Math 1931 – 2000:

909.03048

Simpson, Stephen G.

Subsystems of second order arithmetic. (English)
Perspectives in Mathematical Logic. Berlin: Springer. xiv, 444 p. DM 98.00; oeS 716.00; sFr. 89.50; 37.50; $ 59.95 (1999). [ISBN 3-540-64882-8/hbk]

The main question studied in the book under review is "What are the appropriate axioms for mathematics?" Since the author focuses on the language of second-order arithmetic, he is especially interested in the question of which set existence axioms are needed to prove the known theorems of mathematics. The attention is restricted to ordinary non-set-theoretic mathematics, i.e. to those branches of mathematics which are prior to or independent of the introduction of abstract set-theoretic concepts (such branches as geometry, number theory, calculus, differential equations, real and complex analysis, countable algebra, the topology of complete separable metric spaces, mathematical logic, computability theory). The emphasis is laid on Hilbert's program and the emerging reverse mathematics. The latter attempts to find weakest subsystems of second-order arithmetic Z_2 in which particular results of ordinary mathematics can be proved (this is done by deriving axioms from theorems).

The book consists of an Introduction, two main parts and an Appendix. The extensive Introduction is devoted to the description of the main subsystems of second-order arithmetic studied in the book and of mathematics within them, as well as to the main ideas of reverse mathematics. Part A consists of 5 chapters. Each chapter is devoted to one subsystem of Z_2 and to the development of mathematics in it. Particular chapters focus on: RCA_0 (recursive comprehension), ACA_0 (arithmetical comprehension), WKL_0 (weak Koenig's lemma), ATR_0 (arithmetical transfinite recursion) and $\Pi^1_1 - CA_0$ (Π^1_1 comprehension). In Part B, consisting of 3 chapters, models of Z_2 and its subsystems are studied. In particular, Chapter VII is devoted to β-models, Chapter VIII to ω-models and Chapter IX to non-ω-models. In the Appendix some additional reverse mathematics results and problems are presented (without proofs but with references to the published literature).

The book is supplemented by historical and bibliographical notes (spread throughout the text), the extensive bibliography (279 items) and an index.

This monograph can be studied both by graduate students in mathematical logic and foundations of mathematics and by experts. It provides an encyclopedic treatment of subsystems of Z_2. It can be viewed as a continuation of "Grundlagen der Mathematik" by D. Hilbert and P. Bernays.

R.Murawski (Poznan)

Keywords : recursive comprehension; arithmetical comprehension; weak Koenig's lemma; arithmetical transfinite recursion; Π^1_1 comprehension; β-models; ω-models; non-ω-models; set existence axioms; Hilbert's program; reverse mathematics; subsystems of second-order arithmetic

Classification:

- 03F35 Higher-order arithmetic and fragments
- 03-02 Research monographs (mathematical logic)
- 03C62 Models of arithmetic and set theory