Futures, Forwards, and Options

1) Futures and Forwards for managing
   a) Interest rate risk
   b) FX risk

2) To get intuition compare
   a) Spot Contract
      i) t=0: price agreed and paid and goods delivered
   b) Forward contract
      i) t=0: Price agreed
      ii) t=3: Buyer pays and goods delivered
   c) Futures contract
      i) t=0: Price agreed
      ii) 0<t<3: marking to market
      iii) t=3: Buyer pays and goods delivered
   d) Difference between Futures and Forwards
      i) Marked to market
      ii) Forwards are tailor made contracts while futures are standardized
      iii) Forwards have a higher level of default risk

3) Hedging
   a) Naïve hedge
   b) Suppose you hold a 20-year bond FV = $1mill, PV = $970,000, paying a coupon rate of 7.69%
   c) Forecast the interest rates will rise from 8% to 10% in next three months
   d) What do you stand to loose in terms of the value of your portfolio?
      i) You know the price of the bonds will fall
      ii) The price of the bond under the new interest rate of 10% will be $803,336.68
      iii) So your portfolio will fall in value $166.663.32.

   e) What can you do to cover your position?
      i) Sell a Forward contract
      ii) Find someone who will be willing to buy $1 mill FV of 20 year bonds in three months for $970,000
      iii) If your interest rate prediction becomes reality then you are perfectly hedged!
      iv) In three months if the interest rate has risen then you can go to the spot market and purchase the bonds for $803,336.68 and sell them for $970,000 leaving you with $166,663.32 just what you needed.
      v) You have officially *immunized* your assets against the interest rate risk.

4) Two Types of options
   a) **Call:** a right to buy one share at a specified price, most common type of option
   b) **Put:** a right to sell one share at a specified price
      i) An owner of a call option has a right to buy the stock sometime in the future at a predetermined price.

(1) Owner = holder = long position = bullish on the stock - hope price will increase

ii) Writer of call option has an obligation to sell the stock at the predetermined price.
   (1) Seller = writer = short position = bearish on the stock - hope price will decrease

iii) An owner of a put option has a right to sell the stock sometime in the future at a predetermined price.
   (1) long position = bearish on the stock - afraid the price will fall

iv) Writer of put option is obligated to buy the stock at the predetermined price.
   (1) short position = bullish on the stock - hope the price will increase

5) Two families of options
   a) **European** options may be exercised only at expiration
   b) **American** options may be exercised any time before expiration
      i) American options - most common type
      ii) Expiration - always on the Saturday following the third Friday of the stated month.
6) The Jargon
   a) **Exercising the option**: The act of buying or selling the underlying asset via the option contract
   b) **Strike**: The fixed price specified in the option contract at which the holder can buy or sell the underlying asset.
   c) **Expiration**: The last (or only) day an option may be exercised.
   d) **In the money**: exercise would produce a profit for the holder
   e) **Out of the money**: exercise would be unprofitable
   f) **Premium**: Price of the option

7) Option Quotes

Option Quotes

LISTED OPTIONS QUOTATIONS

Option & NY Close	Strike Price	Calls-Last			Puts-Last		
		Jul.	Aug.	Oct.	Jul.	Aug.	Oct.
Ford							
52 7/8	50	4 1/8	6 1/4	r	1	2 1/4	3
52 7/8	55	1 3/4	3 1/8	4 1/4	r	r	r
52 7/8	60	1/2	1 1/2	2 3/8	r	r	r
Ford o	36 5/8	16 1/2	s	s	r	s	s

   a) First column = Name and closing price of the underlying stock
   b) Second Column = Strike price
   c) Calls listed first, then puts, by maturity

d) Option prices are reported on a per share basis, i.e. actual price is found by multiplying the quoted price by the number of shares per contract (usually a round lot of 100).
e) Prices under $3 trade in sixteenths of a point while those over $3 trade in eighths of a point.
f) r = available for trade but none traded during that day
g) s = not offered
h) o = nonstandard terms from stock splits, dividends, spin-offs, etc.

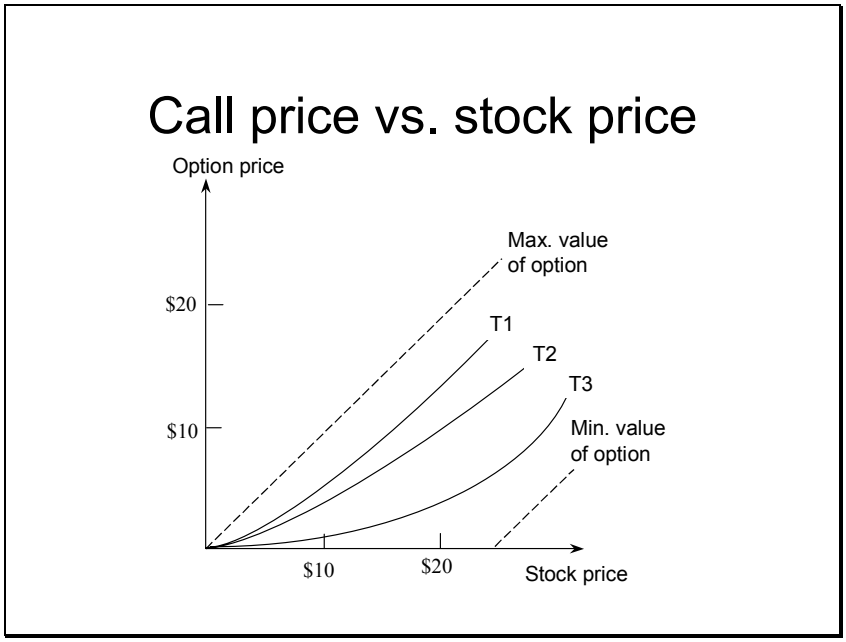
Ex. Want ability to buy 100 shares of Ford for 50 before third Friday in July. Call would cost $4.125*100=$412.5

8) Option Payoffs
   a) Consider purchasing 50 of the July 50 call contracts.
   b) Total cost is 50 * $412.50 = $20,625

	Option position		Stock Value	
Ending stock price	50 contracts	Net profit	390.1 shares	Net profit
$ 40	$ 0	$- 20,625	$ 15,604.0	$- 5,021.0
45	0	- 20,625	17,554.5	- 3,070.5
50	0	- 20,625	19,505.0	- 1,120.0
55	25,000	4,375	21,455.5	830.5
60	50,000	29,375	23,406.0	2,781.0
65	75,000	54,375	25,356.5	4,731.5

   c) If S < $50/ share the option will not be exercised and the loss is $20,625. Limited downside risk.
   d) If the stock price is > $50 the option is "in the money" and will be exercised.
   e) The payoffs are listed in the table.
      i) I.E. if S=$60 then profit is $10/share => option value of $50,000 for the 50 contracts
      ii) NET profit is $50,000 - $20,625 = $29,375
      iii) Compare with purchasing the stock outright: $20,625/52.875 = 390.1 shares.
      iv) NOTE: the variance is much higher for the option position than for the long stock position.
      v) The difference comes from dealing with 5000 shares vs. 390

9) If we plot the prices of an option (given a maturity) against the stock we see several things
   a) Boundaries on the options values
   b) Maximum value of the option is the 45 degree line since the option may not be worth more than the stock. C < S
   c) Minimum value of the option since the options value may not be negative and cannot be less than the stock price minus the exercise price. C > S - X or 0 - point out the strike price



d) These will turn out to be the boundary conditions to differential equation, which must be solved.
e) The value of the option will be higher the further away it is from maturity due to the greater probability of upward movements in the stock price. T1 is the furthest away from maturity
f) Absolute changes in the price of a stock are greater than the price of the option and difference becomes smaller as we move into the money
g) Percentage changes are larger for option prices and again decrease moving into the money
h) Out of the money options are more volatile

10) Option value depends on
    a) Stock price
    b) Strike price
    c) Time to expiration
    d) Volatility of stock price
    e) Risk free rate
    f) Dividend

11) Stock and strike prices:
    a) Call - more valuable as S increase and less valuable as X increases
    b) Put - opposite direction
12) Time to Expiration
    a) For American options more time = more exercise option = more value
13) Volatility
    a) Because options limit downside risk more stock volatility => more value
    b) Call values increase with the risk free rate but put values decrease with the risk free rate.

14) Dividends
    a) reduce the price of a stock on the ex-dividend date. Bad news for calls and good news for put

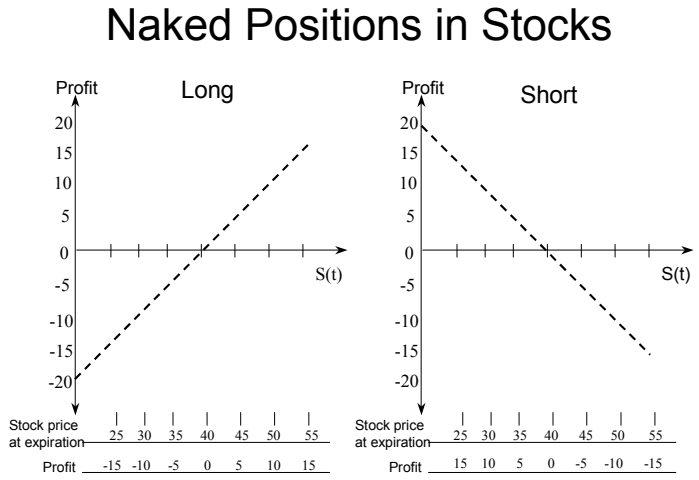
15) Molding payoff patterns:
    a) Investor knows a major news announcement on the horizon but the news itself may be good or bad. Using options profits can be attained if the news is either good or bad. Options allow investors flexibility in designing positions that best reflect their outlook
    b) Timing the market involves smoothing out fluctuations in portfolio value by buying and selling component stock. If the short-term prospects for a stock look grim but long-term prospects still appear good, an investor may want to sell the stock and repurchase it at a later time. Options do this with lower transaction costs.
    c) Hedging: Insurance
    d) Market and price volatility
        i) Cover industry specific risk - unsystematic risks without diversification

16) Pricing Corporate liabilities:
    a) Risky corporate debt and coupon bonds
    b) Warrants and convertibles
    c) Equity (common stock) as an option

17) Pure speculation:

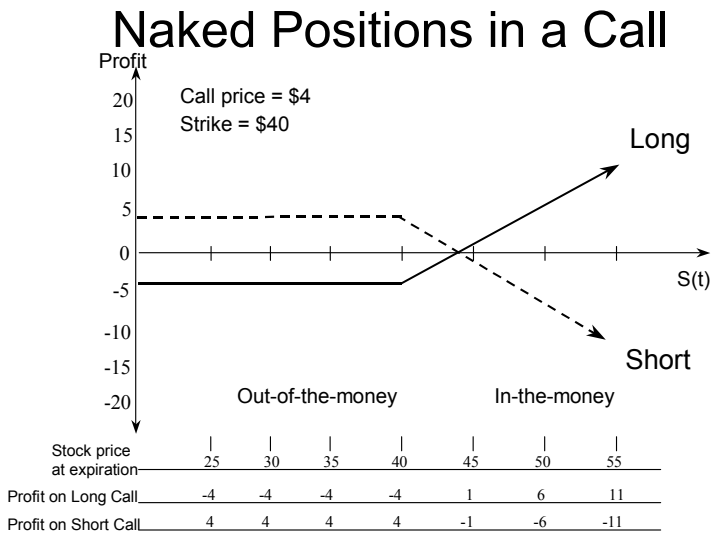
18) Investors can play the market for a fraction of the cost since premiums are usually only a small portion of the stock price.

19) Option Positions
    a) Naked: the stock or option held alone
    b) Hedge: the stock and option held together
    c) Spread: holding a portfolio of options of the same type
    d) Combination: holding a portfolio of options of both calls and puts



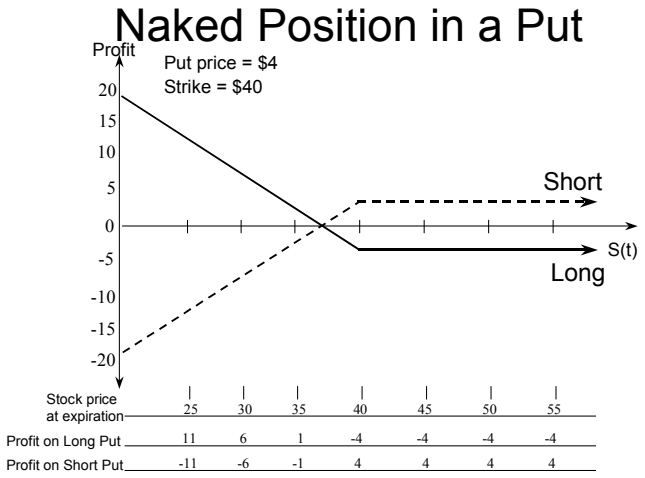
20) Reading the picture
   a) Vertical axis represents the profit
   b) Horizontal axis represents the stock price at any time t
   c) Current stock price = $40

21) Profit graph for a call



   a) Vertical axis represents the profit
   b) Horizontal axis represents the stock price at any time t
   c) Call price (Premium) = $4
   d) Current stock price = $40
   e) Strike price = $44
   f) Owner of the call has the long position
   g) Writer of the call has the short position

22) Profit graph for a put:



a) Vertical axis represents the profit
b) Horizontal axis represents the stock price at any time t
c) Put price (Premium) = $4
d) Current stock price = $40
e) Strike price = $44
f) Owner of the call has the long position
g) Writer of the call has the short positions

Black Sholes

1) The Black-Sholes option pricing model
   a) The formula for the price of a call option on a non-dividend paying stock with a *constant* volatility is

$$c = SN(d1) - Xe^{-rfT}N(d2)$$

$$d_1 = \frac{\ln(S/X) + (rf + \frac{\sigma^2}{2})T}{\sigma\sqrt{T}}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

2) Where:
   i) c = call price
   ii) S = current stock price
   iii) X = strike price
   iv) rf = risk free rate
   v) T = time to maturity
   vi) σ = volatility of the stock price

vii) N(d) = cumulative normal distribution up to point d

b) What the formula says is that the call is equal to the stock price, S, less the discounted value of the exercise price, Xe^{-rfT} where each of these components is weighted by a probability.

c) We can loosely interpret N(d1) and N(d2) as risk adjustments that the call will expire in the money.

d) I.E. If N(d1) and N(d2) are both close to 1, there is a high probability that the option will be exercised.

e) Note:
   i) Investors attitudes towards risk are irrelevant, that is the expected return on the stock does not appear in the equation
   ii) Taking the first derivative of the option value w.r.t. the stock price shows that the option is always more volatile than the stock.
   iii) Since the formula assumes the variance of the optioned asset is constant, it cannot be used to price options on options
   iv) If the stock pays a dividend we may not use the model
   v) The model tends to misprice deep out-of-the-money and deep in-the-money options
   vi) If interest rates or variances of stock prices are not constant or if the stock's returns are not normally distributed, the model will yield inaccurate results

f) **Examples**
   i) Let
      S = $100
      X = $95
      rf = .10
      T = 3 months = .25 year
      σ = 50% per year = .5

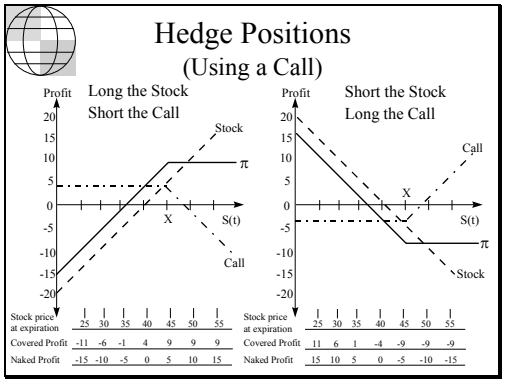
      Plugging all this into the formula gives d1 = .43 and d2 = .18.

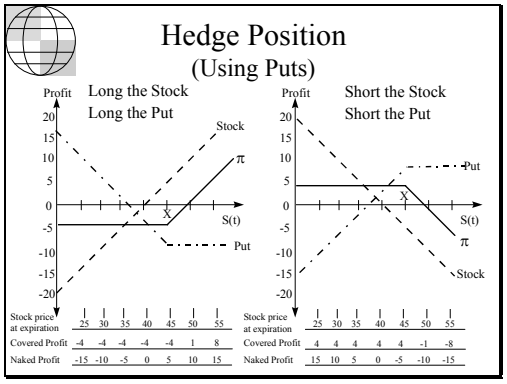
      To find N(d1) and N(d2), take d1 and d2 and plug them into the NORMSDIST function in excel to get N(.43) = .6664 and N(.18) = .5714.

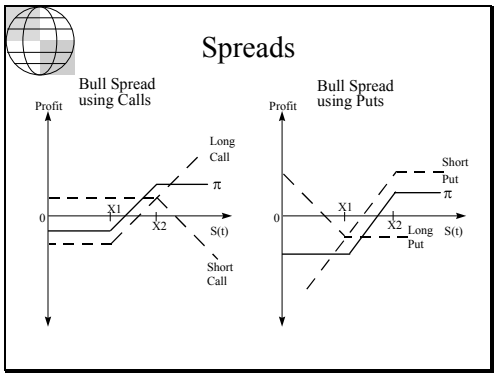
      $c = 100*.6664 - 95(e^{-.1*.25})(.5714) = \$13.70$

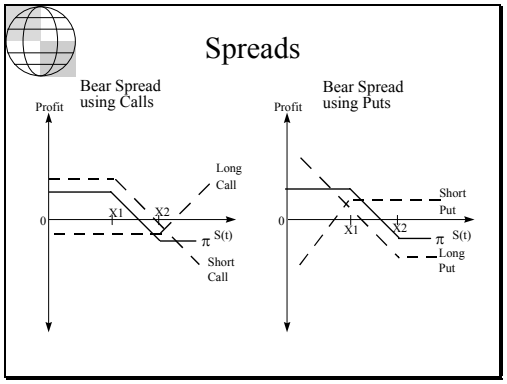
   ii) Keep everything the same as in i) but change X to $100, then C = $11.11
   iii) Keep everything the same as in i) but change T to .5, then C = $18.71
   iv) Keep everything the same as in i) but change S to $110, then C = $21.02
   v) Keep everything the same as in i) but change vol to .8, then C = $19.22

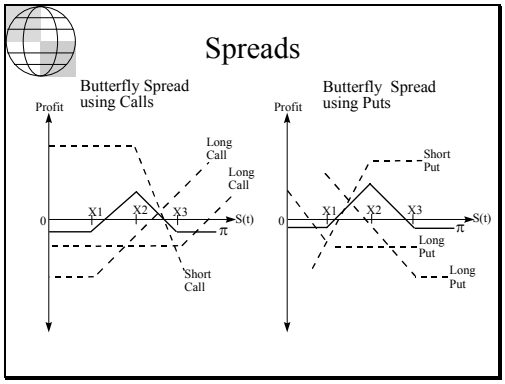
3) Creating synthetic payoffs with options contracts

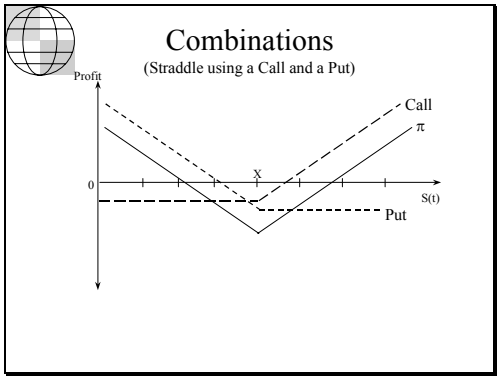

Hedge Positions
(Using a Call)
Profit
Long the Stock
Short the Call
Stock
Call
X
S(t)
π
Short the Stock
Long the Call
Stock price at expiration 25 30 35 40 45 50 55
Covered Profit -11 -6 -1 4 9 9 9
Naked Profit -15 -10 -5 0 5 10 15
Covered Profit 11 6 1 -4 -9 -9 -9
Naked Profit 15 10 5 0 -5 -10 -15


Hedge Position
(Using Puts)
Profit
Long the Stock
Long the Put
Stock
Put
X
S(t)
π
Short the Stock
Short the Put
Stock price at expiration 25 30 35 40 45 50 55
Covered Profit -4 -4 -4 -4 -4 1 8
Naked Profit -15 -10 -5 0 5 10 15
Covered Profit 4 4 4 4 4 -1 -8
Naked Profit 15 10 5 0 -5 -10 -15


Spreads
Bull Spread
using Calls
Profit
Long
Call
Short
Call
X1
X2
S(t)
π
Bull Spread
using Puts
Short
Put
Long
Put


Spreads
Bear Spread
using Calls
Profit
Long
Call
Short
Call
X1
X2
S(t)
π
Bear Spread
using Puts
Short
Put
Long
Put


Spreads
Butterfly Spread
using Calls
Profit
Long
Call
Long
Call
Short
Call
X1
X2
X3
S(t)
π
Butterfly Spread
using Puts
Short
Put
Long
Put
Long
Put


Combinations
(Straddle using a Call and a Put)
Profit
Call
Put
X
S(t)
π

1) *At the money* is when the strike equals the spot.

2) Options profits
   a) Four different ways to use options
      i) buy or sell
      ii) puts and calls

   b) If you buy a call your profit is
      $\pi = S - X - P$

   c) If write a call your profit is
      $\pi = P - S + X$

   d) If you buy a put your profit is
      $\pi = X - S - P$

   e) If you write a put your profit is
      $\pi = P - X + S$

3) Pricing options, Futures, and Forwards
   a) Forwards
      i) Suppose you sell a forward contract so you know you will get the forward price for your commodity. This is a risk-free transaction since you can buy the commodity today, hold it, then sell it at a guaranteed price.
      ii) Hence the forward price must be

      iii) Forward Price = Current price*$(1+Y)^T$

      iv) That is, the forward price is equal to the *future value* of the current commodity price invested at the risk-free yield to maturity for the term of the contract.
      v) We can of course rearrange this to get

      vi) Current price = Forward Price/$(1+Y)^T$

4) Example: What is the forward price to sell a risk-free bond with 15 years to maturity that is currently priced at $1000 with a YTM of 15% and annual payments of $150 in one month?
   a) Find a comparable risk-free bond that is maturing in 1 month. Suppose it is yielding 1% per month.
   b) Calculate the forward price using the formula above.

   Forward Price = Current price*$(1+Y)^T$
   Forward Price = \$1000*$(1 + .01)^{1/12}$ = \$1010.00

5) Example: Suppose we want a forward price to sell in *two years* for the same bond.

a) Find a comparable risk-free bond that is maturing in two years. Suppose it is yielding Y% per month.
b) Calculate the forward price using the formula above.

Since

Current price = $C/(1 + Y)^1 + [C + \text{Forward Price}]/(1+Y)^2$

then

Forward Price = Current price$*(1+Y)^2 - C * (1 + Y) - C$.

c) Keep in mind that the forward price is NOT what the forward costs. It is the agreed upon future price of the commodity.

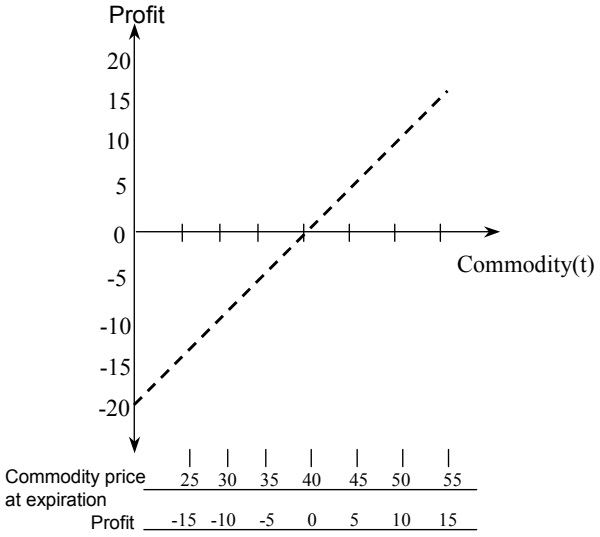
6) Notice that given the equation, the price of the contract at origination is zero.
   a) Of course, after origination the value of a forward will change if the price of the commodity changes.
      i) Since the value of a forward at *expiration* is

         Mkt value = contemporary price – original forward price

      ii) Then the value of a forward after origination but before expiration is

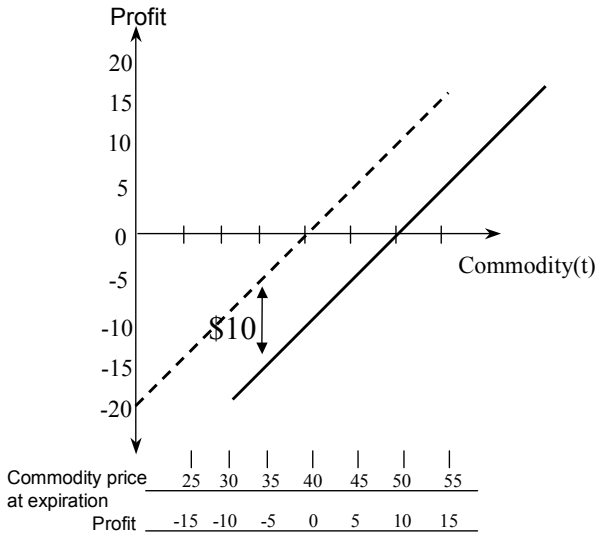
         Mkt value = [contemporary price – original forward price]$/(1+Y)^t$
         where t is the time left until expiration.

7) Another way to think about the value of a forward is to use the graphs we went through last time.
   a) The profit for a forward to buy looks just like a long position in a stock.



b) Here the forward price is \$40. If the commodity price at time (t) is \$45, then the profit from the forward is \$5.

c) Suppose that after a month there is a different forward available for the same commodity with the same expiration date, but it has a forward price of \$50.



d) For every ending commodity price the difference between these two contracts is \$10 so the value of the original contract must be \$10 (or the present value of \$10 discounted by the appropriate risk-free rate).

8) Another way to think about the forward price is in terms of expectations and risk-premia.

Forward price = Expected commodity price – risk premium

a) In general, the risk premium depends on the *covariance* or *correlation* of the commodity price and the level of aggregate wealth.
   i) If the commodity price goes up when the market portfolio goes up, the commodity price and the market portfolio are positively correlated, then a forward contract is insurance for the seller BUT risky for the buyer.
   ii) Why?
      (1) Think of the buyer, the forward contract will pay off big when things are good but will pay off little (or negative) when things are bad. This type of payoff does not add to the utility of the buyer. In fact, this increases the uncertainty of buyers overall portfolio.
      (2) A buyer will demand a lower price then the expected price to be compensated for taking on uncertainty, which implies that the risk premium is positive.
      (3) From the sellers standpoint, the payoffs and marginal utility are the opposite from the buyers. They are going to get big payoffs when the rest of their portfolio is doing poorly and not get much if anything when the rest of their portfolio is doing well. The seller is in effect getting

insurance against bad times and is therefore lowering uncertainty. The seller will be willing to pay the positive risk premium (take a lower price then the expected price) for this insurance.

(a) This situation is called normal backwardation. (I am not making this up)

(4) If the commodity price goes down when the market portfolio goes up, the commodity price and the market portfolio are negatively correlated, and a forward contract is insurance for the buyer BUT risky for the seller.

(a) In this situation we will see a negative risk premium and the forward price will be bigger then the expected commodity price.

(b) This situation is called Contango. (I swear I am not making this up either.)

iii) Think of this stuff in terms of utility and consumption. When you are starving (consumption is very low) a piece of bread and a cup of water will greatly increase your utility. When you are rich and fat (consumption is high) it will take a lot more than a piece of bread and a cup of water to give you the same increase in utility. In other words, rational investors will always prefer an asset that pays off big when things are bad to one that pays off big when things are good.

9) Futures Prices

a) Pricing futures is a lot like pricing forwards but we have to take into consideration two aspects of the differences between forwards and futures.

i) Futures are marked to market

(1) Because of marking to market holders of futures contracts face reinvestment risk.

ii) Futures sometimes stipulate delivery options

(1) Because of delivery options holders of futures must consider a premium for this option.

Futures price = E(commodity price) – risk prem + reinvestment prem – delivery prem.

iii) The risk premium is just like the risk premium in forward contracts.

iv) The reinvestment premium exists because more cash is tied up in margin accounts when investing in futures contracts.

(1) If the commodity price is positively correlated with interest rates, then when the commodity price goes up, buyers will have more cash in their margin accounts, and at the higher interest rates will be earning more on that cash.

(2) This implies that buyers will be willing to take a higher futures price.

(3) The opposite holds true for the sellers who will have less cash earning the higher interest rate and may even have to borrow cash at the higher

interest rate to keep up their margin account. They will want to be compensated for this reinvestment risk with a higher futures price.

v) The delivery premium exists on futures that allow the seller some latitude in what they actually have to produce when the contract comes due.
   (1) I.E. a futures contract for a 10-year Treasury note may stipulate that the seller can produce any Treasury security of the same value at the expiration of the contract. This is an option for the sellers since they may have 5-year notes on hand rather then 10-year notes.
   (2) The option has value to the sellers but is a risk to the buyers who may really want only the 10-year note.
   (3) So the forward price must be lowered to get buyers to take the contract.