Typographical and other errors

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1 Typographical errors in the first edition

Here is a list of typographical errors in *Subsystems of Second Order Arithmetic*, Stephen G. Simpson, Springer-Verlag, 1999, XIV + 445 pages. It contains all errors which were discovered through 2006. All of these errors were corrected in the second edition which was published in 2009.

- Proof of Theorem I.9.1, displayed formula, replace $|c - c_k| < 2^n$ by $|c - c_k| < 2^{-n}$.

- Theorem I.10.3, in item 8, replace "set of sentences" by "consistent set of sentences".

- Proof of Lemma III.2.5, the subscripts need to be repaired. The proof should be as follows:

  > We first consider the case of a finite product $\widehat{A} = \prod_{i=1}^m \widehat{A}_k$. In this case, for each $j \in \mathbb{N}$, let $l_j =$ the smallest l such that $m \cdot 2^{-l} \le 2^{-j}$. Put $n_j = \prod_{k=1}^m (n_{l_j k} + 1) - 1$ and let $\langle x_{ij} : i \le n_j \rangle$ be an enumeration of $\prod_{k=1}^m \{x_{il_j k} : i \le n_{l_j k}\}$. Then $\langle \langle x_{ij} : i \le n_j \rangle : j \in \mathbb{N} \rangle$ attests to the compactness of $\widehat{A}$.
  >
  > In the case of a countably infinite product $\widehat{A} = \prod_{k \in \mathbb{N}} \widehat{A}_k$, for each $j \in \mathbb{N}$ let $l_j =$ smallest l such that $(j+2) \cdot 2^{-l} \le 2^{-j-1}$. Put $n_j = \prod_{k=0}^{j+1} (n_{l_j k} + 1) - 1$ and let $\langle x_{ij} : i \le n_j \rangle$ be an enumeration of $\prod_{k=0}^{j+1} \{x_{il_j k} : i \le n_{l_j k}\}$. Again $\langle \langle x_{ij} : i \le n_j \rangle : j \in \mathbb{N} \rangle$ attests to the compactness of $\widehat{A}$. This completes the proof of the lemma.

- Proof of Theorem III.3.2, main paragraph, replace $h : K \to \mathbb{Q}$ by $g : K \to \overline{\mathbb{Q}}$, and replace $h(a) = b$ by $g(a) = b$. Also, replace all three occurrences of K_1 by M.

- Proof of Theorem III.4.3, last line of third paragraph, replace

  $$\forall n\, (q_{2n} \neq 0 \to f(q_{2n+1}/q_{2n}) = n)$$

  by

$$\forall n\,(q_{2n} \neq 0 \to f(q_{2n+1}/q_{2n}) = n) \text{ and } \forall n\,(q_{2n} = 0 \to q_{2n+1} = 0).$$

- Proof of Theorem III.6.5, middle of page 120, replace $y_{ij} = p_{f(i)} z_{i,j+1}$ by $y_{ij} = p_{f(i)} y_{i,j+1}$.
- Proof of Theorem III.6.5, first line of last paragraph on page 120, replace $h_i : A \to D, i = 1, 2$ by $h_1, h_2 : A \to D$.
- Proof of Lemma IV.1.4, end of proof, replace $f^*(\sum_{i=0}^{j-1} f(i) + k) = 1$ by $f^*(\sum_{i=0}^{j-1} g(i) + k) = 1$.
- Exercise IV.2.10, replace "closed set" by "closed set C".
- Definition V.4.5, replace A by A.
- Remark V.10.1, replace Σ^0_1-AC_0 by Σ^1_1-AC_0.
- Theorem VI.2.6, replace "Over" by "over".
- Proof of Sublemma VI.3.3, in the definition of C_0, replace $X_1 \oplus X_1$ by $X_0 \oplus X_1$.
- Definition VII.3.2.4, replace $u - (v - u)$ by $u - (u - v)$.
- Proof of Theorem VII.3.31, first line, replace T by T.
- Proof of Theorem VII.6.9, first paragraph, second last sentence, replace "The" by "Then".
- Remark VIII.1.16, replace "as do ACA_0 and Σ^1_1-AC_0" by "while ACA_0 and Σ^1_1-AC_0 prove the same Π^1_2 sentences."
- Proof of Lemma VIII.2.16, first line of page 323, replace Y by Z.
- Proof of Lemma IX.2.4, second line, replace $|M| \cup \mathcal{S}_{M'}$ by $|M| \cup \mathcal{S}_M$.
- Theorem X.2.9, items 5, 6, and 7, replace X by X^*.
- Section X.3, just before Definition X.3.1, replace "Ramsey's theorem" by "Ramsey's theorem for exponent 3".
- Definition X.4.1, replace "atomic formula" by "numerical term".
- Bibliography, item 268, replace "borel" by "Borel".
- Index, atomic formula, replace "2, 410" by "2".
- Index, formula, atomic, replace "2, 410" by "2".
- Index, GKT basis theorem, replace "325–326" by "325–326, 354–356".
- Index, numerical term, replace "23" by "2, 23, 410".
- Index, term, numerical, replace "2, 23" by "2, 23, 410".
- Index, add index entry, weak Σ^1_1-AC_0, 342–347.
- Index, universal Σ^1_1, replace "333, 356" by "252, 333, 356".

2 Errors in the second edition

Here is a list of errors in *Subsystems of Second Order Arithmetic*, Second Edition, Stephen G. Simpson, Association for Symbolic Logic, Cambridge University Press, 2009, XVI + 444 pages. This list is current as of today, January 9, 2017.

- The proof of Theorem IV.8.2 contains two typographical errors. In displayed equation (15), $< 2^{-3(n+2)}$ should be $< -2^{-3(n+2)}$. In line 7 from the bottom of the page, $> -2^{-3(n+2)}$ should be $> 2^{-3(n+2)}$.

- David Madore has pointed out that part 18 of Lemma VII.3.7 is incorrect. The error is not typographical but mathematical. Namely, $\mathsf{B}_0^{\mathrm{set}}$ does not include the Axiom of Regularity, so for instance it is consistent with $\mathsf{B}_0^{\mathrm{set}}$ that there are proper-class-many sets x such that $x = \{x\}$, and of course all such sets are hereditarily finite.

  There are two ways to repair this.

  1. Preferred way: Move the Axiom of Regularity from Definition VII.3.8 (the axioms of $\mathsf{ATR}_0^{\mathrm{set}}$) to Definition VII.3.3 (the axioms of $\mathsf{B}_0^{\mathrm{set}}$).
  2. Another way: Change part 6 of Definition VII.3.6 to read as follows:

     Trans$(u) \leftrightarrow u$ is *transitive*, i.e., $\forall x\, \forall y\, ((x \in y \wedge y \in u) \rightarrow x \in u) \wedge \forall v\, ((v \subseteq u \wedge v \neq \emptyset) \rightarrow \exists x\, (x \in v \wedge \forall y\, (y \in v \rightarrow y \notin x)))$.

     Thus regularity is incorporated into the definitions of transitivity and hereditary finiteness.

  Under either 1 or 2, part 7 of Definition VII.3.6 can be simplified by omitting the clause which begins with $\forall v$.

- In line 2 of the proof of Theorem VII.3.9, "Then" should be "The".

- David Madore has pointed out another mathematical error. Namely, in the Notes for §VII.5 on page 293, the stated characterization of α_{k+2}^{Δ} is correct for $k \geq 1$ but incorrect for $k = 0$.

  A correct characterization of α_2^{Δ} reads as follows. α_2^{Δ} = the least admissible ordinal which is the limit of smaller admissible ordinals.

- Andre Kornell has pointed out a typo in Definition VII.4.22. In the displayed formula, $u \subseteq \mathrm{rng}(f)$ should be $u \in \mathrm{rng}(f)$.