Reverse Mathematics and $\Pi^1_2$ Comprehension

Stephen G. Simpson
Department of Mathematics
Pennsylvania State University
http://www.math.psu.edu/simpson/

November 13, 2006

Note: This is an abstract of an invited one-hour talk, to be given November 17, 2006, at the Logic Colloquium in the Department of Mathematics at the University of Florida.

Abstract

This is joint work with Carl Mummert. We initiate the reverse mathematics of general topology. We show that a certain metrization theorem is equivalent to $\Pi^1_2$ comprehension. An MF space is defined to be a topological space of the form $MF(P)$ with topology generated by $\{N_p \mid p \in P\}$. Here $P$ is a poset, $MF(P)$ is the set of maximal filters on $P$, and $N_p = \{F \in MF(P) \mid p \in F\}$. If the poset $P$ is countable, the space $MF(P)$ is said to be countably based. The class of countably based MF spaces can be defined and discussed within the subsystem $ACA_0$ of second order arithmetic. One can prove within $ACA_0$ that every complete separable metric space is homeomorphic to a countably based MF space which is regular. We show that the converse statement, “every countably based MF space which is regular is homeomorphic to a complete separable metric space,” is equivalent to $\Pi^1_2-CA_0$. The equivalence is proved in the weaker system $\Pi^1_2-CA_0$. This is the first example of a theorem of core mathematics which is provable in second order arithmetic and implies $\Pi^1_2$ comprehension.