seller and both bidders. Next, we proceed by using maximum likelihood estimation to identify parameters (distributions of $\epsilon_i$) that best fit the observed experimental data.

### 4.2 Parameter Estimation

In this section we estimate the parameters that define the distribution of $\epsilon$, $(\mu, \sigma)$, that best fit our experimental data. We use maximum likelihood estimation (MLE) technique for this
purpose. We take a progressive approach to the estimation process. We first fit a common set of $(\mu, \sigma)$ across both institutions, the auction and the sequential mechanism, and then we allow $(\mu, \sigma)$ to vary between the auction and the sequential mechanism.

In this section we estimate the model, which assumes that bidder 1 is strategic with respect to bidder 2’s behavior. Therefore, the joint likelihood function incorporates the entry rates of both bidders:

$$L(\mu, \sigma) = \prod_{i \in I} \left( \beta_{E_i} E_2^i (1 - \beta_2)^{(1-E_2^i)} \right) \left( \beta_{E_i} E_1^i (1 - \beta_1)^{(1-E_1^i)} \right)$$

where $EK_i = \begin{cases} 
1 & \text{If bidder } K \text{ enters in decision } i \\
0 & \text{Otherwise}
\end{cases}$.

Table 5 provides the MLEs, LLs, and BIC values for the full model’s estimation when we restrict $\mu$ and $\sigma$, and when we allow them to vary across institutions and bidders:

<table>
<thead>
<tr>
<th>Treatment</th>
<th>MLEs ($\mu, \sigma$)</th>
<th>LL</th>
<th>BIC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Both Institutions</td>
<td>(0.49, 0.40)</td>
<td>-1,059</td>
<td>2,135</td>
</tr>
<tr>
<td>Auction</td>
<td>(0.02, 0.10)</td>
<td>-543</td>
<td>2,131</td>
</tr>
<tr>
<td>Seqmech</td>
<td>(0.57, 0.44)</td>
<td>-506</td>
<td>2,120</td>
</tr>
<tr>
<td>Auction</td>
<td>(0.02, 0.10)</td>
<td>-543</td>
<td>2,120</td>
</tr>
<tr>
<td>Seqmech - bidder 1</td>
<td>(0.07, 0.10)</td>
<td>-170</td>
<td>2,120</td>
</tr>
<tr>
<td>Seqmech - bidder 2</td>
<td>(0.70, 0.59)</td>
<td>-322</td>
<td></td>
</tr>
</tbody>
</table>

As one can see from Table 5, the model allowing parameters to vary between bidders is the preferred estimation (based on the smallest BIC value). For this model, starting from the bottom, $\mu$ and $\sigma$ are quite large for bidder 2, agreeing with our data that second bidders overenter with considerable noise. The first bidder, however, has a relatively low $\mu$ and $\sigma$, indicating a smaller benefit of entry along with less variability. This too agrees with our data, where bidder 1 consistently entered (and standard theory assumes an entry rate of 1). The MLEs for the auction suggest that there is almost no additional fixed benefit of entry, $\mu = 0.02$, with a $\sigma = 0.10$ accounting for the noise we see, pushing entry rates slightly below 1. In short, the MLE results on a whole agree with our experimental results.
We show the predicted seller revenue levels based on the MLEs that vary between bidders for the full model in Table 6. We see that the predicted revenues are close to the actual revenues we observe in our experiment, both in terms of the point predictions, and in terms of the qualitative comparison between the two institutions. In line with our data and contrary to the Bulow and Klemperer (2009) theory, the model predicts higher revenues under the sequential mechanism than under the auction.

### 5. Conclusion

Our paper is the first one to compare the performance of an auction and a sequential mechanism in a controlled laboratory setting. We design our experiments to closely match the setting in the Bulow and Klemperer (2009) model. We find that the average seller revenue in the auction is slightly lower than what the theory predicts, and the average sequential mechanism revenue is significantly higher, especially in the treatment with high entry costs.

These experiments demonstrated that individual behavior can vary significantly from the strong predictions of standard game theory. While individual variations from theoretical predictions are certainly not surprising, we demonstrate that those variations are sufficient to reverse the normative prescriptions of the theory.

The behavior we observe in our experiment differs from the model predictions in three ways. First, bidders do not enter the auction 100% of the time, causing auction revenues to be somewhat lower than predicted by the model. Second, in the sequential mechanism, we find that the first bidders do not set pre-emptive bids according to the threshold strategy. Instead, both the probabilities of setting positive preemptive bids, and the magnitudes of these bids increase with the first bidders’ values. Third, second bidders in the sequential
mechanism tend to over-enter in response to high preemptive first bidder bids. We find that it is the second bidder’s over-entry that accounts for most of the difference between the sequential mechanism revenue we observe, and the revenue predicted by the Bulow and Klemperer (2009) model. We develop a new model that incorporates the noisy entry behavior and use MLE techniques to estimate model parameters for our data. We find that the model organizes our data reasonably well, in that it matches revenues fairly closely. While our model is quite consistent with the data, we recognize that there might be other behavioral factors that we have not accounted for (such as limited rationality regarding the informational content of the preemptive bid) that might also be playing a role in the experiments. Rather, our results are a warning that a mechanism designer might want to consider the robustness of their results to many possible behavioral phenomena. The formal incorporation of non-standard behavior into the design and selection of mechanism is, in our opinion, an exciting and challenging avenue for future research.

A normative implication of our study is that sequential mechanisms may well represent a viable alternative to auctions for a variety of applications. Not only do bidders prefer them, but they are actually better for the bid takers as well. In a setting with costly entry, sequential mechanisms are also more efficient than auctions, because fewer potential bidders end up paying the entry fees unnecessarily. Thus, further theoretical and empirical work is called for to better understand sequential mechanisms.

References


A. Sample Instructions

You are about to participate in a decision-making experiment. If you follow these instructions carefully you can earn a considerable amount of money. Your earnings depend on your decisions and the decisions of other participants. The experiment will last for 30 decision periods. You are NOT allowed to communicate with the other participants, or use any type of electronic device (i.e. cellphones, texting devices, iPods) during the session. If you have any questions, raise your hand and I will come to you.

Game flow

In this experiment, you will play the role of either buyer A or buyer B of some item. Your role will stay the same for the entire session. At the beginning of each round you will be provided with an initial endowment of 20. To buy the item, you must first decide whether or not you want to enter an auction to compete for the item. The cost to enter the auction is 3 in every round.

If you choose not to enter the auction, your earnings for the round are your initial endowment of 20.

If you choose to enter the auction, you will first pay the entry fee of 3 and then be shown your value for the item. Your value will be a random integer between 1 and 100 each round, each integer in that range equally likely.

Each round starts with buyer A making a decision of whether to enter the auction. If buyer A decides to enter the auction, she will then have the opportunity to set the initial bid. This bid will determine the starting point of the bid clock for the auction if buyer B decides to enter the auction, or, it will determine the price buyer A pays for the item if buyer B decides to not enter the auction. Once buyer A has made these decisions, buyer B will then be shown whether buyer A entered the auction, and, if applicable, what buyer A set as the starting bid for the auction. If buyer A does not enter the auction, buyer B can buy the item by entering the auction with a bid of zero. Buyer B will then make a decision of whether to enter the auction.

If both buyers choose to enter the auction, the bid clock will start at the bid set by buyer A, and increase by 1 every second (after 3 rounds it will speed up and increase by 1 every half a second). When the bid clock reaches your maximum willingness to pay, you must click the Drop Out button. The person who clicks the Drop Out button first will lose the auction, and this bid will be the price the other buyer will pay for the item.
Note that if the clock is above your value and the other buyer drops out first, you will win the auction but lose money, so bid carefully. Also, your decision to enter or not enter the auction will not affect the speed at which the session is completed since you will have to wait for all participants to complete each potential auction for the round to end.

You will participate in 30 rounds and will be randomly matched with a different buyer in each round.

**Profit calculations**

The profit calculations are as follows:

Any buyer who does not enter the auction earns 20 for the round, regardless of what the other buyer does.

If buyer A enters the auction and buyer B does not, buyer A earns:

- $20 - 3 + \text{Value} - \text{Initial Bid}.$

If buyer B enters the auction after observing that buyer A does not, buyer B earns:

- $20 - 3 + \text{Value} \text{ (because in this case buyer B wins with a bid of 0).}$

If both buyers enter the auction their profits are:

- **Winning Buyer’s Profit** = $20 - 3 + \text{Value} - \text{Auction Price}$
- **Losing Buyer’s Profit** = $20 - 3 = 17.$

Note that the auction price is the price at which the losing bidder dropped out of the auction.

**Examples**

**Example 1:** Suppose that you and the other buyer compete in an auction, where you are buyer A and decide to start the auction at 0. In this example, you have a value of 60, buyer B has a value of 80 (which you do not know), and both of you have incurred the entering cost of 3. Suppose you drop out of the auction first with a bid of 60. You lose the auction, and the winning bid is equal to 60.

- Your Profit (buyer A) = $20 - 3 = 17$
- The Other Buyer’s Profit (buyer B) = $20 - 3 + 80 - 60 = 37$
**Example 2:** Suppose that buyer A enters the auction and sets the starting price of the auction at 15. Suppose that you are buyer B and decide to not enter the auction. In this case, you do not incur the entry cost, but buyer A does and wins the item at a price of 15. Suppose that the value of buyer A is 30.

- Your Profit (buyer B) = 20
- The Other Buyer’s Profit (buyer A) = 20 - 3 + 30 - 15 = 32

**How the game will progress**

At the beginning of the game you will be told your role (buyer A or buyer B), which will be the same for the duration of the session. Each round you will be randomly matched with a buyer with the other role. After completing each round you will be shown who entered the auction, the outcome of the potential auction, who won the item, and your resulting profits. You will also be shown this information for all past rounds at the bottom of each screen and this information will be for different partners.

**How you will be paid**

At the end of the session the actual earnings from the game will be converted to US dollars at the rate of 60 experimental dollars per 1 US dollar. These profits will be displayed on your screen and paid to you in cash at the end of the session.