BETTING ON LEVERAGE*

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ABSTRACT

We combine classic corporate finance theory of capital structure with an asset pricing theory of leverage-constrained investors to explain why CAPM beta is negatively related to abnormal stock returns. Current theory explaining this anomaly suggests that leverage constrained investors tilt portfolios towards high-beta stocks. With a stylized analytical model and simulation, we show leverage-constrained investors rationally tilt investment, not towards high-beta firms generally but specifically towards those with high financial leverage. The advantage to adding levered firms, rather than an unlevered firms with comparably high betas, comes through lower covariance of the levered assets with the market portfolio. Informed by a continuous-time capital structure model, we estimate the varying impact of firm-level financial leverage on market risk measures and document two novel contributions. First, we find no remaining evidence of the anomalous low returns to high beta stocks. Second, we formally test an adjusted model of leverage-constrained investors and conclude that such constraints have practical implications for investors and for asset pricing models.

JEL Classification: G01, G11, G12, G14, G15

Keywords: beta, betting against beta, CAPM, levered beta, unlevered beta, stock returns.

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Introduction

The well-documented inverse relation between market risk and abnormal returns remains a significant piece of evidence against the Sharpe-Lintner (Sharpe, 1964; Lintner, 1965) capital asset pricing model (CAPM). We offer an explanation for this anomaly by combining classic corporate finance theory of capital structure with a recent asset pricing theory which includes the leverage constraints of investors. This paper offers evidence that accounting for cross-sectional differences in how firm-level financial leverage impacts market risk within a model where investors face leverage constrains helps explain why stocks with high (low) betas produce low (high) CAPM alpha.

We are motivated by Frazzini and Pedersen's (2014, FP hereafter) general equilibrium model of asset prices in an economy with leveraged-constrained investors. In their model, constrained investors seeking higher expected returns bid up the prices of high-beta stocks, thus placing downward pressure on the alpha of these stocks. We show that the non-linear relation between abnormal returns and market risk in their model offers only a partial solution to the anomaly. Alphas from the FP model with leverage constrained investors are smaller, but remain jointly significant and continue to produce a negative relation with beta. Our innovation is to consider the possibility that constrained investors' demand for high-beta stocks is conditional on the financial leverage of the firm.

We appeal to Modigliani and Miller's (1958) Proposition I and hypothesize that investors prohibited from investing with 'homemade leverage' bid up firms for which high CAPM betas reflect high levels of financial leverage rather than firms investing in systematically risky assets.[1] Sorting firms according to CAPM beta, we find that relatively high-asset-risk firms with zero-leverage (e.g. Microsoft) reside in the same market risk portfolios as relatively low-asset-risk-

[1] M&M Proposition I states that the value of the firm is independent of the percentage of debt in its capital structure. Its proof requires investors without leverage constraints. Intuitively, if financial leverage could increase value, then investors could simply invest on margin in the less valuable unlevered firm. Prohibiting investors from investing borrowed funds should thus result in a preference for levered firms among those who prefer levered returns.

high-leverage firms (e.g. Archer Daniels Midland). When they fall into the same CAPM beta portfolio, these different firms are treated identically in the original FP framework. Arguing that traditional CAPM beta reflects measurement error, we find support for the FP model only after separating the varying effects of firm-level financial leverage from the true systematic risk of the firms' underlying assets. After correcting the measurement error in beta, we document two novel contributions to the literature. First, we find no remaining evidence of the negative relation between abnormal returns and market risk. Second, we provide empirical support for the FP model of leverage-constrained investors. We conclude that investor leverage constraints are important for understanding the cross-section of stock returns.

We begin by deriving the benefit to a leveraged-constrained, utility-maximizing investor of investing in levered firms. The advantage to the investor of adding a levered firm to her portfolio, rather than an unlevered firm with a similar traditional beta, comes through the lower covariance of the levered firm with the market portfolio. As suggested by our stylized analytics, our boot-strap experiment reveals that constrained investors will on average achieve a higher Sharpe ratio by tilting towards high-leverage rather than high-beta stocks. Portfolios formed from independent sorts on leverage and traditional beta further reveal patterns in returns and alphas that motivate our empirical methodology for tests of the FP model.

Based on the evidence from our analytical and simulation exercises, we test the FP model with estimates for asset beta undistorted by variation in financial leverage. Because Gomes and Schmid (2010, hereafter GS) find that traditional beta is a function of age, growth options, and default probability as well as financial leverage, we employ their continuous-time model to avoid omitted variable bias.[2] Specifically, we estimate the systematic risk of firm assets with the fitted value of a cross-sectional regression of traditional beta on leverage, growth options relative to firm

[2] To validate the GS model variables for our setting, we run cross-sectional and multivariate regressions and find that the time-series of CAPM alphas across beta-sorted portfolios are significantly related to these firm characteristics after controlling for the Fama-French size, book-to-market and momentum factors, and a measure of the aggregate leverage constraint in Boguth and Simutin (2015). We also find that of these firm characteristics, leverage is the dominant explanatory factor.

age, and default probability relative to the size of the firm. We then rank firms based on these fitted betas and form portfolios. These portfolios exhibit significantly positive, but non-decreasing alphas across the fitted-beta sorted portfolios.

In the FP model, alpha is a non-linear function of a parameter representing the average level of investor leverage constraints and the market risk parameter. To circumvent the problem of estimating non-linear parameters, we follow a method similar to the one-step Gauss-Newton procedure in Gibbons (1982), estimating the procedure within a generalized method of moments (GMM) framework. For 20 portfolios sorted on traditional beta, a likelihood ratio test and the Gibbons, Ross, Shanken (1989) test reject the null hypothesis that alphas are jointly zero for both the CAPM and the FP model. We find that alphas from both the FP model and CAPM decrease with market risk, although the FP alphas are universally smaller. For 20 portfolios sorted on our fitted betas, our tests reject the null hypothesis that alphas are jointly zero for the CAPM, but not for the FP model. Sorting on our fitted betas, we find that CAPM and FP alphas increase with market risk, although the FP alphas are much closer to zero.

Our paper makes the following contributions to the literature. We take the analytic, simulation, and empirical results not only as validation of the FP model, but also as evidence supporting the theory of Gomes and Schmid. Our results indicate that leverage constraints of investors matter for asset prices through corporate capital structure (corporate finance policies), not through systematic asset risk (corporate investment policies), and that utilizing financial leverage (rather than investing in riskier assets) can make a firm more attractive during periods when the average level of investors' leverage constraint is binding. Overall, our results support recent asset pricing and corporate finance theories and suggest that leverage-constrained investors should look specifically to high-beta firms with high-leverage to increase their utility.

Section I motivates our study with prior literature. Section II describes the data and validates our Betting Against Beta (BAB) estimation. Section III contains our analytic justification for considering firm-level leverage demands of leverage constrained investors. Section IV contains our simulation and portfolio sorting results. Section V describes our direct tests of the FP model and our main results. Section VI concludes.

I. Background and Motivation

The high (low) beta and low (high) abnormal stock returns relation is first documented by Black, Jensen and Scholes (1972) and the list of subsequent papers noting the anomaly is long.[3] More recent research by Frazzini and Pedersen (2014), Bali, Brown, Murray and Tang (2016), and Cederburg and O'Doherty (2016) confirm and offer various solutions to this negative risk-return phenomenon. Bali *et al*. (2016) suggest a behavioral explanation showing that controlling for characteristics of lottery stocks, the negative relation between alpha and market risk disappears. They argue that the demand by lottery investors for stocks with high probabilities of large short-term price increases are partially generated by a stock's sensitivity to the overall market beta.[4] Cederburg and O'Doherty (2016) hypothesize that the anomaly is due to a bias in unconditional alpha. They demonstrate that in a version of the CAPM with time-varying parameters, conditioning beta on the cross-sectional dispersion of leverage, along with cross-sectional dispersions of other firm characteristics such as investment and idiosyncratic volatility, helps explain the alphas of beta-sorted portfolios. As noted above, Frazzini and Pedersen (2014) explain the negative relation in a model where investors' face binding funding constraints.

Our paper extends and tests the FP model, but is empirically related to Cederburg and O'Doherty (2016). Cederburg and O'Doherty rely on Lewellen and Nagel's (2006) argument that, if the conditional CAPM holds, unconditional CAPM alphas can be negatively biased when beta is, in part, positively related to market volatility. They show that variables which are related to increases in cross-sectional beta dispersion can be utilized to model time-varying beta which in

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[3] See, e.g., Blume and Friend (1973), Fama and MacBeth (1973), Lakonishok and Shapiro (1986) and Fama and French (1992, 1993).

[4] Bali *et al*. (2016) identify lottery demand as the average of the five highest daily returns of the given stock in a given month. Excess returns to a portfolio long high-beta stocks and short low-beta stocks (High- Low beta portfolio) disappear when the portfolio is constrained to be neutral to lottery demand.

turn provide a solution to the anomaly. Both the Bali *et al.* (2016) paper and the Cederburg and O'Doherty paper can be taken as evidence against the model of FP.

Our approach and conclusions differ from the above work. We hypothesize that rational leverage-constrained investors within a general equilibrium framework generate demand not for high-beta assets, but for high leverage firms. As illustrative examples, consider Microsoft (MS) and Archer Daniels Midland (ADM). Although both firms are classified into the same equity beta portfolio, the source of their systematic risk exposure is very different. MS exhibits a relatively high asset beta,[5] invests heavily in research and development (7% of TA), exhibits low financial leverage (market D/E = 0.087), and holds a cash balance of approximately $100 billion (56% of TA). In contrast, Archer Daniels Midland (ADM) exhibits a low asset beta, invests nothing in R&D (0% of TA), exhibits higher financial leverage (market D/E = 0.31) and holds much less cash (2.6% of TA). To the extent that leverage-constrained investors explain the high price (low return) of high equity beta stocks, as suggested by FP, we hypothesize that it is specifically those with high financial leverage, rather than those with risky assets, that get overpriced.

Statistically, we argue that it is not the unconditional alphas that are biased, but measurement error in unconditional betas due to ignoring theoretically and empirically known relations between beta and cross-sectional differences in leverage. It is well known that estimates of equity beta reflect both financial and operational leverage [e.g. Hamada (1972) and Merton (1974)]. Galai and Masulis (1976) present a model where equity beta is a function of systematic risk and the price elasticity of equity value with respect to changes in asset value, which in turn is a function of leverage. Beta estimates contaminated by firm's leverage bias estimates of true stock risk as suggested by Drobetz, Meier and Seidel (2014) and cause beta instability as shown in the work of DeJong and Collins (1985). Moreover, Ferguson and Shockley (2003) show that betas estimated using an equity-only market proxy are understated and the bias is more pronounced

[5] What we term "asset beta" is also frequently referred to as an unlevered beta; see Hamada (1972).

among high leverage firms. In our setting, accounting for firm-level financial leverage and its varying impact on beta not only explains the anomaly, but provides evidence in favor the FP model.

Finally, Gomes and Schmid (2010) provide a continuous-time model of the relation between expected returns on equity and corporate capital structure where both corporate investment and financing decisions are endogenous. Their model provides a link between equity beta and corporate investment and financial policies. They present a decomposition of equity beta into four components. The first component captures the risk of assets in place. The second represents the risk of levering up equity cash flows. The third part of beta is related to default risk relative to the value of the firm. The final component reflects the risk stemming from growth options relative to the age of the firm. The intuition for the last term is that debt financing is often used for investment in growth options with different amounts of risk. Since the number and risk of growth options depends on the age of the firm, the betas of mature firms with fewer and lower-risk growth opportunities are impacted differently by leverage than the betas of younger firms with higher-risk growth options. The results of their paper indicate that it is crucial to consider growth options when examining the cross-sectional relation between leverage and equity returns. We extend that warning to the relation between firm-level financial leverage and abnormal returns. We employ the variables from the GS model to adjust equity betas in a formal test of the FP model of constrained investors.

II. Data

All data are monthly over the period January, 1963 to December, 2015. We utilize a sample of U.S. equities that is similar to that in FP. The sources and formation methodologies for all of our data are fully described in the appendix. We replicate the non-standard method of estimating a firm's beta employed by FP. To verify that our sample of firms and beta estimation methodology matches that of FP, we compare the summary statistics and alphas from the CAPM, 3- and 4-factor models for our measure of the Betting Against Beta (BAB) factor with those obtained from the BAB data employed by FP and available from AQR Capital Management. In Panel A of Table 1 the distributions of the AQR BAB factor and our BAB factor are almost identical. Our BAB factor has a modestly higher mean, standard deviation, and Sharpe ratio, with slightly fatter tails. Even so, the two BAB factors have a time-series correlation of 96%. In Panel B, we show our BAB factor produces slightly higher measures of alpha across the three models, *i.e.* our BAB alphas are a basis point higher than those produced by FP's BAB factor available from AQR.

[Table 1 Here]

Table 2 contains a replication of FP's Table 3 for our sample period. Qualitatively, we find similar results to those in FP. Our portfolios are rebalanced every month and sorted by the previous month's beta. As in FP, we see in row 1 that beta sorted portfolio returns increase then decrease with market risk. Also consistent with previous literature we see in rows 3, 6, and 9, CAPM, Fama-French 3-factor and Carhart 4-factor alphas fall moving from portfolios of low beta stocks to high beta stocks.

[Table 2 Here]

To adjust beta based on the model by GS, we first calculate the market value debt-to-equity ratio adjusted for taxes using marginal tax rates from Graham (1996a, 1996b) and fixed-cost (non-cancelable) operating leases as suggested by Cornaggia, Franzen, and Simin (2013). We measure growth options using the market value of assets to book value of assets relative to the age of the firms. For the age variable we merge data from Jovanovic and Rousseau (2001) and Jay Ritter's webpage. We use the founding date of the firm, then year of incorporation, then listing date if no founding date is available. We estimate the expected default probability (EDF) derived from the Merton (1974)/KMV model as in Duan and Simonato (2017) and detail the methodology in the appendix. Throughout the paper we measure the size of a firm by its market capitalization. Summary statistics for these data are in the appendix.

III. Analytical and Empirical Evidence

In this section we examine the impact of firm-level financial leverage on the utility of leverage-constrained investors. We start with a stylized analytic comparison of the utility changes associated with a portfolio of assets tilted towards high-leverage stocks compared to the utility changes associated with a portfolio tilted towards high-beta stocks for a rational leverage-constrained investor. We then take the question to the data and bootstrap the increase in Sharpe ratios for a hypothetical leverage-constrained investor. Finally, we examine the abnormal returns of portfolios sorted by both firm-level financial leverage and traditional beta.

III.A. Tilting Towards Leverage

Let the investor have a standard quadratic utility function as in FP. The expected return and variance of the utility-maximizing portfolio for an investor who invests a percentage of her endowment, *y*, in the tangency portfolio *m* is

$$E(r_c) = y * E(r_m) + (1 - y) * rf \tag{1}$$

$$\sigma_c^2 = y^2 * \sigma_m^2.$$

In addition, suppose the investor faces a leverage constraint such that $(1 - y) \geq 0$, but the investor's risk aversion is small enough to indicate her utility would be maximized at some $y > 1$. That is, the investor would like to earn an expected return greater than $E(r_c)$ and is willing to take on more risk than σ_c^2 to achieve that return. FP suggest that the investor adds assets to her portfolio with a beta greater than β_m to increase her utility.

To demonstrate the importance of firm-level leverage, let the investor have a choice between investing in one of two firms, firm *A* or firm *B* where $\beta_A = \beta_B > \beta_m > 0$. Suppose that firm *A* utilizes debt while firm *B* does not. These firms are identical only in their market risk as measured by traditional beta. This comparison is a departure from the more common practice of comparing otherwise identical firms with different market risk measures. Here, we compare firms with equivalent traditional beta but derived from different types of risk: investment risk associated with systematically risky assets versus financial risk associated with fixed-cost financing.

If we ignore taxes, assume that default risk is idiosyncratic, and assume that firm *A* fixes its debt-to-equity ratio, then β_A = β_{Au}*(1+*D*/*E*) as in Hamada (1972), where β_{Au} is firm *A*'s unlevered beta reflecting the systematic risk of firm *A*'s assets. Because firm *B* utilizes no debt financing, its equity beta is equivalent to its asset beta. We can write

$$\beta_A = \beta_{Au}*(1+D/E) = \beta_B \tag{2}$$

$$\sigma_{A,m} * (1+D/E) = \sigma_{B,m}$$

to show that cov(R_A^{assets},R_m) < cov(R_B^{assets},R_m), since (1+*D*/*E*) > 0 for firm *A*.

The advantage to the investor of adding a levered firm to her portfolio, rather than an unlevered firm with equivalent equity beta, comes through the lower covariance of the levered firm with the portfolio m. To see this, suppose the above investor sells a small portion, δ, of her holdings in the tangency portfolio and buys δ amount of firm *K*, *K* ∈ {*A*, *B*}. The increase in her expected return is $\Delta E(r_c) = \delta\big(E(r_k) - E(r_m)\big)$, and is the same whether she tilts towards stock *A* or *B* since $\beta_A = \beta_B \Rightarrow E(r_A) = E(r_B)$ in equilibrium.

The variance of the investors original portfolio $\sigma_c^2 = \sigma_m^2$ since she invests all of her endowment in portfolio *m*, *i.e.* when *y* =1. Her variance after shifting δ of her endowment from the tangency portfolio to asset *K* is

$$\sigma_{cnew}^2 = (1-\delta)^2\sigma_m^2 + \delta^2\sigma_k^2 + 2(1-\delta)(\delta)\sigma_{m,k}. \tag{3}$$

If δ is small, then $\delta^2 \to 0$ and we assume away the δ^2 terms since they have a negligible impact. This means

$$\sigma_{cnew}^2 \cong (1-2\delta)\sigma_m^2 + 2\delta\sigma_{m,k}. \tag{4}$$

The increase in the variance of the investors complete portfolio is

$$\Delta\sigma_c^2 = 2\delta\big(\sigma_{m,k} - \sigma_m^2\big). \tag{5}$$

Because both A and B deliver the same increase in expected returns, but the levered firm produces a smaller increase in the variance of her portfolio since $\sigma_{A,m} < \sigma_{B,m}$, the constrained investor will seek out firms with high beta attributable to high leverage rather than firms with high asset risk in order to improve her utility maximizing portfolio.

III.B Simulating Changes in Sharpe Ratio

The intuition provided above suggests that constrained investors produce a better risk-return tradeoff by adding high-beta/high-leverage stocks, rather than high-beta/low-leverage stocks to their portfolios. Even so, the assumptions leave the assertion that leverage-constrained investors should favor high-leverage firms as an empirical question. To further test this hypothesis, we perform a bootstrap experiment. In our exercise we use all firms in our sample from the CRSP-Compustat database between 1970 and 2015 with at least 60 months of data. For each of the 493 60-month windows, we sort firms independently based on their average traditional beta and average financial leverage defined as the natural logarithm of one plus the tax-and-operating-lease-adjusted market value of debt to market value of equity ratio (as defined in the data appendix). *HL* firms fall in the highest leverage quintile based on their average leverage over the 60 months and *HB* firms fall in the highest beta quintile based on their average beta over 60 months. The rankings are done separately so a firm may be designated as both *HL* and *HB* in any 5-year period.

In each five-year period we calculate the Sharpe ratio, S_{MKT}, of the market proxy obtained from Ken French's website. In each period we also calculate the Sharpe ratios S_{HL}, and S_{HB} for the portfolios where the investor invests 99% of her wealth in market proxy and 1% in each firm in both *HL* and *HB*, respectively. For the bootstrap experiment we do the following:

1) Randomly draw with replacement 500 Sharpe ratios, (S_{MKT}, S_{HL}, S_{HB}) from the 493 60-month samples.
2) Calculate the average percentage change in the Sharpe ratio between a portfolio of investing only in the market proxy and
   a) the new portfolios with $w_{Mkt} = 0.99$ and $w_{HL} = 0.01$ and
   b) the new portfolios with $w_{Mkt} = 0.99$ and $w_{HB} = 0.01$.

3) Repeat steps (1) and (2) 10,000 times.

Figure 1 plots the distribution of the 10,000 average percentage changes in the constrained investors Sharpe ratio when tilting towards high beta or high leverage firms and Table 3 contains summary statistics of the distributions.

[Figure 1 Here]

[Table 3 Here]

Tilting away from fully investing in the market towards *HL* or *HB* improves Sharpe ratios on average. Tilting towards *HB* stocks produces an expected 4.07% increase in the Sharpe ratio and tilting toward HL stocks improves the Sharpe ratio by 4.50% on average. The addition of *HL* stocks also reduces the Sharpe ratio less often, compared to adding *HB* stocks. In order to test whether the empirical probability distribution of possible outcomes from the constrained investor tilting towards *HL* can be ranked as superior to tilting towards *HB* stocks, we use a test of stochastic dominance proposed by Linton, Maasoumi and Whang (2005). We are unable to reject the null that *HL* dominates *HB* over the whole support at the 5% level with a p-value = 0.73.[6] This bootstrap result confirms the analytic result that constrained investors should favor levered firms.

To highlight the combined importance of the level of beta and leverage, we repeat the bootstrap experiment by double sorting stocks into two portfolios: *HBLL* are stocks with highest beta and lowest leverage rankings and *HBHL* are stocks with the highest beta and highest leverage rankings. Figure 2 presents the distributions of the percentage changes in Sharpe ratios for an investor that tilts towards either *HBHL* or *HBHL* stocks. The figure also contains the distribution of percentage changes in Sharpe ratios when tilting towards *HB* for reference.

[Figure 2 Here]

[6] Linton, Maasoumi and Whang (2005) propose a procedure for estimating the critical values of the extended Kolmogorov-Smirnov tests of Stochastic Dominance of arbitrary order in the general K-prospect case. Their test allows for serially dependence and accommodates general dependence among the prospects being ranked. P-values are based on a centered boot-strap.

Figure 2 reveals that tilting towards *HBLL* stocks provides a 1.15% increase in Sharpe ratios on average, while tilting towards *HBHL* stocks produces an average 4.37% increase which exceeds the average increase of 4.07% from tilting towards *HB* stocks without considering leverage. Here we fail to reject null that *HBHL* dominates *HB* (*p-value* = 0.928), but we strongly reject that *HBHL* dominates *HL* (*p-value* =0.000). The plots and the stochastic dominance test indicate that tilting towards stocks with both high-beta and high-leverage produces a larger increase in Sharpe ratios than tilting towards high-beta stocks. Because the *HBHL* does not dominate the simpler *HL* portfolio, we infer that firm financial leverage is first order. For a rational utility-maximizing investor facing binding leverage constraints, favoring leverage appears beneficial.

III.C. Sorting on Leverage and Beta

The impact of firm-level financial leverage can also be seen in the abnormal returns of portfolios sorted by traditional beta and leverage. Table 4 contains the alphas for double-sorted equally-weighted (Panel A) and value-weighted (Panel B) portfolios. For equally-weighted portfolios moving from low-beta to high-beta stocks generally produces a monotonically decreasing trend in alpha (except for firms in the fourth leverage portfolio, where the correlation is non-linear). The last row of Panel A shows that on average alpha decreases as beta increases consistent with the results in Table 2. The last column of Panel A shows that on average, alpha increases monotonically with leverage. This is true except for the lowest beta firms where alpha increases with leverage, but not monotonically.

[Table 4 Here]

Value weighting has a significant impact on Table 4 results. In the last row of Panel B, we observe no trend in alphas, on average, across beta quintiles. However, firm leverage again matters: alpha trends down with beta only for firms with low leverage. Alpha trends up for firms with greater financial leverage. Looking across leverage quintiles, although value-weighting shifts the distribution of alpha upwards relative to the equally-weighted portfolios, abnormal returns continue to increase with leverage especially for higher beta firms. The interaction between market risk, leverage, and size becomes apparent when comparing the equal and value weighted results. Giving less weight to small firms removes the downward trend in alpha across beta sorted portfolios. Because FP present results only for equally-weighted portfolios, this is a novel insight relative to their paper.

IV. Incorporating Firm Leverage into the FP Framework

Motivated by the results above, we estimate the systematic risk of firm assets (corporate investment policy) distinguishing the increased market risk that stems from leverage (corporate financial policy). We then formally test the FP model in Section V with betas reflecting true asset risk.

We first establish here the persistence of the negative relationship between CAPM alpha and beta that we aim to explain. We generate a time-series of CAPM alpha for each of 20 portfolios sorted on the lagged traditional beta. The alpha time-series are intercepts from 60-month rolling window estimations of the market model using the S&P500 excess return as the market proxy. As reported in Panel A of Table 5, regressing each cross-section of alphas on a linear trend, we find that 65% of the 433 resulting cross-sections of alpha exhibit a negative slope across the 20 beta-sorted portfolios, with 52% of the cross-sections having a significantly negative slope based on White's standard errors to correct for heteroscedasticity. This percentage indicates that the negative relation between CAPM alpha and beta is persistent.

Next, we estimate a series of cross-sectional and multivariate seemingly unrelated regressions to test whether these CAPM alphas are related to our measures of leverage, growth options, and default probability – and to test the relative importance of leverage in this regard. In cross-sectional regressions of alpha reported in Panel B of Table 5, we observe that leverage is most often significantly related to alpha with *p*-values less than 0.05 in 56% of the cross-sections, again based on White's standard errors. Growth options and default probability are significant in 39% and 35% of samples, respectively. The average of the absolute value of the *t*-statistics is 3.20 for leverage, 1.80 for the growth option proxy, and 1.89 for the default risk measure. We infer that of the variables shown previously by GS to explain beta, leverage appears the most relevant for explaining conditional CAPM alpha.

As an additional test, we regress the portfolio alphas on our measures of leverage, growth options, and default probability within a seemingly-unrelated-regression equation (SURE) using GMM. Using the GMM SURE to estimate the multivariate system produces standard errors that are adjusted for cross-equation correlation and correct for heteroscedasticity using White's standard errors. As controls we include the Fama-French risk-factors, momentum, and a measure of the aggregate leverage constraint (LCT) from Boguth and Simutin (2015)[7]. Panel C of Table 5 contains the average of the absolute value of the *t*-statistics across the 20 portfolios. In this experiment, each of the firm characteristics is statistically related to alpha; controlling for ubiquitous risk-factors ignored by the CAPM and a measure of the aggregate level of leverage constraints does not mitigate their relevance. As before, the magnitude of leverage is greater than the magnitudes of growth of default risk.

[Table 5 Here]

The panel regression results in Table 5 reveal that the cross-section of alphas is significantly related to our measures of firm leverage, growth and default risk. We therefore adjust betas for these firm characteristics. For our primary version of adjusted beta (β^{GS}), we first regress the cross-section of traditional $\hat{\beta}_i$ at time *t* on the time-series average over the previous 60-months of the asset-specific leverage; the ratio of market value of assets to book value of assets, relative to the age of the asset; and the default probability relative to the log of the assets' market capitalization.[8] Using the coefficients from that cross-sectional regression we form adjusted betas as

[7] LCT is a measure of leverage constraint tightness that strongly and significantly predicts returns of FP's betting-against-beta factor. Boguth and Simutin (2015) find LCT alone explains 19% of the variation in future annual BAB returns.

[8] In several of the cross-sectional regressions, the lack of variability in the measure of default produces a singular covariance matrix. To avoid this problem, we do not include an intercept in the cross-sectional regressions.

$$\beta_{i,t}^{GS} = \hat{\gamma}_{1,t} \ln\left(1 + \frac{D}{E}_{i,t-1}\right) + \hat{\gamma}_2\left(\frac{GO_{i,t-1}}{Age_{i,t-1}}\right) + \hat{\gamma}_3\left(\frac{100 * DefProb_{i,t-1}}{ln(Size_{i,t-1})}\right). \quad (6)$$

Here

$\frac{D}{E}_i$ = leverage defined by the average lagged market value debt-to-equity ratio adjusted for taxes and operating leases of the firms in portfolio i,

GO_i = growth options defined by the average market value of assets to book value of assets of the firms in portfolio i,

Age_i = the average age of the firms in portfolio i,

$DefProb_i$ = the average default probability from the Merton (1974)/KMV model of the firms in portfolio i, and

$Size_i$ = the average market capitalization of the firms in portfolio i.

Because the second and third component of the above decomposition roughly coincide with the risks that are accounted for in the standard textbook method of un-levering beta first described in Hamada (1972), we also create an adjusted beta (β^H) every month as

$$\beta_{i,t}^{H} = \frac{\beta_{i,t}^{Levered}}{\left[1 + (1 - Tax) * \frac{Debt}{Equity}_{t-1}\right]}. \quad (7)$$

Using β^{GS} and β^H, separately, we sort firms into two additional sets of 20 equally-weighted portfolios rebalanced every month. In the last two columns of Table 3 we report the results for the Sharpe ratio bootstrap experiment using β^{GS} and β^H to identify high beta stocks. Tilting towards high β^{GS} stocks increases Sharpe ratios by 4.68%, the largest increase in average Sharpe ratios in Table 3. Tilting towards high β^H stocks improves Sharpe ratios by 1.92%. We again generate time-series of alpha as the intercepts from 60-month rolling window estimations of the market model using the S&P500 excess return as the market proxy. Regressing each cross-section of alphas on a linear trend, we find that only 18% (28%) of the 433 resulting cross-sections of alpha exhibit a significantly negative slope across the 20 β^{GS} (β^H) sorted portfolios compared to 52% of the cross-sections having a negative slope when sorting on traditional beta.

The full sample market model alphas for the three sets of beta adjusted portfolios are displayed in Figure 3.

[Figure 3 Here]

As expected, the CAPM alpha decreases moving from the low- to the high-tranditional-beta portfolio. However, adjusting beta for leverage per Hamada (β^{H}) produces sorted portfolios with alphas that first increase with market risk, then decrease across the 20 portfolios. Finally, the portfolios sorted on β^{GS} exhibit clearly non-decreasing alphas. We therefore elect the β^{GS} adjustment in our formal tests of the FP model in the next section.

We compare the fit of our estimations of the CAPM to restricted versions of the CAPM without an intercept using a large sample likelihood ratio test (LRT). The LRT compares the statistical fit of the unrestricted model with the fit of the restricted model. If the model fits are close, the null hypothesis is not rejected. Since we are comparing the CAPM with and without an intercept, a *p*-value less than 0.05 indicates that we can reject that the alphas of the 20 sorted portfolios are jointly different from zero. The LRT is defined as

$$-2ln\lambda = T\left[ln\left|\hat{\Sigma}_r\right| - ln\left|\hat{\Sigma}_u\right|\right],\ -2ln\lambda \sim \chi^2_{N-1}, \qquad (8)$$

where:

$\left|\hat{\Sigma}_r\right|$ = the determinant of the contemporaneous covariance matrix estimated from the residuals of the restricted model and

$\left|\hat{\Sigma}_u\right|$ = the determinant of the contemporaneous covariance matrix estimated from the residuals of the unrestricted model.

We also use the finite sample test of Gibbons, Ross, Shanken (1989), (GRS).

$$GRS = \left(\frac{T-N-1}{N}\right)\left[1 + \frac{E(mkt)^2}{\sigma^2_{Mkt}}\right]^{-1} \hat{\alpha}'\hat{\Sigma}^{-1}\hat{\alpha}, \quad GRS \sim F_{N,T-N-1} \qquad (9)$$

where $\hat{\Sigma} =$ a consistent estimate of the covariance matrix of the residuals. The test statistics and associated *p-values* are presented in Table 6. Both the asymptotic and small sample tests reject that the alphas are jointly zero for traditional beta, β^{GS}, and the β^{H} sorted portfolios.

[Table 6 Here]

To summarize this section, we show that CAPM alphas are statistically related to firm characteristics (leverage, growth options, and default probability) shown previously by GS to components of traditional beta, and that leverage appears the most important of these characteristics in this regard. Based on these results, we adjust the traditional beta to account for the varying impact of leverage on market risk as suggested by Hamada (1972) and by our estimates of the GS variables. We find that accounting for firm leverage does not spare the CAPM, since our adjusted betas sort assets into portfolios that produce significant abnormal returns. However, the patterns of alpha across adjusted-beta portfolios indicate the importance of growth-options and default risk, as suggested by GS. The alphas of the β^{GS} -sorted portfolios increase with market risk while the β^{H} -sorted portfolios exhibit alphas that increase then decrease with market risk. In the next section, we formally test the leverage-constrained investor model suggested by FP, employing our primary adjusted beta (β^{GS}) to account for the effects of financial leverage on market risk.

V. Direct Tests

In this section, we outline our estimation framework and formal testing procedure of the FP general equilibrium model. We report our test results in Section V.B. below.

V.A. Estimating the non-linear FP Model

Equilibrium in the Frazzini and Pedersen (2014) model produces the following linear relation between expected returns and risk.

$$E_t(r_{i,t+1}) - r_f - \varphi_t = \beta_t[E_t(r_{M,t+1}) - r_f - \varphi_t] \tag{10}$$

Where $r_{i,t+1}$ is the return on asset i, $r_{M,t+1}$ is the return on the market factor, r_f is the risk-free return with the parameters β_t, which represents market risk, and φ_t which is the average Lagrange multiplier from the portfolio constraint having the economic interpretation of a measure of the tightness of leverage constraints in the economy.

The theory of FP implies a CAPM represented by the following statistical model.

$$R_{i,t} = \varphi(1 - \beta_i) + \beta_i R_{M,t} + \eta_{i,t}. \quad (11)$$

where:

$R_{i,t}$ = the return on firm i in period t in excess of the return on a risk-free asset,

$R_{m,t}$ = the excess return on the market portfolio at time t,

β_i = cov($R_{i,t}$, $R_{m,t}$)/var($R_{m,t}$), and

$\eta_{i,t}$ = a random disturbance, $\eta \sim N(0,\sigma^2)$.

Compared with the classic Sharpe-Lintner version of the CAPM,

$$R_{i,t} = \alpha_i + \beta_i R_{M,t} + \eta_{i,t}\ ,\ i = 1, \ldots, N, \text{ and } t = 1, \ldots, T, \quad (12)$$

the FP model places a non-linear restriction on the CAPM alpha. In particular, $\alpha_i = \varphi(1 - \beta_i)$. Non-linearity in the parameters, namely the multiplication of φ and β, make the FP model difficult to estimate. To circumvent the non-linearity problem, we follow an estimation method similar to the one-step Gauss-Newton procedure of Gibbons (1982).

Using a Taylor series expansion about consistent estimates of φ and β, we are able to linearize the non-linear term as,

$$\varphi\beta_i \cong \hat{\varphi}\hat{\beta}_i + \hat{\beta}_i(\varphi - \hat{\varphi}) + \hat{\varphi}\left(\beta_i - \hat{\beta}_i\right) = \varphi\hat{\beta}_i + \hat{\varphi}\beta_i - \hat{\varphi}\hat{\beta}_i\ . \quad (13)$$

Here we use a consistent estimate of $\hat{\beta}$ from the unrestricted model in equation (12) estimated by Ordinary Least Squares (OLS). Note that the functional form of equation (13) is identical to the statistical model representing the CAPM without a risk-free rate developed by Black (1972). In Black's model φ represents the return on a zero-beta asset. Given the similarity of the FP and

Black models, we estimate $\hat{\varphi}$ using the functional form for an estimate of the zero-beta rate in the Black version of the CAPM found in Black, Jensen and Scholes (1972). We find our consistent estimate of $\hat{\varphi}$ using a generalized least squares (GLS) version of the Black, Jensen, and Scholes (1972) estimator, $\hat{\varphi} = \frac{\hat{\alpha}'\hat{\Sigma}^{-1}(\iota_N - \hat{\beta})}{(\iota_N - \hat{\beta})'\hat{\Sigma}^{-1}(\iota_N - \hat{\beta})}$ where $\hat{\Sigma}^{-1}$ is the covariance matrix of the errors from the OLS regressions of (12).

Substituting equation (13) into equation (11) allows for linearization of the FP model,

$$\left(R_i - \hat{\varphi}\hat{\beta}_i\iota_T\right) \cong \varphi\left(1 - \hat{\beta}_i\right)\iota_T + \beta_i(R_M - \hat{\varphi}\iota_T) + \hat{\eta}_i \text{ , } i = 1, \ldots, N. \tag{14}$$

Using equation (14) we estimate φ and β with a multivariate seemingly unrelated regression model (SURM) using a general linear hypothesis across equations. Specifically, we estimate

$$\begin{bmatrix} R_i - \hat{\varphi}\hat{\beta}_1\iota_T \\ R_i - \hat{\varphi}\hat{\beta}_2\iota_T \\ \vdots \\ R_i - \hat{\varphi}\hat{\beta}_N\iota_T \end{bmatrix} = \begin{bmatrix} R_M - \hat{\varphi}\iota_T & 0 & \cdots & 0 & \left(1-\hat{\beta}_1\right)\iota_T \\ 0 & R_M - \hat{\varphi}\iota_T & \cdots & 0 & \left(1-\hat{\beta}_2\right)\iota_T \\ \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & & R_M - \hat{\varphi}\iota_T & \left(1-\hat{\beta}_N\right)\iota_T \end{bmatrix} \begin{bmatrix} \beta_1 \\ \vdots \\ \beta_N \\ \varphi \end{bmatrix} + \begin{bmatrix} \eta_1 \\ \vdots \\ \eta_N \end{bmatrix}$$

or more compactly,

$$y^* = Z\delta + \hat{\eta}^* \tag{15}$$

where:

$y^{*\prime} = \left(R_1 - \hat{\varphi}\hat{\beta}_1\iota_T, R_2 - \hat{\varphi}\hat{\beta}_2\iota_T, \ldots, R_N - \hat{\varphi}\hat{\beta}_N\iota_T\right)$,

$Z = \left[I_N \otimes (R_M - \hat{\varphi}\iota_T) \,\vdots\, \left(\iota_N - \hat{\beta}\right) \otimes \iota_T\right]$,

$\delta' = (\beta_1, \beta_2, \ldots, \beta_N, \varphi)$.

The SURM estimator for δ is then defined by

$$\hat{\delta} = \left[Z'\left(\hat{\Sigma}^{-1} \otimes I_T\right)Z\right]^{-1} Z'\left(\hat{\Sigma}^{-1} \otimes I_T\right)y^*. \tag{16}$$

We use the generalized method of moments (GMM) with a Newey-West spectral density estimate with $T^{1/3}$ lags to estimate equations (12), (15), and (16). The estimation procedure is attractive since it produces consistent and asymptotically efficient estimates. The procedure also avoids the errors-in-variables problem by simultaneously estimating φ and β. Finally, the procedure produces more precise estimators since we use the full contemporaneous covariance matrix of the errors.

V.B. Results

In this section, we focus on the traditional beta and β^{GS} sets of monthly beta-sorted portfolio returns. The FP model does not explicitly produce an estimate of alpha. For the FP model we form alphas based on the restriction, $\hat{\alpha}_i = \hat{\varphi}\left(1 - \hat{\beta}_i\right)$ where $\hat{\varphi}$ and $\hat{\beta}_i$ are from our estimation of equation (16). These alphas along with the CAPM alphas in Figure 4 are presented in Figure 5.

[Figure 5 Here]

For both sets of portfolios, the FP model produces smaller alphas on average than the CAPM alphas. When applied to the beta-sorted portfolio the alphas, labeled FP Alphas, are shifted down with half falling below zero. As can be seen in the figure, when applied to beta-sorted portfolios where beta does not account for the GS model leverage variables the FP alphas continue to decrease with market risk and are significantly different from zero based on the LRT and GRS test results in Table 7.

[Table 7 Here]

When the FP model is applied to beta sorted portfolios where beta accounts for the GS model leverage variables (β^{GS}), the FP alphas marginally increase with market risk and we fail to reject that they are jointly different from zero using either the LRT or GRS test results in Table 7. We then split our sample into two equally sized sub-samples and repeat our experiment. Alphas for the beta sorted portfolios are presented in Figure 5.

[Figure 5 Here]

We find similar patterns for alpha across the traditional-beta- and β^{GS} - sorted portfolios in both sub-samples. CAPM alphas decrease across portfolios sorted on the traditional measure of market risk. CAPM alphas increase with the market risk adjusted for the impact of leverage and are not significantly different from zero based on the LRT tests results in Table 8. Alphas implied by the FP model are smaller in both sub-sample for portfolio sorted on either traditional or adjusted measures of risk, but the FP alphas on the adjusted-beta-sorted portfolios display a slight increase in both sub-samples and are not statistically different from zero based on the LRT tests in Table 8.

[Table 8 Here]

VI. Conclusion

Motivated by the general equilibrium model of Frazzini and Pedersen (2014) and the assumptions underlying M&M Proposition I, we consider the possibility that leverage-constrained investors affect asset prices not by bidding up high-beta stocks in general, but specifically bidding up stocks for which high beta reflects high firm-level financial leverage. We first present a stylistic analytic result and bootstrap experiment to confirm our intuition. The advantage to the investor of adding a levered firm to her portfolio, rather than an unlevered firm with equivalent equity beta, comes through the lower covariance of the levered firm with the market portfolio. We find that constrained investors will on average achieve a higher Sharpe ratio by tilting towards high-leverage rather than high-beta stocks.

Motivated to separate the market risk that stems from firm leverage (corporate finance policy) from systematic asset risk (corporate investment policy), we un-lever traditional betas based on variables suggested by Gomes and Schmid's (2010) continuous-time capital structure model. We then sort stocks into portfolios based on an adjusted measure of market risk that more cleanly captures the systematic risk of firm assets. We reject the classic CAPM, which ignores the leverage constraints of investors, when we sort stocks into portfolios using leverage-adjusted betas. We also reject the FP model, a CAPM that includes the leverage constraints of investors, when applied to portfolios sorted on traditional (unadjusted) betas. Yet when we price leverage-adjusted beta sorted portfolios (sorted based on systematic asset risk) using the FP model with leverage constrained investors, we find abnormal returns are positively associated with market risk and we are unable to reject that abnormal returns are different from zero.

Our contributions to the literature are novel empirical support for the FP model and an explanation for the persistent anomalous relationship between CAPM alpha and beta. Beyond the implications for asset pricing models, our results are further relevant to leverage-constrained investors. Blume and Keim (2012) show that the proportion of U.S. public equities managed by institutions has risen from about 7% of market capitalization in 1950, to about 67% in 2010. While by no means conclusive, our work does suggest that in light of the increased trading of equity by institutions, many of which face leverage constraints, accounting for leverage at the firm level may be an important consideration for the development of new asset pricing models.

We do not rule out the importance of increases in high frequency trading which may be related to the lottery-based explanation of Bali, *et al*. (2014). And our leverage adjustments do imbue some time-variation in beta consistent with the findings of Cederburg and O'Doherty (2016). However, because our results provide support for a rational explanation, rather than behavioral or statistically motived explanation, of the long-standing anomaly that sorting firms on beta produces portfolios of assets that exhibit higher (lower) expected abnormal returns with lower (higher) market risk, we believe our paper provides an important contribution to the literature.

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Table 1: Comparison Statistics

Panel A contains univariate summary statics for the betting-against-beta (BAB) factor available from AQR Capital Management, LLC and our replication of the factor, as well as their Sharpe ratios and the correlation between the two measures. Panel B contains the intercept coefficient, t-statistics and R-squared from regressing the BAB factors on the excess market return, the Fama-French three factors, and the Fama-French three factor model plus momentum respectively. All data are monthly over the period January, 1963 to December, 2015 and all the explanatory variables and the risk free rate are from Ken French's web page.

Panel A: Summary Statistics		
	AQR BAB	BAB
Mean	0.007	0.008
Median	0.007	0.008
Standard Deviation	0.031	0.032
Skewness	-0.747	-0.726
Kurtosis	8.952	10.957
Autocorrelation	0.136	0.129
Sharpe Ratio	0.262	0.284
Correlation		0.96
Panel B: Regress BAB on CAPM and FF models		
	AQR BAB	BAB
CAPM alpha	0.007	0.008
t-stat	7.537	7.909
R^2	0.013	0.006
3-factor alpha	0.007	0.008
t-stat	7.484	7.886
R^2	0.014	0.006
4-factor alpha	0.005	0.006
t-stat	5.591	6.016
R^2	0.087	0.078

Table 2: U.S. Equities Returns, 1963 - 2015

This Table shows average monthly portfolio returns. The first row across columns 1 through 10 contain average returns of beta sorted portfolios. Portfolios are formed each month by ranking all stock using the past month's beta. Betas used for sorting are calculated as in Frazzini and Pederson (2014). The ranked stocks are sorted into equally sized portfolios each month and the returns are equally (Panel A) or value (panel B) weighted. Each panel contains the intercept coefficient, t-statistics (in parentheses) and R-squared from regressing the portfolio monthly excess return on the excess market return, the Fama-French three factors, and the Fama-French three factor model plus momentum respectively. All data are monthly over the period June, 1963 to December, 2015 and all the explanatory variables and the risk free rate are from Ken French's web page. The parentheses contain t-statistics. Alphas are scaled by 100.

Panel A: Equally-Weighted Returns

	Low Beta	2	3	4	5	6	7	8	9	High beta
Mean	0.009	0.009	0.010	0.010	0.010	0.009	0.009	0.009	0.008	0.008
	(6.15)	(5.37)	(5.52)	(4.99)	(4.57)	(4.04)	(3.82)	(3.15)	(2.68)	(1.92)
CAPM alpha	0.639	0.516	0.546	0.512	0.404	0.341	0.320	0.220	0.105	-0.153
	(5.96)	(5.02)	(5.35)	(5.20)	(3.90)	(3.09)	(2.70)	(1.66)	(0.61)	(-0.63)
R^2	0.491	0.622	0.682	0.747	0.762	0.765	0.772	0.765	0.708	0.625
3-factor alpha	0.439	0.290	0.323	0.295	0.181	0.119	0.092	-0.009	-0.106	-0.356
	(5.03)	(3.66)	(4.45)	(4.48)	(2.66)	(1.77)	(1.24)	(-0.11)	(-0.93)	(-2.00)
R^2	0.674	0.783	0.845	0.891	0.901	0.917	0.914	0.911	0.876	0.807
4-factor alpha	0.465	0.344	0.364	0.327	0.266	0.216	0.240	0.196	0.202	0.105
	(5.21)	(4.28)	(4.93)	(4.88)	(3.92)	(3.27)	(3.41)	(2.59)	(1.99)	(0.65)
R^2	0.675	0.786	0.847	0.892	0.906	0.922	0.926	0.930	0.907	0.850

Panel B: Value-Weighted Returns

	Low Beta	2	3	4	5	6	7	8	9	High beta
Mean	0.010	0.008	0.009	0.009	0.010	0.010	0.010	0.011	0.011	0.015
	(6.38)	(6.16)	(6.47)	(5.77)	(6.12)	(5.38)	(5.13)	(4.82)	(4.23)	(4.41)
CAPM alpha	0.791	0.559	0.586	0.546	0.573	0.506	0.503	0.538	0.431	0.646
	(5.88)	(5.75)	(6.35)	(6.31)	(6.78)	(6.17)	(5.94)	(5.91)	(4.00)	(3.75)
R^2	0.324	0.489	0.584	0.692	0.743	0.800	0.823	0.838	0.824	0.749
3-factor alpha	1.020	0.798	0.882	0.938	1.024	1.049	1.123	1.268	1.297	1.845
	(6.25)	(5.88)	(6.17)	(6.02)	(6.18)	(5.78)	(5.69)	(5.75)	(5.27)	(5.74)
R^2	0.004	0.004	0.000	0.005	0.011	0.025	0.036	0.053	0.083	0.132
4-factor alpha	1.088	0.883	0.967	0.988	1.049	1.078	1.145	1.327	1.296	1.781
	(6.62)	(6.48)	(6.77)	(6.33)	(6.29)	(5.89)	(5.73)	(5.97)	(5.12)	(5.25)
R^2	0.003	0.011	0.017	0.015	0.012	0.023	0.026	0.051	0.041	0.046

Table 3: Bootstrapping Sharpe Ratios

This table contains summary statistics of the distributions of the 10,000 average percentage changes in the constrained investors Sharpe ratio when tilting towards high beta or high leverage firms from the bootstrap experiment described in section III.B. The first four columns contain the results from tilting towards high traditional beta and/or different rankings of market value debt-to-equity ratio adjusted for taxes using marginal tax rates from Graham (1996a, 1996b) and fixed-cost (non-cancelable) operating leases (Lev). The last two columns use rankings of stocks based on classic unlevered betas (β^H) and our cross-sectional adjustment based on GS (β^{GS}).

	High Lev	High Beta	High Beta/ Low Lev	High beta/ High Lev	High β^H	High β^{GS}
Mean	4.50	4.07	1.15	4.37	1.92	4.68
Median	4.29	3.96	1.08	4.23	1.76	4.53
Std. Dev.	3.02	3.37	3.07	3.43	2.93	3.42

Table 4: Sorted Portfolio CAPM Alphas

Alphas for the double sorted portfolios. Panel A contains the alphas for the equally weighted portfolios and Panel B contains the value weighted alphas. Portfolios are sorted each month on last month's beta then on last month's leverage defined as the average lagged market value debt-to-equity ratio adjusted for taxes and operating leases of the firms in portfolio. Alphas are the intercepts from a regression of excess portfolio returns on the excess returns of a market proxy. The market proxy and the risk free rate are both from Ken French's data base. All alphas are scaled by 100. Bold alphas are significant at the 5% level using Newey-West standard errors with 12 lags.

Panel A: EW CAPM Alphas *100						
	Low Beta	2	3	4	High Beta	Mean
Low Leverage	**0.530**	**0.380**	0.180	-0.040	-0.380	0.134
2	**0.600**	**0.520**	**0.320**	0.130	-0.130	**0.288**
3	**0.650**	**0.530**	**0.340**	**0.320**	0.030	**0.374**
4	**0.410**	**0.510**	**0.510**	0.320	0.050	**0.360**
High Leverage	**0.760**	**0.710**	**0.620**	**0.610**	0.250	**0.590**
Mean	**0.590**	**0.530**	**0.394**	0.268	-0.036	

Panel B: VW CAPM Alphas * 100						
	Low Beta	2	3	4	High Beta	Mean
Low Leverage	**0.810**	**0.570**	**0.480**	**0.420**	**0.370**	**0.530**
2	**0.700**	**0.550**	**0.560**	**0.520**	0.350	**0.536**
3	**0.570**	**0.730**	**0.660**	**0.570**	**0.580**	**0.622**
4	**0.510**	**0.620**	**0.750**	**0.600**	**0.730**	**0.642**
High Leverage	**0.840**	**0.940**	**0.840**	**1.070**	**1.280**	**0.994**
Mean	**0.686**	**0.682**	**0.658**	**0.636**	**0.662**	

Table 5: Explaining Alphas

This table contains regressions using the CAPM alpha for each of 20 portfolios sorted on the lagged FP version of beta as the left-hand-side variable. The alpha time-series are intercepts from 60-month rolling window estimations of the market model using the S&P500 excess return as the market proxy. Panel A contains the percentage of times we find a negative coefficient (percentage significant at 5%) in the 433 cross-sectional regressions of portfolio alphas on a linear trend. Panel B contains the percentage of times we find a significant coefficient in the cross-sectional regressions of portfolio alphas on our leverage adjustment variables based on the GS model. Panel C (a) contains the average of the absolute value of the *t*-statistics from regressing the 20 portfolio alphas on our measures of leverage, growth options, and default probability within a seemingly-unrelated-regression equation. Columns (b) and (c) add controls from the four-factor model and the measure of aggregate leverage constraints LCT. All standard errors are adjusted for cross-equation correlation and correct for heteroscedasticity using White's standard errors.

Panel A: Percentage of Significant Slopes

FP alphas	GS alphas	Unlevered Alphas
65%	24%	44%
(52%)	(18%)	(28%)

Panel B: Percentage of Significant Coefficients

Leverage	Growth Options	Default Probability
56%	39%	35%

Panel C: Average |*t*-statistic|

	(a)	(b)	(c)
Intercept	1.998	1.942	1.453
Lev	5.782	5.382	5.085
GO	2.318	2.26	2.441
PD	4.636	4.68	4.950
Mkt		0.301	0.360
SMB		2.129	1.988
HML		2.262	2.113
UMD		0.697	0.654
LCT			0.431

Table 6: Joint Tests of Alpha

This table contains test statistics and associated *p-values* comparing the fit of our estimations of the CAPM to restricted versions of the CAPM without an intercept using a large sample likelihood ratio test (LRT) and the finite sample Gibbons, Ross, Shanken (1989) test. The LRT compares the statistical fit of the unrestricted model with the fit of the restricted model while the GRS is a joint test that the alphas are equal to zero. A *p*-value less than 0.05 indicates that we can reject that the unrestricted and restricted model fits are close for the LRT test and that the alphas of the 20 sorted portfolios are jointly different from zero for the GRS test. The column labelled β^{CAPM}-Alphas uses portfolios sorted on traditional betas. The column labelled β^{GS}-Alphas uses portfolios sorted on GS adjusted betas. The column labelled β^{H}-Alphas uses portfolios sorted on unlevered betas.

	β^{CAPM}-Alphas	β^{GS}-Alphas	β^{H}-Alphas
LRT	97.76	46.65	60.69
p-value	0.000	0.000	0.000
	β^{CAPM}-Alphas	β^{GS}-Alphas	β^{H}-Alphas
GRS	4.03	2.34	3.09
p-value	0.000	0.001	0.000

Table 7: Joint Tests of FP Alpha

This table contains test statistics and associated *p-values* comparing the fit of our estimations of the FP version of the CAPM

$$R_{i,t} = \varphi(1-\beta_i) + \beta_i R_{M,t} + \eta_{i,t}$$

to restricted versions of the FP CAPM without an intercept using a large sample likelihood ratio test (LRT) and the finite sample Gibbons, Ross, Shanken (1989) test. The LRT compares the statistical fit of the unrestricted model with the fit of the restricted model while the GRS is a joint test that the alphas are equal to zero. A *p*-value less than 0.05 indicates that we can reject that the unrestricted and restricted model fits are close for the LRT test and that the alphas of the 20 sorted portfolios are jointly different from zero for the GRS test. The column labelled β^{CAPM}-FPAlphas uses portfolios sorted on traditional betas. The column labelled β^{GS}-FPAlphas uses portfolios sorted on GS adjusted betas.

	β^{CAPM}-FPAlphas	β^{GS}-FPAlphas
LRT	77.28	17.30
p-value	0.000	0.570
	β^{CAPM}-FPAlphas	β^{GS}-FPAlphas
GRS	1.566	0.284
p-value	0.056	0.992

Table 8: Joint Tests of FP Alpha, Sub-Samples

This table contains test statistics and associated *p-values* comparing the fit of our estimations of the FP version of the CAPM

$$R_{i,t} = \varphi(1 - \beta_i) + \beta_i R_{M,t} + \eta_{i,t}$$

to restricted versions of the FP CAPM without an intercept using a large sample likelihood ratio test (LRT) and the finite sample Gibbons, Ross, Shanken (1989) test splitting our sample into two equally sized sub-samples. The LRT compares the statistical fit of the unrestricted model with the fit of the restricted model while the GRS is a joint test that the alphas are equal to zero. A *p*-value less than 0.05 indicates that we can reject that the unrestricted and restricted model fits are close for the LRT test and that the alphas of the 20 sorted portfolios are jointly different from zero for the GRS test. The column labelled β^{CAPM}-Alphas contains the results from the CAPM when the portfolios are sorted on FP betas. The column labelled β^{CAPM}-FPAlphas uses portfolios sorted on traditional betas. The column labelled β^{GS}-FPAlphas uses portfolios sorted on GS adjusted betas. The column labelled β^{H}-Alphas uses portfolios sorted on unlevered betas.

	β^{CAPM}-Alphas	β^{CAPM}-FPAlphas	β^{GS}-FPAlphas
	First Sub-Sample		
LRT	2.87	36.73	22.11
p-value	0.000	0.009	0.279
	Second Sub-Sample		
LRT	2.25	45.74	28.89
p-value	0.002	0.001	0.068

Figure 1: Tilting Towards High Beta or High Leverage Stocks

This figure contains the distributions of the 10,000 average percentage changes in the constrained investors Sharpe ratio when tilting towards high beta or high leverage firms (away from an all market portfolio) from the bootstrap experiment described in section III.B. The solid line is the distribution when tilting towards firms in the highest market value debt-to-equity ratio adjusted for taxes using marginal tax rates from Graham (1996a, 1996b) and fixed-cost (non-cancelable) operating leases (Lev) quantile. The dashed line is the distribution when tilting towards firms in the highest beta (traditional) quantile.

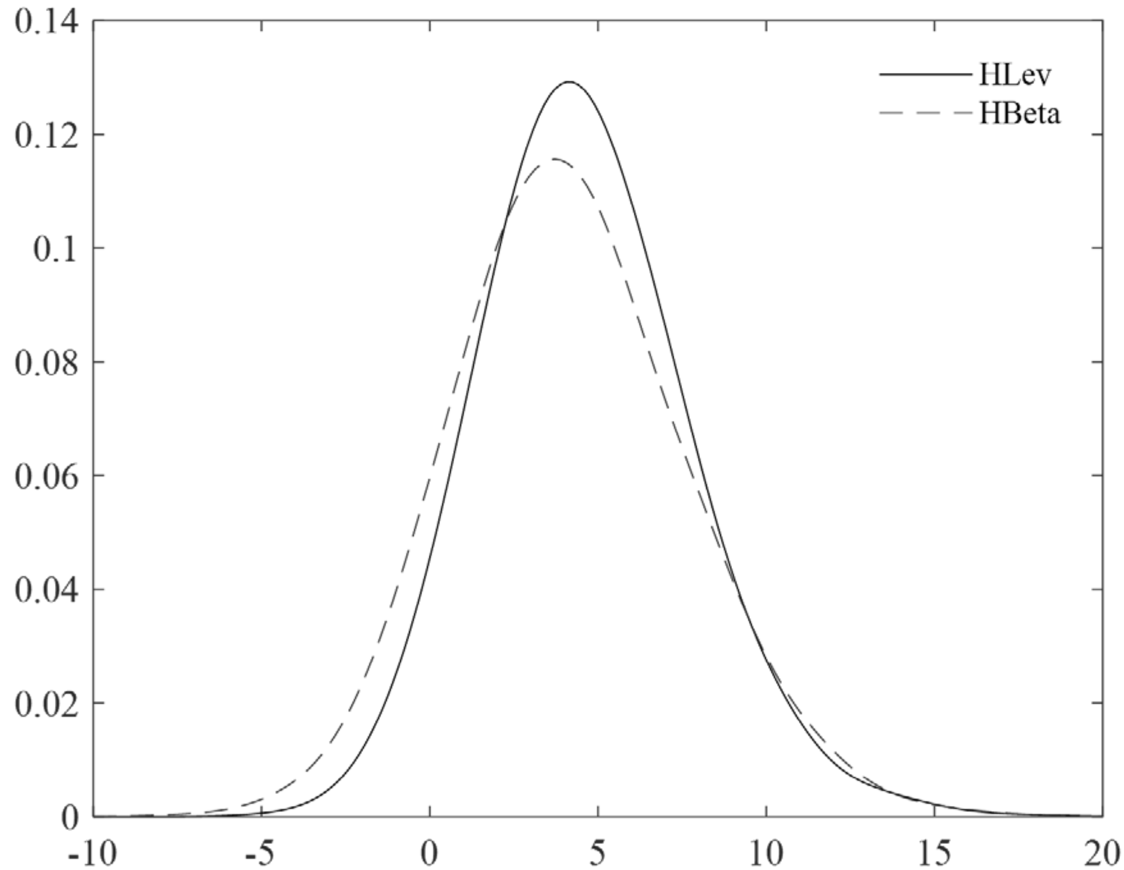

Figure 2: Tilting Towards High Beta/High Leverage or High Beta/Low Leverage Stocks

This figure contains the distributions of the 10,000 average percentage changes in the constrained investors Sharpe ratio when tilting towards high beta and/or different ranked leverage firms (away from an all market portfolio) from the bootstrap experiment described in section III.B. The solid line is the distribution when tilting towards firms in the highest market value debt-to-equity ratio adjusted for taxes using marginal tax rates from Graham (1996a, 1996b) and fixed-cost (non-cancelable) operating leases (Lev) quantile and in the highest beta (traditional) quantile. The dashed line is the distribution when tilting towards firms in the highest beta quantile and the lowest leverage quantile. The dashed/dotted line is the distribution when tilting towards firms in the highest beta quantile.

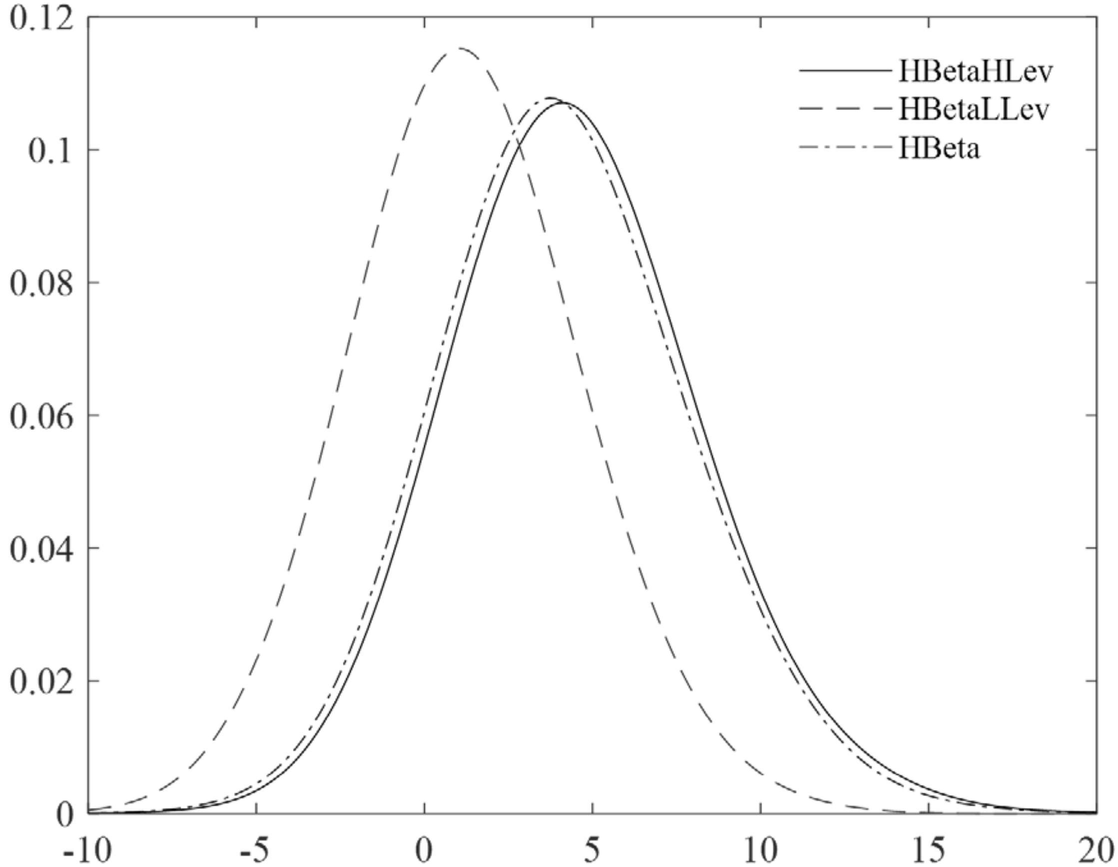

Figure 3: Alphas of Beta Sorted Portfolios

This figure contains the alphas of the 20 sorted portfolios estimated from the CAPM over our sample. The solid line represents the alphas when the portfolios are sorted on lagged traditional betas. The dashed line represents the alphas when the portfolios are sorted on lagged betas adjusted by the GS variables. The dashed/dotted line represents alphas when the portfolios are sorted on lagged unlevered (Hamada) betas.

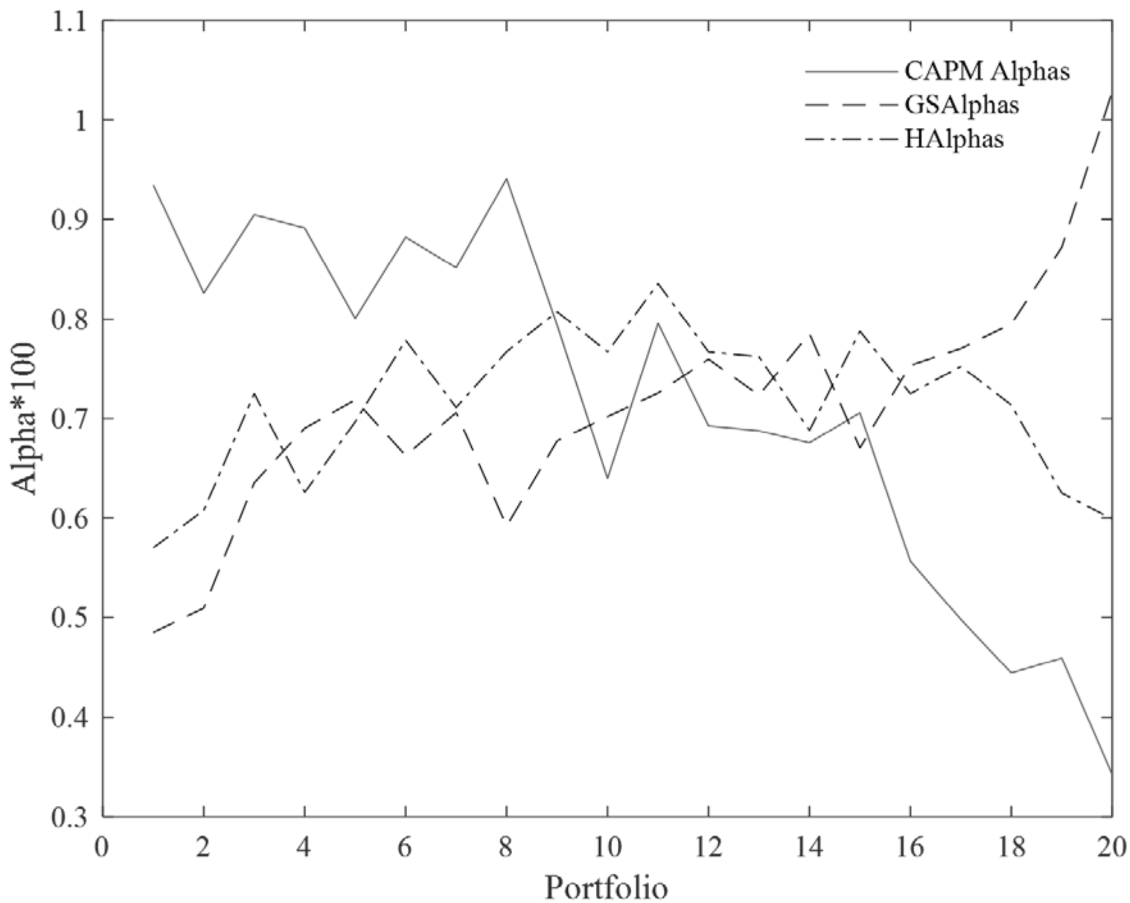

Figure 4: Alpha from the FP Model

This figure contains the alphas of the 20 sorted portfolios estimated from the CAPM and the FP model over our sample. The dotted line represents the alphas from the CAPM when the portfolios are sorted on lagged traditional betas. The dashed line represents the alphas from the FP model when the portfolios are sorted on lagged traditional betas. The dashed/dotted line represents alphas from the CAPM when the portfolios are sorted on lagged betas adjusted by the GS variables. The solid line represents alphas from the FP model when the portfolios are sorted on lagged betas adjusted by the GS variables.

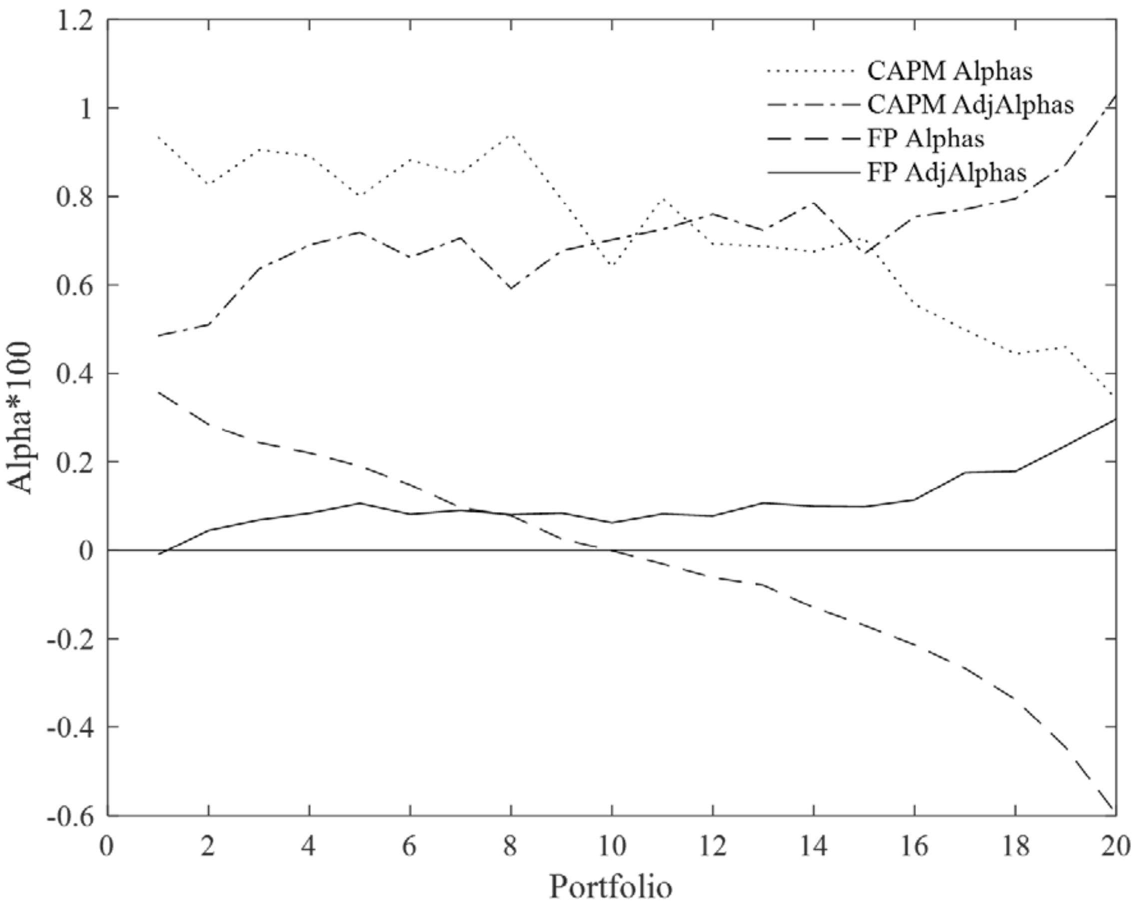

Figure 5: Alphas from the CAPM and FP Model

This figure contains the alphas of the 20 sorted portfolios estimated from the CAPM and the FP model two evenly sized subsamples. The dotted line represents the alphas from the CAPM when the portfolios are sorted on lagged traditional betas. The dashed line represents the alphas from the FP model when the portfolios are sorted on lagged traditional betas. The dashed/dotted line represents alphas from the CAPM when the portfolios are sorted on lagged betas adjusted by the GS variables. The solid line represents alphas from the FP model when the portfolios are sorted on lagged betas adjusted by the GS variables.

Panel A: First Sub-Sample

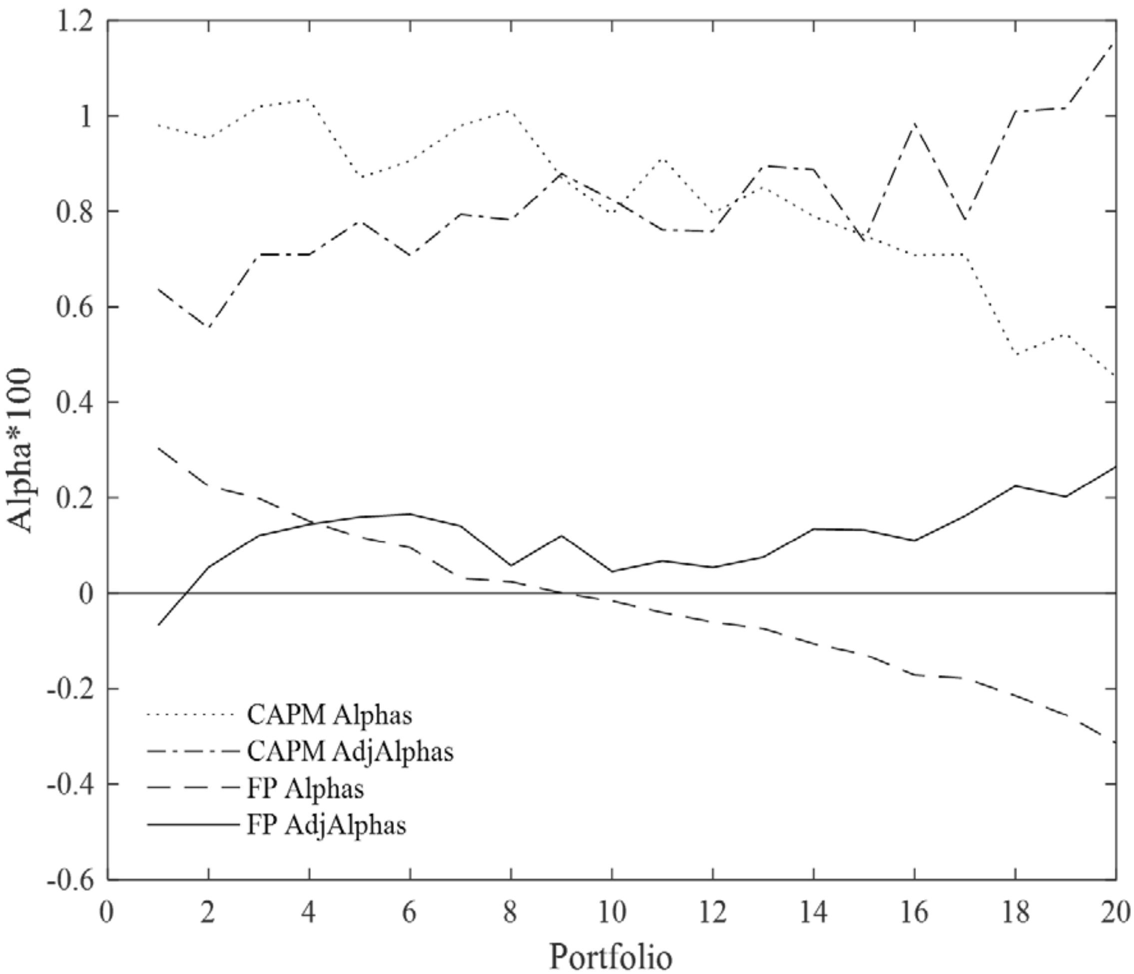

Figure 5: Alphas from the CAPM and FP Model (continued)

Panel B: Second Sub-Sample