Manpower Planning with Limited Hiring Opportunities

The Value of Stochastic Modeling

(004-0145)

Extended Abstract

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We consider the issue of capacity planning in professional services organizations where the policy is to hire and train entry level resources. We briefly review the literature on manpower planning, highlighting and contrasting the two primary methods applied to the problem - stochastic decision models and deterministic math programming. We develop a base model formulated as a stochastic linear program, and illustrate the model through a numerical experiment. We solve the problem with a straight-forward mean value model and also a more complex stochastic model of demand and attrition. We find that although the stochastic model is substantially more complex to solve, it yields a higher valued solution and provides better insight into the dynamics of the staffing decision. In particular the stochastic model develops a more finely tuned hedging strategy. Our model indicates a clear trade off between computational complexity and solution quality and provides a starting point for modeling more complex manpower planning environments

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1. Introduction

This paper addresses the manpower planning problem in the context of a professional service firm, i.e. a firm that generates revenue by deploying resources on billable engagements. The professional services industry is a rapidly growing segment of both developed and developing economies. IBM Global Services provided $46.2 billion of revenue in 2004, 48% of the company's total (IBM 2004). Also in 2004, India-based Infosys grew its headcount 37%, hiring 11,597 new employees from an applicant base of over 13 million (Infosys 2005). For our purposes professional services includes business such as traditional management consulting, technical support, and IT outsourcing. The heavy reliance on human resources as the primary determinant of productive capacity implies that a key challenge for professional service managers is the acquisition, training, and termination of resources (Dietrich and Harrison 2005). In this paper we focus on the long term capacity planning problem in a professional services firm whose policy objective is to staff operations by hiring and training new university graduates.

In Section 2 we briefly review the literature related to strategic manpower planning. Section 3 presents a hiring model formulated as a stochastic linear program with recourse represented in extensive form. Section 4 provides an analysis of the model results. Section 5 addresses future research.

2. Manpower Planning Literature Review

The strategic manpower planning literature is in general divided into two complementary approaches. One approach is based on a stochastic process formulation; the other is based on an optimization formulation. Bartholomew (1982) provides a general review of the application of stochastic modeling to social systems, while Bartholomew and Forbes (1979) develops a more specific application of these principals to the manpower planning problem. A basic model defines a number of discrete manpower grades, with employees advancing or leaving with fixed transition probabilities. The state of the system is defined as the number of employees in each grade and the system is analyzed as a Markov chain. Grinold develops a stochastic model motivated by the demand for naval aviators (Grinold 1976). Anderson (2001) develops a model where demand is driven by a continuous nonstationary seasonal process meant to approximate a business cycle. Gaimon and Thompson (1984) develop a model that looks at an organization in terms of cohorts, i.e. employees with the same length of service, using an objective function that measures the "effectiveness" of the organization.

An alternative approach to manpower planning is based on optimization theory. The theoretical foundations of the optimization approach were developed in Holt *et al.*(Holt *et al.* 1960) Holt develops a cost model that includes both the costs of maintaining and changing the workforce. Holt uses a quadratic cost model that allows a closed form solution to be developed and finds that optimal staffing levels are based on the weighted values of forecasted demand. Holt's quadratic cost model is converted to a linear cost model in Hanssmann and Hess (1960) and solved as an LP. The Holt model is also extended in Ebert (1976) with the inclusion of time varying productivity. Ebert solves this non-linear program using a search heuristic. A long rang planning system for the U.S. Army is described in 2 papers which outline successive generations of the system (Holz and Wroth 1980) (Gass *et al.* 1988). The systems tracks manpower by skill, grade, years of service, and quality, and are formulated as goal programs where the goal variables define force targets. Bres, *et al*. (1980) describe a similar model developed for the U.S Navy in the 1970s.

The two approaches to manpower planning outlined above emphasize different aspects of the system and as such have different applications. The stochastic models are generally high level abstractions useful for identifying system phenomenon or developing general policies, but are not designed to be implemented as management control systems. The optimization models are crafted to identify specific management actions but tend to ignore variability, typically representing stochastic parameters by their expectation. We summarize the key literature below in Table 2-1.

In this paper we attempt to develop an actionable model that explicitly incorporates variability by formulating the problem as a stochastic linear program with recourse. Stochastic programs are likely to yield solutions that are superior to their deterministic counterparts, but solutions to these stochastic programs are difficult to find (Kall and Wallace 1994; Birge and Louveaux 1997; Kall and Mayer 2005).

	1	2	3	4	5	6	7	8	9	10	11
Planning Approach											
Stochastic Modeling	X	X	X	X	X						
Optimization						X	X	X	X	X	X
Foundation											
Theory	X	X	X	X	X	X	X	X			
Practice									X	X	X
Resource Indexing											
Skill	X	X							X		X
Grade	X	X			X				X	X	
Length of Service	X	X	X		X				X		X
Trained				X							
Transitions											
Hiring/Quitting	X	X	X	X	X	X	X	X	X	X	X
Layoffs	X	X		X	X	X	X	X			
Skill Redeployment									X		
Promotions	X	X							X		
Demand											
External-deterministic						X			X	X	X
External-variable			X	X	X		X				

1-Bartholomew (1982). 2-Bartholomew and Forbes (1979), 3-Grinold (1976), 4-Anderson (2001), 5-Gaimon and Thompson (1984), 6-Holt *et al*. (1960), 7-Ebert (1976), 8-Hanssmann and Hess (1960), 9-Gass *et al*. (1988), 10-Holz and Wroth (1980), 11-Bres *et al*. (1980)

Table 2-1

3. The model

In this section we develop a base manpower planning model for a professional services firm. The primary variable in our model is the number of resources in the organization which we index by grade, skill set, and time period, X_{sgt}. We specifically motivate this model by considering a firm whose preferred policy is to hire only new graduates and enroll them in a quarter long training program. We also assume the firm can supplement full-time employees with a limited number of sub-contractors. Because the firm hires only new graduates they have only two hiring windows per year.

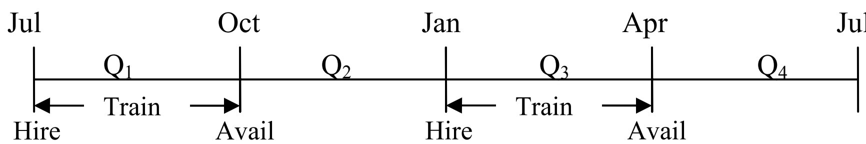

Figure 4-1 – Hiring Timeline

Note that resources hired in July become available to support billable work in October and that the next class of recruits does not become available until April. The firm's July hiring decision must therefore account for 9 months of uncertain demand.

We develop an extensive form model with the following definitions:

Sets

s: skill sets
g: grades
t: time periods
m: demand scenarios
n: attrition scenarios

Deterministic Parameters

w_{sg}: wage rate
h_{sg}: hiring cost
b_{sgt}: billing rate per period
f_{sg}: termination cost
c_{sg}: subcontractor cost
e_{sg}: training cost
α: maximum proportion of subs

Stage 1 Variables

X_{sgtmn}: number of resources
H_{sgt}: number of hires
B_{sgtmn}: billable resources
T_{sgtm}: resources in training

Recourse Variables

Y_{sgtmn}: subcontractors by skill, grade, and time for each scenario
F_{sgtmn}: terminations by skill, grade, and time for each scenario
A_{sgtmn}: attrition at time t for each scenario
RF_{sgtmn}: resources reskilled from skill set s
RT_{sgtmn}: resources reskilled to skill set s

Stochastic Parameters

d_{sgtm}: demand forecast in resources
a_{sgtn}: per period attrition rate

Probabilities

p_m: probability of demand scenario *m*
q_n: probability of attrition scenario n

$$Max\sum_{s=1}^{S}\sum_{g=1}^{G}\sum_{t=1}^{T}\sum_{m=1}^{M}\sum_{n=1}^{N}\left[b_{sgt}B_{sgtmn}-(w_{sg}p_mq_nX_{sgtmn}+h_{sg}H_{sgt}+e_{sg}T_{sgtm})+p_mq_n(c_{sg}Y_{sgtmn}+f_{sgt}F_{sgtmn})\right] \quad (2.1)$$

subject to

$$X_{sgtmn}=X_{sg,t-1,mn}+H_{sgt}-F_{sgtmn}-a_{sgtn}X_{sgtmn}+RT_{sgtmn}-RF_{sgtmn} \quad \forall sgtmn \quad \text{(C1)}$$

$$T_{sgtmn}=H_{sgtmn}+RT_{sgtmn} \quad \forall sgtmn \quad \text{(C2)}$$

$$B_{sgtm}\le d_{sgtm} \quad \forall sgt \quad \text{(C3)}$$

$$B_{sgtm}\le X_{sgtmn}-T_{sgtmn}+Y_{sgtmn} \quad \forall sgtm \quad \text{(C4)}$$

$$Y_{stgmn}\le \alpha d_{sgtm} \quad \forall sgtmn \quad \text{(C5)}$$

$$\sum_{s=1}^{S}RF_{sgtmn}=\sum_{s=1}^{S}RT_{sgtmn} \quad \forall gtmn \quad \text{(C6)}$$

The objective function of this program maximizes the difference between billable revenue, and payroll costs, recruiting costs, training costs, and the expected cost of firing and subcontracting. Constraint 1 is the overall staff balance constraint; it states that in every skill and grade category the net change in manpower is equal to the number of new hires, less the number lost to attrition or termination. The model also allows for retooling, the retraining of resources from one skill set to another. Constraint 2 specifies that all new hires and retooled resources are designated as in training for one period. Constraints 3 and 4 constrain the number of billable resources according to demand and supply. Note that these constraints are expressed as inequalities which allows for solutions in which less then 100% of demand is satisfied. Constraint 5 sets an upper limit on the proportion of billable resources that can be subcontracted. Constraint 6 balances the number of retrained resources.

4. Numerical Analysis

We illustrate the use of the model with a numerical example. Consider an aggregate plan for resources at a single skill/grade level. Assume the firm is experiencing rapid growth and estimates quarterly growth of 6% per with a standard deviation of 1.5%. Attrition is forecast at 2% per quarter with a standard deviation of .5%.

We first solve the mean value problem; that is the problem where demand and attrition are each represented by a single scenario at the expected value of each parameter. Using representative baseline values for each parameter, listed in the appendix, we find an optimal hiring policy of 690 resources, with an expected value of 580 subcontractors-periods. (Note: a subcontractor-period is the use of one subcontractor for one period). To evaluate the value of using a stochastic model, we now solve the problem using the same parameters but with a discrete approximation for the distribution of demand and attrition. We assume demand and attrition are independent of each other and independent across time. We approximate each variable in each period with a 4 point approximation as per Miller and Rice (1983) resulting in a total of 4,096 scenarios. Solving the stochastic program to optimality yields a first stage hiring decision of 936 for the baseline scenario, in contrast to the mean value solution of 690.

Sensitivity Analysis

We now evaluate the influence of parameter values on the stage 1 hiring decision. Specifically we wish to assess how the hiring decision is impacted by the level of the hiring cost, the margin on billable resources, the margin on subcontract resources, and the policy limit on subcontractors. We perform a straightforward 2^n factorial analysis, setting our analysis variables to levels above and below the baseline values utilized above. At each parameter setting we calculate the number of new hires, the expected number of subcontractor periods, the contribution margin, and the expected level of unsatisfied demand. We calculate this for both the mean value and stochastic (multi-scenario) problem. We also calculate the Value of the Stochastic Solution (VSS); the difference between the stochastic solution and the solution to the stochastic problem when hiring is forced to the mean value solution (Birge and Louveaux 1997)). The results are summarized in Table 1. In Table 2 we calculate the main effects, the average impact of shifting each variable from its low value to its high value (Box *et al.* 1978).

	Hiring Cost	Staff Cost	Sub Cost	Max Subs	Mean Value Model: New Hires	Exp Sub periods	Objective ($M)	Unsat'd Demand	Stochastic Model: New Hires	Exp Sub periods	Objective ($M)	Unsat'd Demand	Diff in Hires	VSS ($K)
B	B	B	B	B	689.9	579.8	915.5	0.93%	936.5	531.4	912.6	0.47%	247	1,851
R1	+	+	+	+	689.9	894.3	742.3		755.8	994.8	737.9	0.05%	66	15
R2	+	+	+	-	689.9	111.3	741.5	2.32%	773.4	148.7	737.0	2.50%	83	71
R3	+	+	-	+	689.9	894.3	750.4		618.8	1,053.9	747.4	0.11%	-71	47
R4	+	+	-	-	689.9	111.3	742.5	2.32%	706.5	158.1	738.4	2.68%	17	9
R5	+	-	+	+	1,639.0	0.0	1,077.8		1,200.5	436.6	1,077.0		-438	2,262
R6	+	-	+	-	1,639.0	0.0	1,077.8		1,202.7	108.7	1,076.6	0.96%	-436	2,012
R7	+	-	-	+	689.9	894.3	1,085.7		1,023.7	600.2	1,082.5	0.01%	334	1,051
R8	+	-	-	-	1,512.6	119.1	1,078.8		1,202.7	108.7	1,077.6	0.97%	-310	1,192
R9	-	+	+	+	689.9	894.3	747.5		850.0	908.7	743.8	0.02%	160	769
R10	-	+	+	-	689.9	111.3	746.7	2.32%	861.0	139.1	743.0	2.27%	171	895
R11	-	+	-	+	689.9	894.3	755.6		706.5	1,039.9	752.5	0.06%	17	19
R12	-	+	-	-	689.9	111.3	747.7	2.32%	850.0	140.4	744.3	2.30%	160	720
R13	-	-	+	+	1,639.0	0.0	1,090.1		1,524.8	194.8	1,087.2		-114	228
R14	-	-	+	-	1,639.0	0.0	1,090.1		1,548.4	69.3	1,087.1	0.33%	-91	188
R15	-	-	-	+	689.9	894.3	1,090.9		1,130.0	500.3	1,089.9		440	3,315
R16	-	-	-	-	1,512.6	119.1	1,090.2		1,505.3	76.3	1,087.8	0.38%	-7	1
Avg					1,030.0	378.1	916.0		1,028.8	417.4	913.1		-1	800

Table 4-1

Main Effects

		Mean Value Model: New Hires	Exp Sub periods	Objective ($M)	Stochastic Model: New Hires	Exp Sub periods	Objective ($M)	Diff in Hires	VSS ($K)
1	Hiring Cost	0.0	0.0	-7.7	-186.5	67.6	-7.7	-186.5	66
2	Staff Cost	-680.2	249.4	-338.4	-527.0	311.1	-257.7	153.2	-963
3	Sub Cost	268.9	-253.4	-3.5	121.7	-84.6	-3.8	-147.2	11
4	Max Subs	-205.7	585.3	3.1	-105.0	597.5	3.3	100.7	327
	Average	1,030	378	916	1,029	417	913	-1	800

Table 4-2

Observations

This relatively simple experiment summarized above allows us to make several observations concerning the difference between the mean value and stochastic models. First, we note that the optimal hiring decision is substantially different under the stochastic model; sometimes higher and sometimes lower. We also note that the optimal value to the stochastic model is always lower then the optimal value of the mean value model, but the VSS is always positive. The obvious implications are that the mean value problem overstates profit by ignoring the variability of demand and that the objective value found using the expectation of stochastic parameters is not equal to the expected profit. The upward bias in the mean value solution is true in general (Birge 1982).

We also observe that because the feasible region of the mean value model is defined by a relatively small number of constraints, the optimal solution across a wide range of parameter values occur at only 3 extreme points and the shift in optimal policy can be quite dramatic.

Consider for example runs 7 and 8; relaxing the constraint on the maximum proportion of subcontractors causes the hiring level to more than double with only a small change in objective value. The more detailed feasible region of the stochastic solution allows for a more nuanced response with every combination of parameter values resulting in a distinct hiring level. The finer granularity of the stochastic solution is further illustrated when we analyze the main effects in Table 2. In the mean value model the hiring level is independent of the hiring cost, a questionable result. The stochastic solution on the other hand provides a more realistic picture of the hiring policy; as recruiting becomes more expensive the firm decreases the hiring level and relies more on subcontractors to satisfy peak demand.

A clear understanding of the difference between the two models can be found by comparing the unsatisfied demand figures. The stochastic solution allows for a fine grained analysis of the optimal service level; the degree to which the firm must be positioned to satisfy the extreme values of demand. The model finds that in most cases the potential gain of increased revenue is offset by the added cost of excess capacity. The model selects the combinations of high demand and high attrition that are not profitable to staff. Only in the case of high margins and high spare capacity is it optimal to staff so as to satisfy all demand scenarios. The mean value model attempts to make the same calculations, but because of the single demand scenario the result is a crude approximation. Only in the case of the low margins does the model specify a service level of less than 100%, this occurs when the cost of carrying excess capacity for two periods is greater the margin earned in the final period, and therefore the model specifies a hiring plan that staffs to the level of the second period. We also observe that the VSS varies substantially. (Note: it can be shown that applying the stochastic model can not yield answers worse then the mean value solution (Birge and Louveaux 1997)). The value varies from a negligible level to more than 3 million dollars. The VSS is naturally greatest when the difference in hiring between the models is the greatest.

Finally we observe that the optimal solution of the stochastic solution always includes a level of subcontractors that is non-zero and often large, while the mean value model specifies a zero level when sub contractor margins are low. The mean value model is essentially evaluating the cost of carrying excess capacity in the early periods against a "known" demand in the later periods. The stochastic solution in contrast evaluates the probability of carrying excess capacity in all periods given an uncertain demand. This result verifies the value of contingent capacity, a result obscured in the mean value solution.

In summary we find that stochastic model is a more realistic representation, and that the solution is more finally tuned to specific parameter values. The solution to the stochastic model is always as good as the solution to the mean value problem, is usually different, and often superior to the mean value solution. This improved quality of solution however, comes at a price in terms of computational complexity; an issue we briefly address next.

Computational Considerations

The mean value version of (2.1) is a very simple linear program. A typical run has less than 110 non-zero elements and is solved in a few iterations, typically in about .03 seconds of CPU time on a laptop. The complexity of the problem increases dramatically as we move to the stochastic model. With 4,096 scenarios the number of non-zero elements in the problem increases to over 330,000. A representative problem required 15,487 iterations. We solved the problem on an IBM p650 in about 4.2 seconds of CPU time. The increase in computational requirements from the mean value problem to the stochastic problem is dramatic given the extensive form model, but the problem is still easily solved in this form. As the size of the problem grows, in terms of number of skills, grades, or scenarios, the extensive form model becomes intractable and a more complex formulation based on a decomposition approach is necessary.

5. Conclusion and Future Research

Our research objective is to develop manpower planning models applicable to the service sector that explicitly incorporate uncertainty in demand and attrition. We develop a flexible model and test it using a simple scenario. As expected our analysis confirms that the recognition of variability creates a significant shift in the optimal policy. We also confirm that this policy optimizes profit by reserving contingent capacity for less likely demand scenarios. Our analysis shows that the straight-forward extensive model is practical for small problems. Moving forward our research will focus on more complex instances. We seek to extend the model to a longer range, multi-period model with multiple skill/grade combinations.

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7. Appendix

The following table summarizes the parameter values used in the numerical experiment.

	Baseline	-	+
Hiring Cost	5,000	2,500	10,000
Staff Cost	16,250	12,500	20,000
Sub Cost	45,000	40,000	49,000
Max Sub %	5%	1%	10%
Firing Cost	5,000	5,000	5,000
Billing Rate	50,000	50,000	50,000
Initial Staff	11,000	11,000	11,000
Initial demand	10,000	10,000	10,000