1) Portfolio performance measures
   a) You may have heard that a particular mutual fund has consistently "beat" the market.
   b) In fact there are many funds or more precisely, portfolio managers, who have consistently beaten the market in terms of earning a higher expected return.
   c) BUT, look at our utility function for the mean-variance optimizing risk averse investor

   $U = E(R) - .005A\sigma^2$

   d) Is expected return all that this investor (or you for that matter) care about?
      i) The answer is obviously no. We all like higher returns but we dislike variance or risk.
   e) So the question is really, how do we "risk adjust" returns so that we can compare the performance of any particular asset or portfolio with the market?

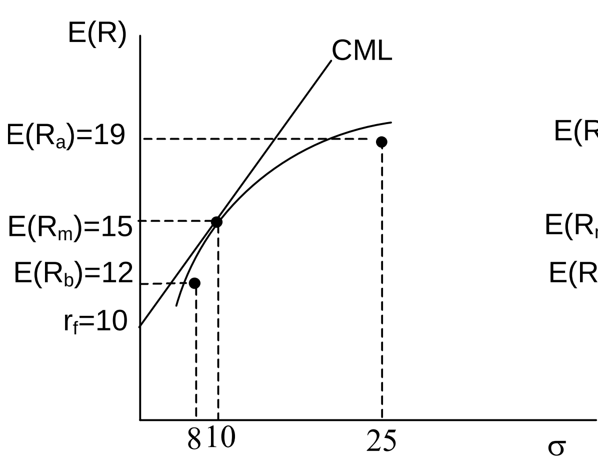
2) There are two general ways we can come up with a risk-adjusted return or benchmark for comparing portfolios. We can use measures based on asset pricing models or we can forget about the models and come up with a measure based on relative performance

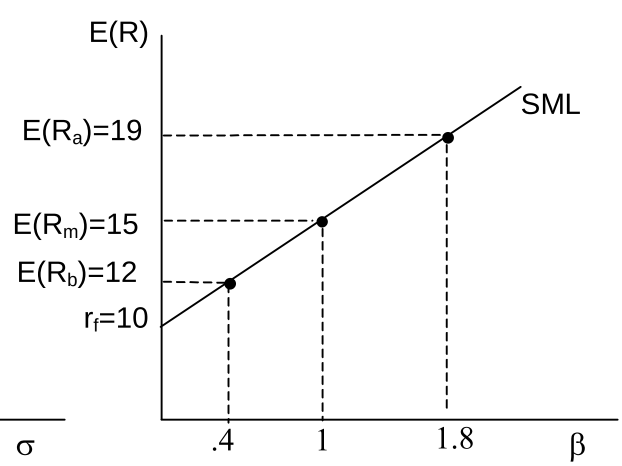
3) Three methods of portfolio performance measurement based on asset pricing models
   a) Jensen's alpha
   b) Treynor index
   c) Sharpe index

   d) Before we go through these, lets consider two mutual funds with the following characteristics

Fund	Expected Return	Std. Dev.	Beta
A	19	25	1.8
B	12	8	.4
Market	15	10	1

4) When we base the fund comparisons on the CAPM we will end up using either CML or the SML. Let's see how they compare.





5) You can check that the returns in the graph are consistent with the CAPM by

   $E(R_i) = r_f + \beta[E(R_m) - r_f]$

   $19 = 10 + 1.8[15-10]$

   and

   $12 = 10 + .4[15-10]$

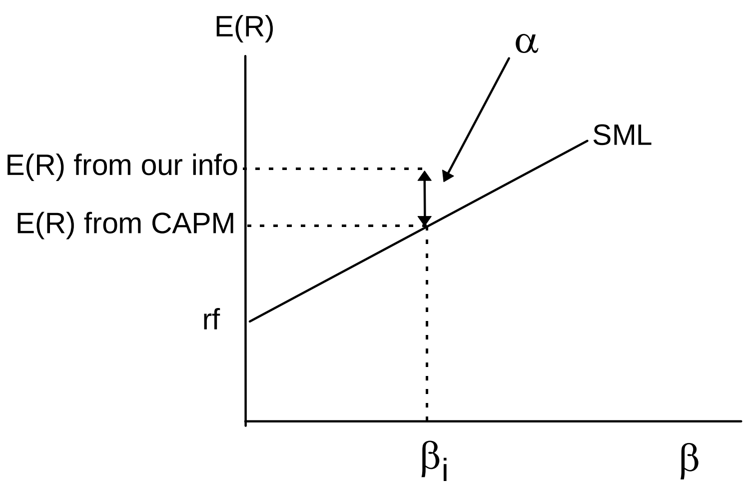
6) Jensen's alpha
   a) We have already seen this measure. Suppose we have inside information on a particular stock that causes us to expect a return that is higher than that predicted by the CAPM. Or suppose that we calculate the β for a mutual fund manager and then observe the return the fund has generated. In particular the CAPM predicts that

      $E(R_i) = r_f + \beta[E(R_m) - r_f]$

   b) But if we suspect that the funds performance will be slightly better/worse than the CAPM then we have

      $E(R_i) = r_f + \beta[E(R_m) - r_f] + \alpha$

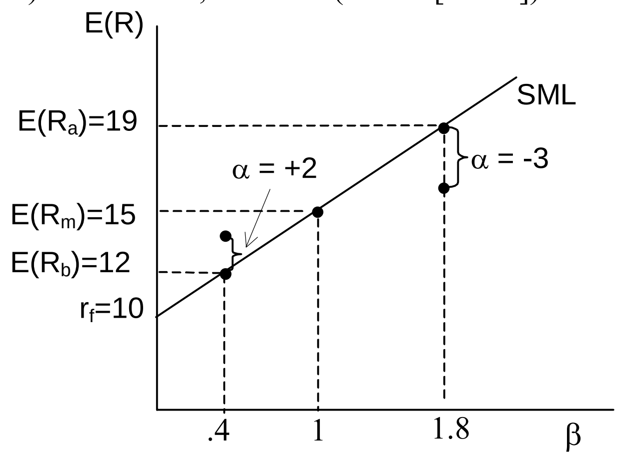
   c) Graphically we have the following result for a positive alpha.



      i) The difference between what the CAPM predicts and what the fund actually produced is known as **Jensen's alpha**.
      ii) If we have inside information and we predict a higher return than the CAPM we may consider the assets as being a good buy (under priced) since the "market" prediction of returns is below what we believe will happen. In this case we would have a positive alpha. If we see a negative alpha then we would think that the stock is a bad buy since it is overpriced by the market.

   d) For example, suppose we believe (expect) the returns for funds A and B are going to be 16% and 14%.
      i) For fund A, $\alpha = 16 – (10 + 1.8[15\text{-}10]) = -3$

ii) For fund B, $\alpha = 14 - (10 + .4[15-10]) = +2$



iii) Note that the alpha of the market is 0 so it looks like fund B will outperform the market by this measure.

e) So we can use Jensen's alpha in a predictive sense, but we can also use it to compare how the funds have done. In this case we can use the historical estimates of the expected return on the fund and the market and the risk-free rate along with the estimated beta to estimate alpha. The bars over the variables in the equation below signify that we are using historical means as the estimates.

$$\hat{J}_p = \overline{r}_p - \hat{\beta}_p \overline{r}_m$$

i) We will do this in the homework so mark this equation!

f) The idea of using Jensen's alpha is that we have taken out the market risk through beta. The measure is risk adjusted since we have removed the market risk.

7) Another way to "take out" market risk is to use the Treynor index.

a) This is similar to Jensen's alpha but instead of subtracting the predicted returns from the CAPM we divide the risk premium by beta.

$$T_p = \frac{E(r_p)}{\beta_p}$$

b) Notice that if our expectations of the returns for both funds match the CAPM predictions then we get the same value.

$T_A = (19-10)/1.8 = 5$
$T_B = (12-10)/.4 = 5$

c) If our expectations are different we get different numbers. For example, suppose we believe (expect) the returns for funds A and B are going to be 16% and 14%.

$T_A = (16\text{-}10)/1.8 = 3.33$
$T_B = (14\text{-}10)/.4 = 10$

i) The asset with the higher Treynor index is the better buy. This index takes the opportunity to lever a position into account when comparing assets. It is essentially the slope of a line running through the risk-free rate and the asset in mean-beta space. The line with the steepest slope offers the best investment opportunity.
ii) Note that the T of the market is 5 so it looks like fund B will again outperform the market by this measure.

d) So we can use the Treynor index in a predictive sense, but we can also use it to compare how the funds have done. In this case we can use the historical estimates of the expected return on the fund and the risk-free rate along with the estimated beta to estimate alpha. The bars over the variables in the equation below signify that we are using historical means as the estimates.

$$\hat{T}_p = \frac{\overline{r}_p}{\hat{\beta}_p}$$

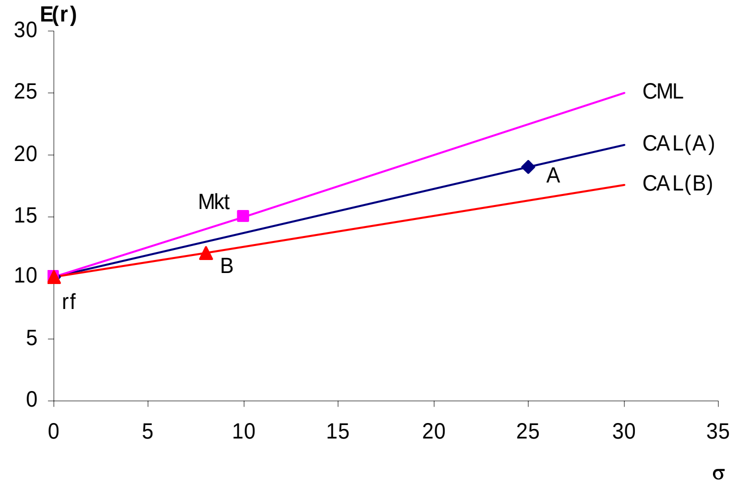
8) The Sharpe index is slightly different then the above index measures. It uses the CML as a benchmark rather then the SML. The Sharpe index is the reward to variability ratio.
a) The Sharpe index does not "take out" market risk, but it does account for diversification.

$$S_p = \frac{E(r_p)}{\sigma_p}$$

b) For example, suppose we believe the expected returns for funds A and B above. Then we have.

$S_A = (19\text{-}10)/25 = .36$
$S_B = (12\text{-}10)/8 = .25$

i) The asset with the higher Sharpe index is the better buy. This index also takes the opportunity to lever a position into account when comparing assets. It is essentially the slope of a line running through the risk-free rate and the asset in mean-standard deviation space. The line with the steepest slope offers the best investment opportunity.
ii) Notice that by this measure B is NOT the better buy and neither asset is better then investing in the market.



c) What happens if our expectations are different? For example, suppose we believe (expect) the returns for funds A and B are going to be 16% and 14%.

$S_A = (16\text{-}10)/25 = .24$
$S_B = (14\text{-}10)/8 = .5$

d) So we can use the Sharpe index in a predictive sense, but we can also use it to compare how the funds have done. In this case we can use the historical estimates of the expected return on the fund and the risk-free rate along with the estimated beta to estimate alpha. The bars over the variables in the equation below signify that we are using historical means as the estimates.

$$\hat{S}_p = \frac{\overline{r}_p - \overline{r}_f}{\sigma_p}$$

9) Which measure is best?
   a) The measures all depend on knowledge of the true Market portfolio or how close our proxy is to the portfolio.
   b) The first two measures also depend on the CAPM being the truth! This is certainly not the case so there will be error from that assumption.
      i) Estimated Betas using different market indexes will be different and could change the ordering of the rankings.
   c) Furthermore, the measures all assume we have accurate measures of EXPECTED return and standard deviations. How accurate our expectations are will affect the measures also.

10) There are other methods of comparing assets that are covered in the text. I encourage you to read that chapter even though I will not cover the material in class.

APT + Interest rate theory

11) <u>Arbitrage</u>: *What is arbitrage?*

a) There are two different types of arbitrage
   i) Pure Arbitrage
   ii) Risk Arbitrage

b) **Pure arbitrage** occurs when an investor can create a *zero net investment portfolio* that provides a sure (riskless) return.
   i) We can define this type of arbitrage via the *law of one price*.
      (1) The law of one price stipulates that securities with identical risk and payoff characteristics must sell for the same price.
      (2) Violation of this "law" allows for arbitrage. A simple example of pure arbitrage is when an investor can simultaneously purchase an asset at a given price in one market and sell it for a higher price in another market. Because the transactions take place simultaneously, the investor does not put out any money and hence has a costless means to make a sure profit.
      (3) If market participants are rational and information flow is uninhibited, then this costless/risk-free type of arbitrage should *never* exist.
         (a) Hence the name, "*the law of one price*".

c) Risk Arbitrage:
   i) This type of arbitrage involves an in investor looking for mispriced securities in specific areas such as merger targets.
   ii) *This does not mean that the investment will be self-financing or risk free.*

The idea that equilibrium market prices ought to be rational in the sense that prices will move to rule out arbitrage opportunities is the arguably the most fundamental concept in capital market theory. Violation of this restriction would indicate serious market inefficiency.

12) It is important to note that:
   a) The CAPM is based on a ***mean-variance dominance argument***, which is *weaker* than a ***no arbitrage argument***.
   b) One of the assumptions of the CAPM is that all investors have the same information and expectations and that everyone processes that information in the same way.
   c) If assets are mispriced in the CAPM world *ALL* investors will move out of the overpriced assets and into the underpriced assets until equilibrium is achieved.
   d) In a no-arbitrage world, it only takes *ONE* investor to restore equilibrium. This implies that asset-pricing theories based on no-arbitrage arguments are much stronger than those based on mean-variance dominance arguments.

13) The Arbitrage Pricing Theory (APT)
   a) The APT assumes that returns are generated by a single index model as in the previous set of notes and on the assumption that in *well functioning markets* there are *no arbitrage opportunities*.
   b) As before, we define the returns on any asset by

$$r_i = E(r_i) + \beta_i F + e_i$$

i) The factor F is some macro economic variable explaining uncertainty in an asset's returns.
ii) In this model, the F is assumed to have an expected value of zero which means we should think of F as being the deviation of the common factor from its mean. (subtract the average value of F from all the observations of F)
iii) This allows the interpretation of F as representing the affect of *new* information. Think of unexpected changes in GNP as being a possible F for the model.

c) Here we also have the e_i being firm specific information, which has an expected value of zero and is uncorrelated with other firm specific shocks.
   i) And as before, if we form an equally weighted portfolio of *n* stocks we can see that diversification works by noting that for a portfolio *p* we have

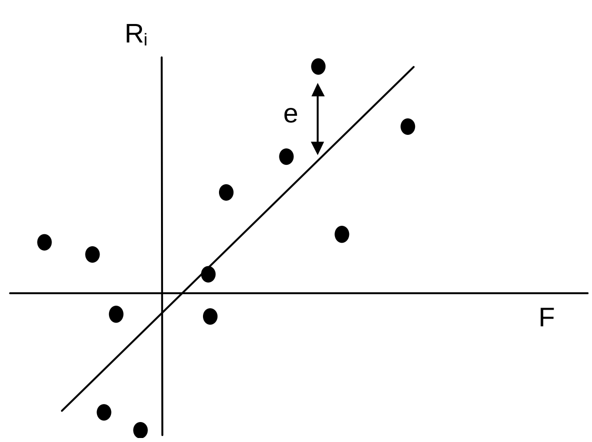
$$var(e_P) = (1/n)^2 var(\Sigma e_i) = 1/n(\overline{\sigma}_e^2)$$

   ii) The variance of the unexpected part of returns goes to zero as n goes to infinity in a well-diversified portfolio.

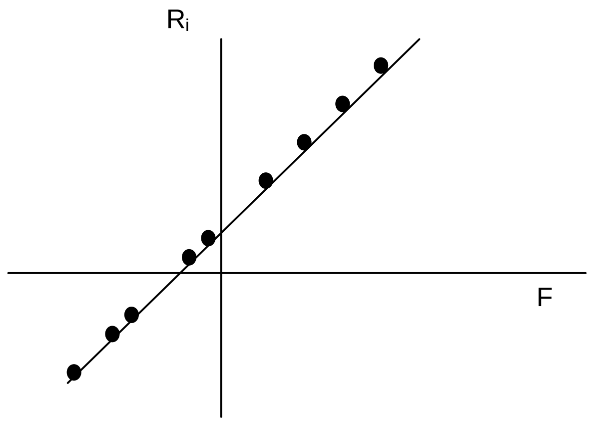
d) Note:
   i) The text notes that the weighting on a portfolio does not have to be equal to get diversification to work. (w_I does not have to equal 1/n)
   ii) It is sufficient to have the weights on each asset in the portfolio getting smaller and smaller as we add more assets to the portfolio to get the idiosyncratic risk to go to zero.
   iii) What this implies is that for a sufficiently large portfolio of assets we have
      1. $r_p = E(r_p) + \beta_p F$
      2. and
      3. $\sigma^2_p = \beta^2_p \sigma^2_p$

This elimination of firm specific risk through diversification implies that only factor risk commands a risk premium. *(This is consistent with the single factor CAPM)*

14) Remember the plot of an assets return against a factor from the single index model?



a) The idiosyncratic risk shows up in the graph as the difference between the individual points and the characteristic line.
b) For a well diversified portfolio the picture will look like this then



c) That is, diversification eliminates the idiosyncratic component of returns so that portfolio returns are completely determined by the factor.
d) Think of the above graph as plotting the returns for a well-diversified portfolio for different realizations of the factor.

15) The risk-return relationship:
a) To derive the risk-return relationship for well-diversified portfolios consider two portfolios P and Q with returns defined as
$r_p = E(r_p) + \beta_p F$
$r_q = E(r_q) + \beta_q F$

b) Let's form a new portfolio C from these two assets with w in P and (1-w) in Q. The returns on that portfolio are then

$$r_c = w(E(r_p) + \beta_p F) + (1-w)(E(r_q) + \beta_q F)$$
$$= wE(r_p) + w\beta_p F + E(r_q) + \beta_q F - wE(r_q) - w\beta_q F$$
$$= w[E(r_p) - E(r_q)] + [w(\beta_p - \beta_q) + \beta_q]F + E(r_q)$$

c) Now if we choose a $w^* = \beta_q/(\beta_q - \beta_p)$ we have

$$r^*_c = \beta_q[E(r_p) - E(r_q)]/(\beta_q - \beta_p) + E(r_q)$$

d) But r^*_c is a constant and so $r^*_c = r_f$. That is

$$r_f = \beta_q[E(r_p) - E(r_q)]/(\beta_q - \beta_p) + E(r_q)$$
$$r_f(\beta_q - \beta_p) = \beta_q[E(r_p) - E(r_q)] + E(r_q)(\beta_q - \beta_p)$$
$$r_f\beta_q - r_f\beta_p = \beta_q E(r_p) - \beta_q E(r_q) + E(r_q)\beta_q - E(r_q)\beta_p$$
$$r_f\beta_q - r_f\beta_p = \beta_q E(r_p) - E(r_q)\beta_p$$
$$\beta_p E(r_q) - r_f\beta_p = E(r_p)\beta_q - r_f\beta_q$$
$$\beta_p(E(r_q) - r_f) = \beta_q(E(r_p) - r_f)$$

$(E(r_q) - r_f)/\beta_q = (E(r_q) - r_f)/\beta_p$

e) This is true for any two well-diversified portfolios P and Q.

16) These ratios are just a number, call it K, so we can write

$(E(r_q) - r_f)/\beta_q = (E(r_p) - r_f)/\beta_p = K$

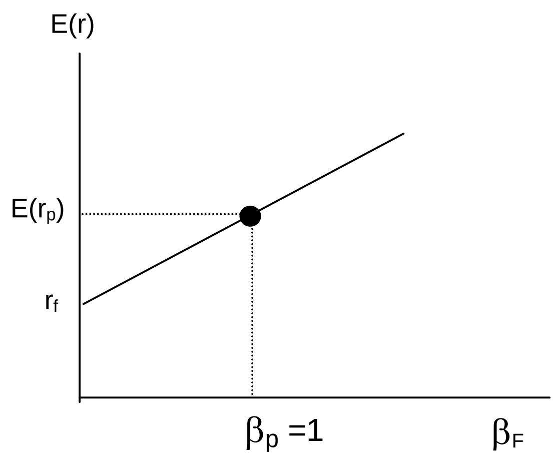
or just

$(E(r_p) - r_f)/\beta_p = K$

or

$E(r_p) = r_f + \beta_p K$

a) In words we have ***K as the excess return per unit of risk associated with the factor***.
b) If we choose a portfolio P that is *perfectly correlated with the factor F* so that its beta = 1, then we can interpret K easily. In particular, $K = E(r_p) - r_f$.
c) If we plot the expected return of the portfolio against its factor beta we get a picture like this.



i) At the intercept we have the expected return on the portfolio when the systematic factor is zero (*no macro economic surprises*).
ii) Note also that the point at the intercept must be the risk-free rate since in this model there are only two sources of risk. The idiosyncratic risk, which we *eliminated* by diversification, and the factor risk. If the factor risk is zero then the return on the portfolio must be the risk free rate to eliminate arbitrage opportunities.

17) Given that F is the *only factor* in the economy, is it possible for any other well diversified portfolio to have an expected return–beta relationship that is not on this line? *No, this would provide an arbitrage opportunity!*

We can thus conclude that in the absence of arbitrage opportunities, ALL well-diversified portfolios must lie on the straight line from the risk-free asset. The equation of this line determines the expected return on ALL well diversified portfolios.

18) This also implies that the risk premium for a well-diversified portfolio is proportional to its factor beta.

a) In particular we have

$$E(r_p) = r_f + [E(r_m) – r_f]\beta_p$$

or

$$[E(r_m) – r_f] = [E(r_p) - r_f]/\beta_p$$

b) Note that because all well diversified portfolios lie on the same line that it must always be the case that for any two well-diversified portfolios P and Q

$$[E(r_p) - r_f]/\beta_p = [E(r_Q) - r_f]/\beta_Q$$

19) Consistency of the APT and the CAPM

a) To show the consistency of the APT with the CAPM pick the well diversified portfolio to be the market portfolio and let the factor of interest be the unexpected return on the market portfolio.

i) In this situation, we have a line that is identical to the SML from the CAPM.

ii) The cool thing is that we can derive the SML and the CAPM relationship between risk and return without relying on the assumption that the well-diversified portfolio is the market and that all people hold the market.

iii) In fact, any well-diversified portfolio that lies on the SML will serve as a benchmark portfolio as long as it is *the most highly correlated portfolio with whatever systematic factor* thought to affect stock returns.

iv) **The APT is therefore a more powerful and flexible view of an asset pricing equilibrium than is the CAPM.**

v) We have shown that precluding arbitrage implies a linear relationship between the expected returns on a well-diversified portfolio and the factor risk as described by the factor beta

20) Does this relationship hold for individual assets too?

a) To show this must be the case, begin by assuming that for any two assets *i* and *j* that

$$[E(r_i) - r_f]/\beta_i = [E(r_j) - r_f]/\beta_j = K$$

b) But this implies that for any single asset

$$[E(r_j) - r_f]/\beta_j = K$$
$$[E(r_j) - r_f] = \beta_j K$$
$$E(r_j) = r_f + \beta_j K$$

c) And if we put a bunch of assets into a portfolio with weights w_i we have

$$E(r_p) = \Sigma w_i E(r_p) = r_f \Sigma w_i + K\Sigma w_i \beta_i$$

d) But since

$\Sigma w_i = 1$ and $\Sigma w_i\beta_i = \beta_p$

e) We have

$E(r_p) = r_f + K\beta_p$

f) And for all portfolios we have

$[E(r_p) - r_f]/\beta_p = K$

g) Which implies that

$[E(r_p) - r_f]/\beta_p = [E(r_q) - r_f]/\beta_q = K$

And so it must be the case that if the expected return-beta relationship holds for all single assets, then it will hold for all portfolios.

21) But, what we want to show is that the relationship $[E(r_i) - r_f]/\beta_i = [E(r_j) - r_f]/\beta_j$ between individual assets holds.
   i) Consider that the above relationship does not hold for two individual assets but that when we combine them into two well-diversified portfolios that it is still true that $[E(r_p) - r_f]/\beta_p = [E(r_q) - r_f]/\beta_q$ is true.
   ii) Now, consider a third well-diversified portfolio. The chance that the violation of the no arbitrage relationship for the individual securities exists while the relationship between the well-diversified portfolios still holds is very small.
   iii) Now consider an infinite number of well-diversified portfolios, which satisfy the no-arbitrage relationship. The probability that two assets exist that do not satisfy the relationship is virtually zero.
   iv) However, it is not strictly zero!

Therefore we may conclude that imposing the no-arbitrage condition on a single-factor security market implies maintenance of the expected return-beta relationship for all well diversified portfolios and for all but, possibly, a small number of individual securities.

22) APT, Single Index Model, and the CAPM
   a) The really nice thing about the APT is that it is based on the powerful no-arbitrage principle and does not rely on the assumption of a "market portfolio".
      i) The problem with the APT is that does not provide as strong a conclusion as the CAPM. The APT only provides that the return-beta relation holds for all but a small number of securities.
   b) The CAPM on the other hand, provides that this relation holds for ALL assets. This is an important difference.
   c) If we consider the Single Index Model we have to add the assumptions that a specified market index is virtually perfectly correlated the theoretical market portfolio and that past

realized returns provide valid estimates of expected returns and variances. Even here though, we need all investors behaving in a like manner. That is the Single Index Model, like the CAPM relies on mean-variance dominance arguments.

23) The Multi-factor APT

a) It should be obvious that there are more than one macro economic factor that affects the returns of an asset. Possible factors that might be considered are things such as interest rate fluctuations, oil price shocks, inflation rates, business cycles and such.

b) The factor model is easily generalized to a two factor model as

$$r_i = E(r_i) + \beta_{i1}F1 + \beta_{i2}F2 + e_i$$

c) The model provides **no** guidance as to what good factors will be though.

1) Determinants of the level of interest rates

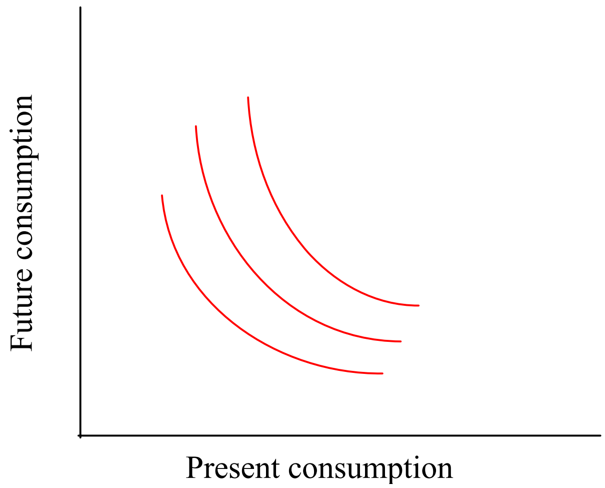
a) We will talk about "the interest rate" but or course there is no *single* interest rate
   i) **Federal Funds rate:** overnight rate that banks charge other banks for loans
   ii) **Discount rate:** rate the Fed charges to banks who need to borrow money
   iii) **Prime rate:** interbank lending rate
   iv) **Commercial paper rate:**
   v) **Certificate of deposit rate:**
   vi) **T-bill rate:**
   vii) When we say the "interest rate" in class, what we really mean is the appropriate rate given the level of risk for that particular investment.

b) Real vs. Nominal interest rates
   You may say that the interest rate is the "*cost of money*"
   i) But
      (1) No one buys money
      (2) You can't eat money
      (3) You can't make a car out of money
   ii) At the most abstract level, what is an interest rate?
      (1) We use money to consume or produce things.
         (a) One way to think of the interest rate is in terms of trading some real amount of consumption in one time period for some real amount of consumption in another time period.
      (2) However, borrowing and lending contracts are stated in *nominal* terms.

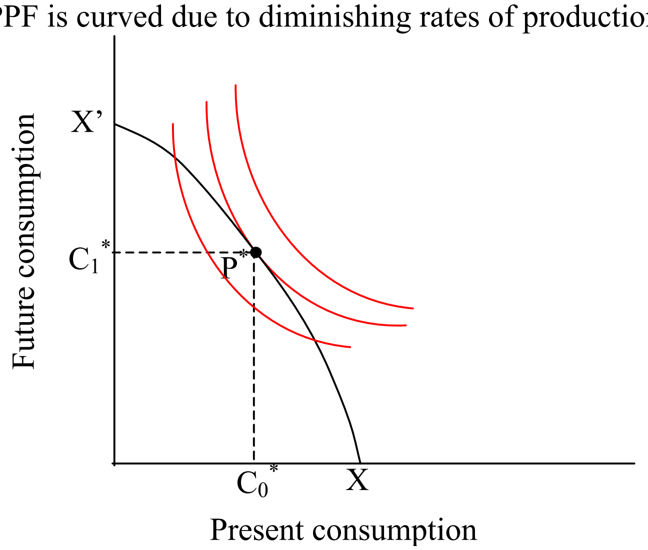
   iii) The nominal rate of interest

$$\mathbf{i \approx real\ rate + anticipated\ inflation}$$
$$\mathbf{i \approx r + g}$$

c) What determines the real interest rate?
   i) Starting at the level of abstraction that the interest rate is the price of future consumption in terms of consumption today, let's build a model and see if we can figure out how interest rates are determined.
      (1) Consider an individual with the following attributes
         (a) Two-period time horizon
         (b) Only cares about one commodity, say corn
         (c) Endowment of corn equals X
         (d) Convex preferences for present and future consumption of corn
         (e) No opportunities to invest across time periods
         (f) The individual is rational implying she maximizes intertemporal utility
         (g) Knows the future with certainty.



(2) The curves are indifference curves relating present and future consumption.
(3) If the individual can produce corn, then she will also have an intertemporal production possibilities frontier (PPF).
   (a) This is the curve X'X below.
   (b) X on the horizontal axis is the endowment of corn – the harvest from today
   (c) If all of the harvest is replanted then next period she will have X'.
   (d) The PPF is curved due to diminishing rates of production given a plot of land.



(4) Since there are no opportunities to trade, the PPF also plots out the combinations of present and future consumption.
   (a) If the entire endowment of corn is consumed today then $C_0 = X$.
(5) Given that our individual is maximizing utility, she will decide to consume C_0* today and save $X - C_0*$ so she may consume C_1* next period. She will produce at the P^* point along her PPF. Why?

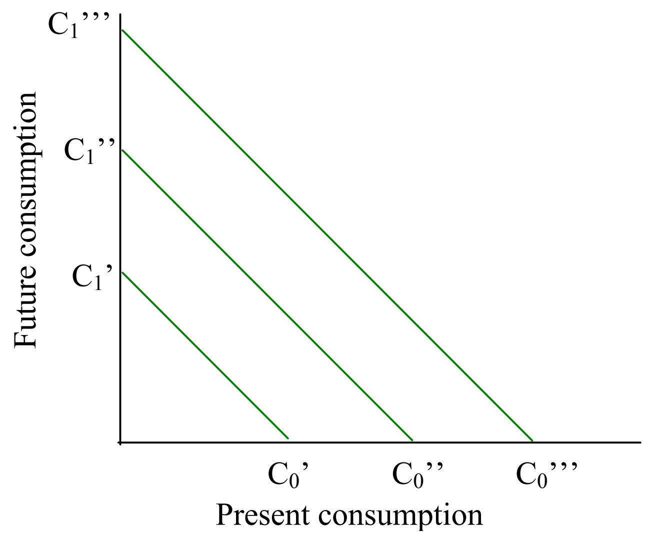
ii) Now, relax the assumption of no investment and allow for the possibility of exchange between current and future consumption and suppose that the rate of exchange across time periods is called r.
   (a) Now suppose the agent takes the endowment and simply consumes it all today. Call that level of current consumption C_0'. If we think about the total

wealth of the agent over both today and next periods as W, then in this case total wealth today $W = C_0$'

(b) What if the agent consumes nothing today but instead invests that C_0' today? She we then get to consume $C_1' = C_0' + C_0'r = C_0'(1+r)$ next period so that total wealth over both periods today $W = C_1'/(1+r)$.

(c) Of course she could consume some today and invest some so that total wealth today is $W = C_0 + C_1/(1+r)$

(d) Given the rate of exchange r, this function shows us the levels of current and future consumption that will give us a particular level of wealth.



(e) The lines above represent different levels of wealth with the slope of the lines being the rise over the run

$$\frac{C_1^{'}}{C_0^{'}} = 1 + r$$

(i) r is the rate at which your investment grows.

(ii) At this rate, trading in the market is possible between present and future consumption.

(iii) As r increases, the line will become steeper.

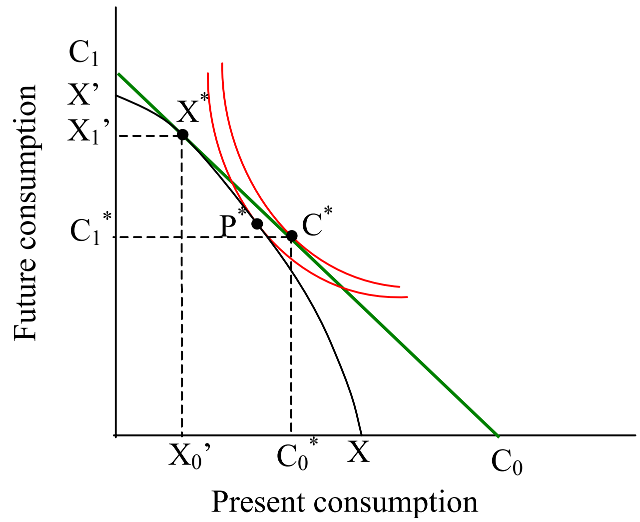
(2) With investment the agent can reach a higher level of utility

(i) Previously our agent would have taken her endowment of corn X, consumed some and planted some so that she would be at point P*.

(ii) With investment, our agent faces a two step procedure

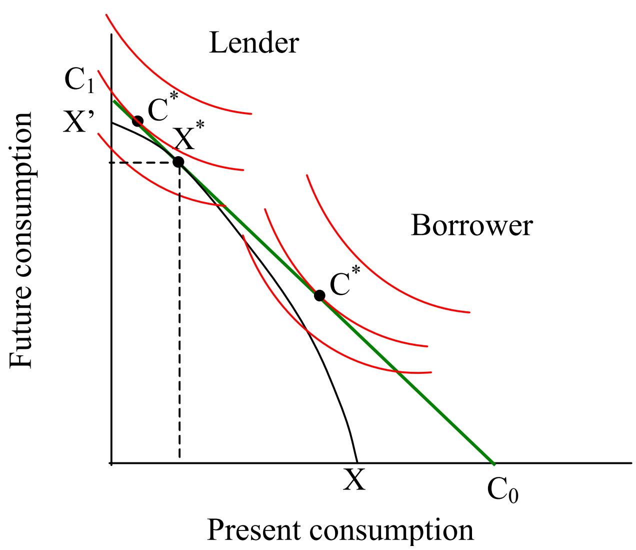
1. First, maximize wealth by producing at X^*, leaving X_0' corn to consume this period and X_1' corn to consume next period.
2. But instead of doing that she will give up $X_1' - C_1^*$ of the future consumption to gain $C_0^* - X_0'$ today, i.e. she will maximize utility of consumption.

(iii) In other words she will be able to reach point C^*, which is on a higher indifference curve than P^* by *borrowing*.



(iv) Note that the determination of how much to produce is independent of the utility structure of the individual.

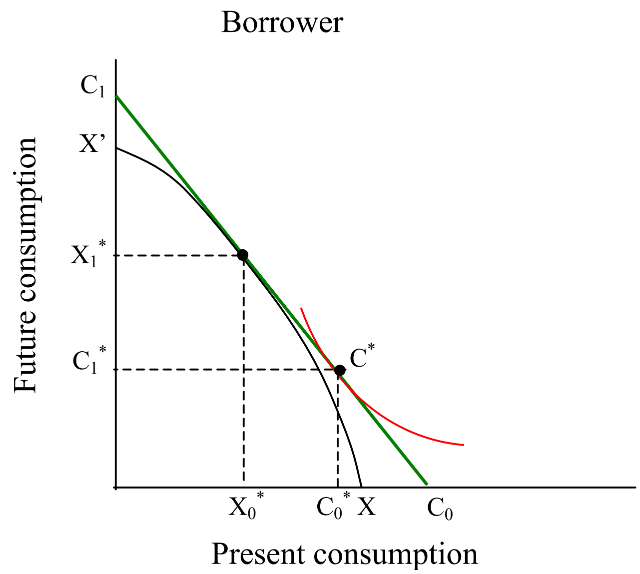
1. That means that both borrowers and lenders can be better off when investment is allowed.



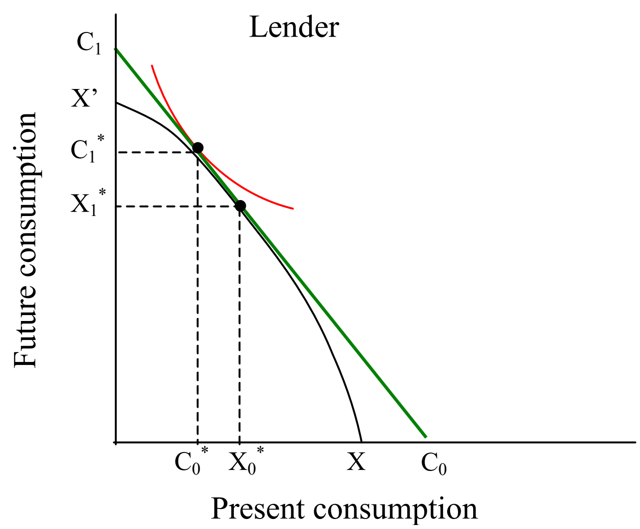
(v) The idea that people first maximize wealth and then rearrange consumption to maximize utility is known as the Fisher Separation Theorem. The decision to maximize wealth is separate from the decision of how to maximize utility.

d) Now let's see how the level of the *real* interest rate is determined.
   i) Suppose we have two agents in the economy where one is a borrower and one is a lender.
   ii) Start with some given rate of exchange in the economy so that the wealth lines for both agents have the same slope.
   iii) The agents want to maximize utility and will choose the optimal level of production to do so.

iv) Below, the borrower is willing to give up $X_1^* - C_1^*$ in future consumption to gain $C_0^* - X_0^*$ in current consumption.



v) While the lender is willing to give up $X_0^* - C_0^*$ of current consumption to gain $C_1^* - X_1^*$ in future consumption.



vi) The problem in this situation is that the borrower would like to borrow more then the lender can be enticed to give up given the current market rate of exchange.

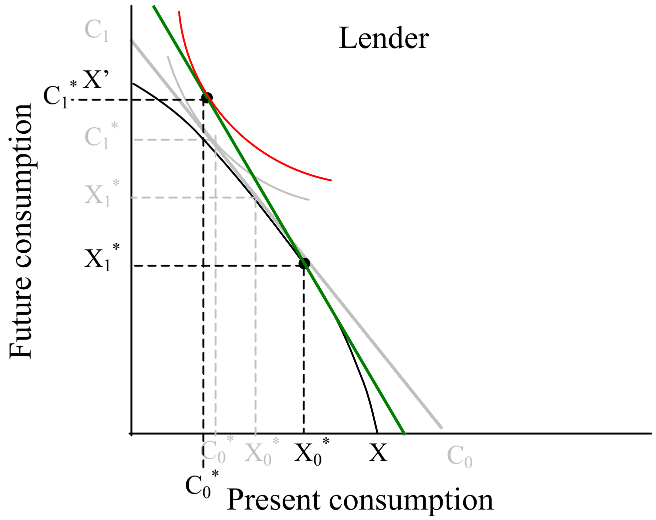
vii) For the lender to be willing to give up the amount that the borrower wants, the lender must be compensated for giving up more current consumption.

(1) That means the interest rate must be increased.

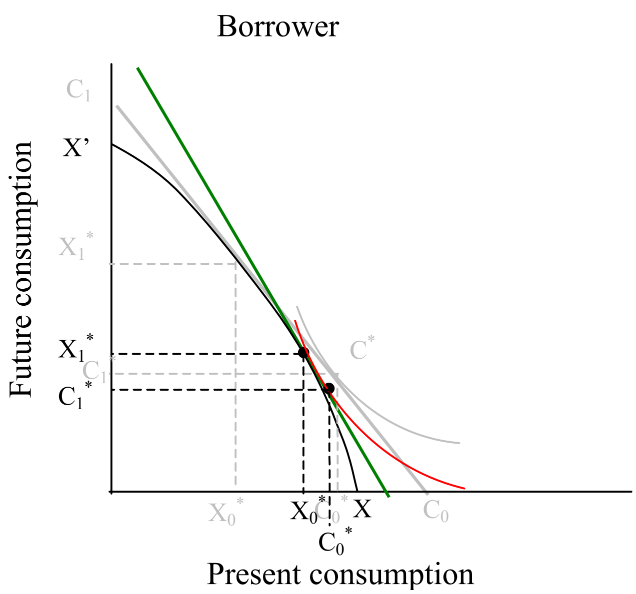
(2) If we increase the interest rate then we have a steeper wealth line faced by both individuals.

(3) Let's increase the interest rate and see how the two agents end up.

e) The lender will adjust the amount to produce and move to a lower point along his PPF so as to maximize wealth. This yields an intersection of the new wealth curve and a higher indifference curve than before.



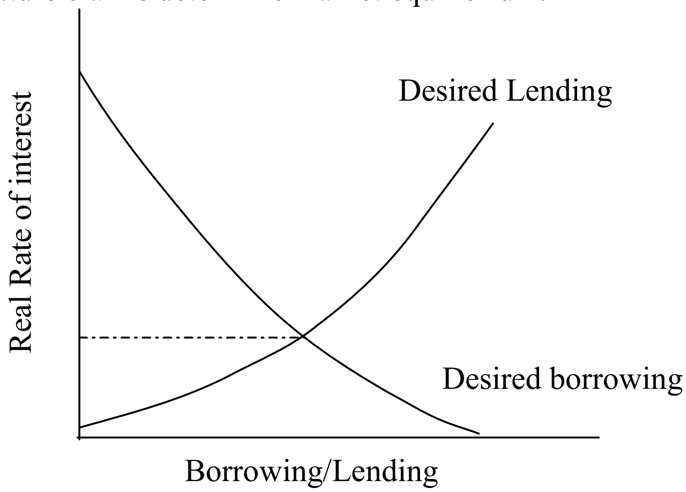
f) The borrower will also adjust the amount to produce and move to a lower point along her PPF so as to maximize wealth. This yields an intersection of the new wealth curve and a lower indifference curve than before.



i) Notice that this drastically reduces the amount the borrower would like to borrow while at the same time increasing the amount the lender would like to lend.
   (1) Hence we are still not at equilibrium.
   (2) This process of changing the slope of the wealth lines, the interest rate, will continue until the amount the borrower would like to borrow and the amount the lender would like to lend are equal.

g) If we extend the analysis to more then two agents it will still work
   i) Market equilibrium is achieved when desired lending equals desired borrowing across all economic units.

ii) Hence, the forces of supply and demand for current consumption claims vis-à-vis future claims determine market equilibrium.



iii) Here the lending curve represent an aggregation of the amounts of desired lending for all economic units at various interest rates
   (1) That is, any point on the curve represents the horizontal sum of different individuals desired lending at the particular interest rate.

iv) What we have been describing is the neoclassical loanable funds theory:
   (1) Supply of funds from savers – households
   (2) Demand of funds from businesses
   (3) The government's net supply and/or demand of funds.

2) Money
   a) So far we have not included money in the discussion of interest rate determination. What is money?
      i) Medium of exchange
         (1) Better than using a barter system.
      ii) Unit of account
         (1) Measure of value
      iii) Store of value
         (1) Money is an asset since it has purchasing power
         (2) Most *liquid* of all assets
            (a) The liquidity of money is the primary reason individuals hold this asset.
      iv) M1
         (1) Monetary aggregate measuring the total amount of currency and checking deposits held by households and firms.
         (2) In 1997 M1 accounted for 13.3% of GNP.

   b) Who controls the money supply?
      i) We will talk about the effects of monetary policy in a couple of days so for now we simply assume that the money supply is controlled by the central bank.

   c) Aggregate money demand
      i) Define aggregate money demand as the demand for money by all households and firms in the economy.

ii) Determinants of the aggregate demand for money
   (1) The interest rate
      (a) Demand for money falls as interest rates rise….why?
   (2) The price level
      (a) If the price level is a measure of a "basket" of goods and services in the economy then as the price level rises, demand for money increases…why?
   (3) Real national income
      (a) Remember that national income is GNP. So as GNP increases, more consumption is taking place and the demand for money increases.

iii) Define

P = price level
R = nominal interest rate
Y = real GNP
M^d = aggregate demand

$M^d = P*L(R,Y)$ or
$M^d / P = L(R,Y)$

   (1) The function L() is called aggregate *real* money demand and should be interpreted as the amount of purchasing power people would like to hold in liquid form.
   (2) L(R,Y) falls when R increases and increases with Y
   (3) M^d / P is the desired money holdings measured in terms of a typical reference basket of commodities.

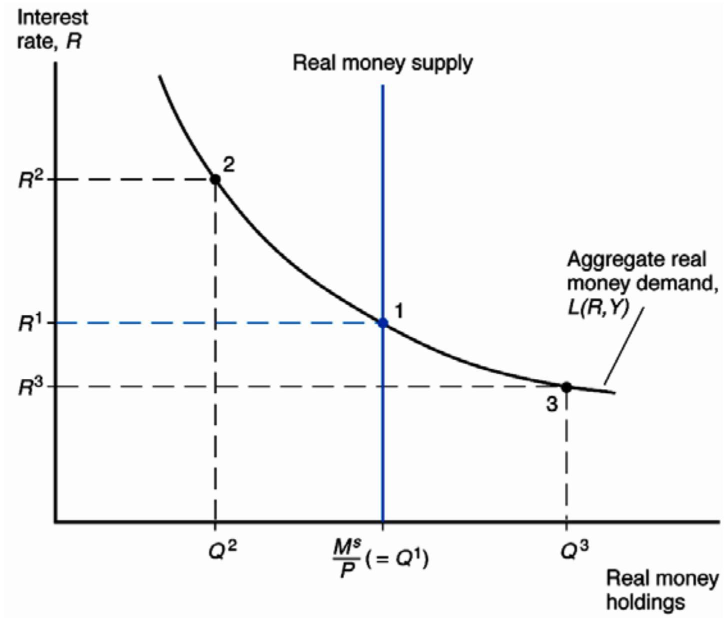
d) Aggregate money supply
   i) Define

   M^s = aggregate money supply

   ii) In equilibrium

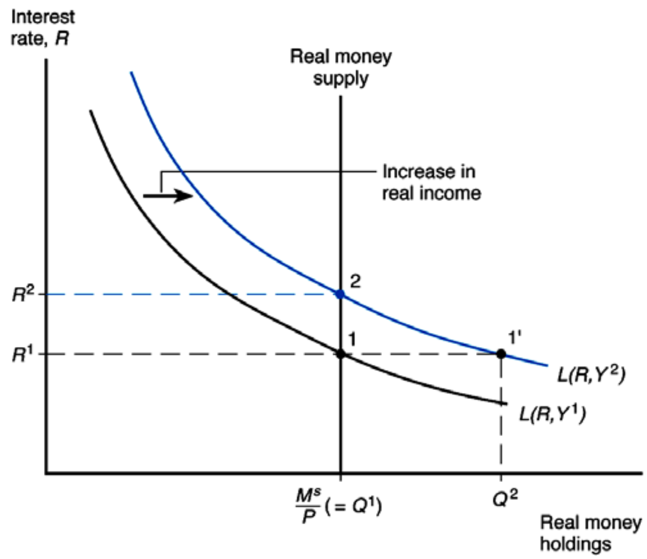
   $M^s = M^d = P*L(R,Y)$ or
   $M^d / P = L(R,Y)$

e) Equilibrium
   i) If we plot aggregate real money demand and aggregate real money supply against the interest rate we get

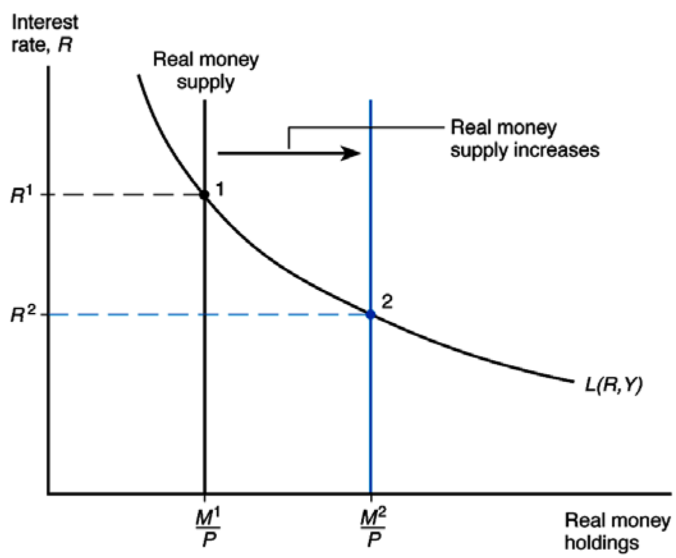


ii) Notice that
 (1) Money demand is downward sloping since an increase in interest rates decreases real money demand
  (a) So changes in the interest rate cause a movement along the curve for a given level of GNP
 (2) Money supply is vertical because at any given time the money supply will be fixed by the central bank.
 (3) At the point labeled 1, the money supply equals the money demand and the economy is in equilibrium. At the interest rate R1 all those wishing to lend can find borrowers of their money and all those wishing to borrow can find lenders.
 (4) At the point labeled 2, there is too much money in the economy. There is an excess supply of money. Because the interest rate is at R2, those with cash will try to invest it. But given a limited number of investments in the economy the demand for assets will push the price of assets up and reduce the interest rate forcing the economy back to equilibrium.

f) What happens to interest rates as GNP increases given the money supply?
 (1) As GNP increases the money demand curve shifts out and interest rates increase.
  (a) $Y^2 > Y^1$
  (b) Too little cash in the economy
  (c) People sell assets pushing asset prices down
  (d) Falling asset prices increases the interest rate.



g) What happens to interest rates as the money supply increases given the price level and output?
   (1) As GNP increases the money demand curve shifts out and interest rates increase.
      (a) $M^2 > M^1$
      (b) Too much cash in the economy
      (c) People buy assets pushing asset prices up
      (d) Increasing asset prices decreases the interest rate.



h) Why do interest rates change?
   i) Unexpected changes in GNP.
   ii) Unexpected changes in Monetary policy
   iii) Unexpected changes in Fiscal policy
   iv) Unexpected changes in expected inflation
   v) Unexpected changes in the term premium