

1. Two questions for today.
 - A. Why do bonds with the same time to maturity have different YTM's?
 - B. Why do bonds with *different* times to maturity have different YTM's?

2. To answer the first question lets look at the *risk structure of interest rates*.
 - A. There are three reasons that bonds with the same time to maturity have different YTM's.
 - i. Different probability of default
 - ii. Liquidity
 - iii. Taxes

 - B. Bonds with default risk will always have a positive risk premium.
 - i. Define the risk premium here as the difference (spread) between the interest rates on bonds with default risk and the interest rate on default free bonds.
 - a) Treasuries are essentially default risk free.
 - b) But...corporate bonds and municipals are certainly not default risk free.
 - (1) How do we know how much default risk a corporate bond has?
 - (2) There are two major firms that rank the debt of corporations based on default risk. Moody's and S&P.
 - (a) The best bonds (least likely to default) are those rated AAA, then there are AA, A, BBB, and anything rated below BBB is considered junk.

 - C. Liquidity should obviously have an affect on interest rates. The harder it is to sell the bond the higher the interest rate must be to compensate investors.
 - i. Note that Treasuries are the most liquid of bonds. They have the most active and largest market. It is easy and fast to sell a T-bond.
 - ii. The same cannot be said for corporate bonds or muni's. These markets are much thinner and it takes much more time to convert a bond in these markets into cash.

 - D. Taxes – if a bond gets special tax treatment then it may have a lower interest rate than bonds of similar risk, liquidity, and maturity.
 - i. Muni's – are exempt from federal taxes
 - ii. Flower bonds – treasuries that can be cashed in at par value to pay estate taxes if the owner dies.

 - E. Hence,

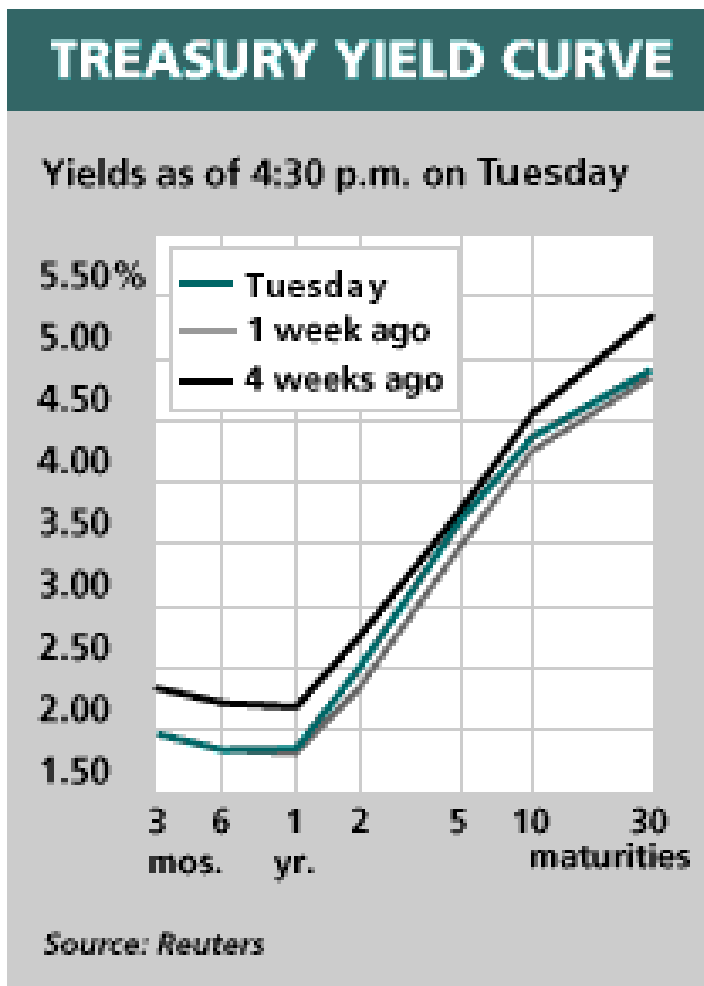
YTM is positively related to default risk.

YTM is positively related to liquidity risk.

YTM is negatively related to special tax treatment.

3. So, why do bonds with *different* times to maturity, but similar risk characteristics have different YTM's?

- A. Another way to put the same question is: Why does the yield curve (term structure) have a given shape?
4. What is the yield curve?
- A. The yield curve is also called the “term structure of interest rates”
- B. The yield curve is simply a plot of yields to maturity versus times to maturity.
- i. The time to maturity is also called the term – hence the name term structure of interest rates.
 - ii. Curve is usually drawn for bonds of equal quality
 - a) Like Treasuries
 - b) AAA or Junk (bonds with ratings below BBB) for corporate bonds.
 - iii. Here is the term structure for Treasury securities from yesterday's WSJ.



- iv. How do you read this?
 - a) For Treasuries with 3 months left to maturity the yield to maturity is just below 2%. For Treasuries with 12 months left to maturity the yield to maturity is about 1.75%. For Treasuries with 10 years left to maturity the yield to maturity is just below 5%.
- C. How does one draw a yield curve?

- i. The way the WSJ has done it is to take the YTM of the Treasury closest to the maturity. Plot the points and connect the dots.
 - ii. The way the book does it is a bit more sophisticated.
 - a) Here they calculate all the YTM's and times to maturities for all available Treasuries.
 - b) Then fit a non-linear spline through the data. This is probably more accurate but is certainly not the only way to estimate a curve through the data.
 - D. Typically, the yield curve is upward sloping – that is long rates are typically higher than short rates. Why?
5. Three theories to explain the term structure
- A. Expectations hypothesis
 - B. Market segmentation theory
 - C. Liquidity preference or Preferred habitat theory.
- D. Expectations hypothesis: Interest rates on long term bonds will equal an average of short-term interest rates that people expect to occur.
- i. This theory assumes that bonds of different maturity are *perfect* substitutes.
 - ii. I.E. investors are indifferent between:
 - a) Buying a one-year bond and holding it to maturity, then buying another one-year bond.
 - b) Buying a two-year bond.
 - iii. If these portfolios are perfect substitutes then the interest rate on the two-year bond should be the average interest rate of the two one-year bonds.
- $$i_{2t} = \frac{i_t + i_{t+1}^e}{2}$$
- iv. And we could extend this out for longer-term bonds too.
 - v. Notice what this implies though.
 - a) If the yield curve slopes up, then short-rates are going to rise
 - b) The yield curve typically slopes up.
 - c) That means if this hypothesis is correct then we would typically see short rates increasing. Since short rates often go up as well as go down, then we have a problem with this theory.
6. Market segmentation theory (the other extreme)
- A. Market for different maturity bonds are completely separate.
 - B. Bonds of different maturities are not substitutes at all.
 - i. Here, people are assumed to only want to purchase bonds that match their investment horizon. - Maybe because they are immunizing their portfolio – we will get to this shortly.
 - ii. At each maturity the supply and demand for bonds determines the interest rate for that maturity.

- iii. More demand for short bonds implies higher prices of bonds and that implies lower interest rates.
- iv. Here we have a theory that is consistent with an upward sloping yield curve that does not suggest that short-term rates will always be rising.
- C. BUT... in this theory there is no reason for bonds of different maturities to be related at all. So, why do long rates typically move up when short rates move up?
 - i. This theory cannot account for the fact that interest rates at different maturities are related.

7. Liquidity preference theory (preferred habitat)
- A. This theory is a combination of the two theories above.
 - B. Interest rates on long-term bonds are an average of expected short-term rates PLUS a term or liquidity premium that depends on the supply and demand for bonds at that maturity.

$$i_{2t} = \frac{i_t + i_{t+1}^e}{2} + k_2$$

- 8. Here, if an investor prefers the one period “habitat” they will normally buy one period bonds, but if the premium for the two-period bond is big enough then they can be induced to substitute a two-year bond for a one-year bond.
- 9. Now we have a theory that explains why rates move together and we can explain why the term structure typically slopes up without the need to say that short rates must be expected to increase.
 - A. The problem with this theory is what exactly determines the term premium.

- 1) What is duration?

$$D = -\frac{(1+R)}{V} \cdot \frac{\partial P}{\partial R}$$

$$P = \sum_{t=1}^T c_t (1+R_t)^{-t}$$

$$\frac{\partial P}{\partial R} = -\sum_{t=1}^T t c_t (1+R_t)^{-t-1}$$

$$\Rightarrow D = \frac{1}{P} \sum_{t=1}^T \frac{t c_t}{(1+R_t)^t}$$

- 2) Well that's nice but what does it mean?
- a) D = present-value weighted average of the maturity of each cash flow.
 - b) D measures the effect of unexpected changes in interest rates on an asset's rate of return.
 - c) Think of it like this.
 - i) Total risk = market risk plus unique (idiosyncratic) risk
 - ii) The risk premium depends on market risk, not unique risk

- iii) Standard deviation measures total risk
 - iv) Beta measures market risk – which is nothing more than changes in the rate of return of an asset due to *unexpected* changes in general economic conditions such as national income or interest rates.
 - v) Duration is the part of beta that depends on unexpected changes in interest rates.
- 3) Duration is simply a measure of the interest rate sensitivity of an asset (or liability).
- a) Large D means that the price of that asset is highly sensitive to changes in interest rates.
 - b) In fact, it is common to see the following approximation.

$$\frac{dP}{P} \approx -D \left[\frac{dR}{1+R} \right]$$

- which says that the percentage change in price is equal to the percentage change in the interest rate times negative duration.
- 4) A reasonable question to ask is just how accurate is that approximation?
- a) The answer is it depends on the size of the interest rate change or shock as it is sometimes called.
 - b) Duration accurately measures the price sensitivity of fixed-income securities for interest rate changes on the order of one basis point. Is that good enough for most interest rate changes?
 - i) The approximation is definitely good enough for the typical movements in interest rates.
 - ii) But what if Federal Reserve Chairman Greenspan decides to up the Fed Funds rate? These changes are almost always 25 or 50 basis point changes.
 - iii) For large interest rate *increases*, duration will *over predict* the fall in bond prices.
 - iv) For large interest rate *decreases*, duration will *under predict* the increase in bond prices.
- 5) Put differently, for rate increase the capital loss effect tends to be smaller than the capital gain effect from rate decreases.
- i) This phenomenon is known as convexity.
 - ii) Let's look at an example.

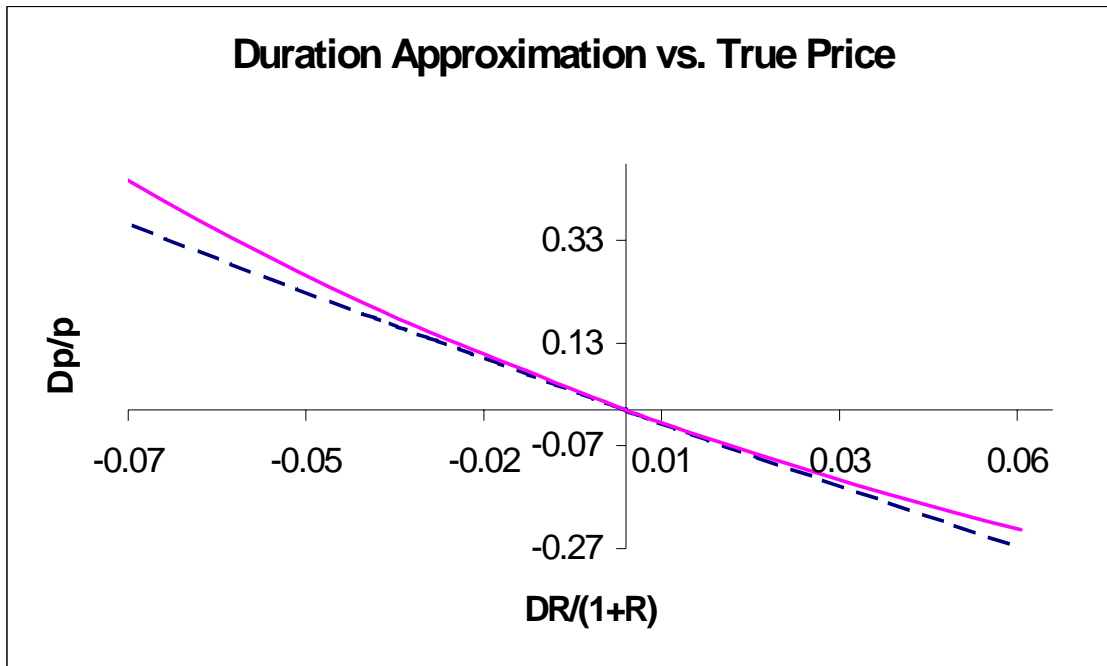
Suppose we have an 8%, \$1000 bond selling at par with six years till maturity and paying annual coupons. This information is in the first row of Table 1.

- 6) I used the Excel duration function to calculate $D = 4.99$.
- a) What happens to the price as the interest rate changes?
- 7) This can be seen in the first two columns of Table.
- a) If the interest rate suddenly goes to zero then the price will become \$1480.

- b) If the interest rate jumps to 14% then the price will become \$766.68. These prices are based on the standard bond pricing equation: the present value of level stream of cash flows and the present value of the face.
- c) The last column of the table shows the percentage change in the price given the changes in interest rates.
- d) Remember that we are looking at the change in price when the interest rate changes from 8%.

Table 1: Duration Approximation

N	Coupon	Rate	Price	Duration			
6	\$80.00	8%	\$1,000.00	4.99			
New Rate	New Price	$-D \cdot dR / (1+R)$	New Duration based price		$dR / (1+R)$	Duration dP/P	True dP/P
0.0%	\$1,480.00	0.384	\$1,384.38		-0.074	0.384	0.480
1.0%	\$1,405.68	0.335	\$1,334.82		-0.065	0.335	0.406
2.0%	\$1,336.09	0.286	\$1,285.67		-0.056	0.286	0.336
3.0%	\$1,270.86	0.237	\$1,236.95		-0.046	0.237	0.271
4.0%	\$1,209.69	0.189	\$1,188.66		-0.037	0.189	0.210
5.0%	\$1,152.27	0.141	\$1,140.80		-0.028	0.141	0.152
7.9%	\$1,004.64	0.005	\$1,004.63		-0.001	0.005	0.005
8.0%	\$1,000.00	0.000	\$1,000.00		0.000	0.000	0.000
8.1%	\$995.39	-0.005	\$995.38		0.001	-0.005	-0.005
9.0%	\$955.14	-0.046	\$954.01		0.009	-0.046	-0.045
10.0%	\$912.89	-0.091	\$908.51		0.019	-0.091	-0.087
11.0%	\$873.08	-0.136	\$863.51		0.028	-0.136	-0.127
12.0%	\$835.54	-0.181	\$819.01		0.037	-0.181	-0.164
13.0%	\$800.12	-0.225	\$775.02		0.046	-0.225	-0.200
14.0%	\$766.68	-0.268	\$731.55		0.056	-0.268	-0.233



- e) We can also estimate that percentage change in price using the duration formula above.
 - i) Column 3 has the percentage change in price given the change in the interest rate in column 1.
 - ii) Again I used the Excel function for duration.
 - f) The prices based on the duration approximation are in column 4.
- 8) Already you can see that the prices implied by the duration approximation are different from the true price changes.
- a) The last three columns of the table have the percentage change in the interest rate, the percentage change in price according to duration, and the true percentage change in price respectively.
 - b) The figure plots the duration estimated percentage change in price and the actual percentage change in price against the percentage change in interest rates.
 - c) In the graph it is easy to see that the larger the interest rate change the worse the estimate of price change predicted by D.
- 9) Remember the curved or convex line is the truth here.
- 10) The amount of curvature in the line what we mean by the term *convexity*.
- a) Attributes of convexity
 - i) Convexity is a good thing.
 - (1) The greater the convexity of a security or a portfolio of securities, the more interest rate protection against interest rate increases and the greater the gains for interest rate decreases.
 - (2) The higher the level of convexity, the bigger the error made by using the duration approximation.

- b) All fixed income securities exhibit convexity.
- c) Convexity increases with maturity
- d) Longer term bonds have more convexity
 - i) This is a positive attribute of long term bonds
- e) Convexity varies with coupon
 - i) For two bonds that are exactly alike except one has a lower coupon, the lower coupon bond will have *more* convexity and higher duration
 - ii) For two bonds with equal duration and yields, the lower coupon bond will have *less* convexity.