1 Typographical errors in the first edition

Here is a list of typographical errors in *Subsystems of Second Order Arithmetic*, Stephen G. Simpson, Springer-Verlag, 1999, XIV + 445 pages. It contains all errors which were discovered through 2006. All of these errors were corrected in the second edition which was published in 2009.

- Proof of Theorem I.9.1, displayed formula, replace $|c - c_k| < 2^n$ by $|c - c_k| < 2^{-n}$.
- Theorem I.10.3, in item 8, replace “set of sentences” by “consistent set of sentences”.
- Proof of Lemma III.2.5, the subscripts need to be repaired. The proof should be as follows:

  We first consider the case of a finite product $\hat{A} = \prod_{k=1}^{m} \hat{A}_k$. In this case, for each $j \in \mathbb{N}$, let $l_j$ be the smallest $l$ such that $m \cdot 2^{-l} \leq 2^{-j}$. Put $n_j = \prod_{k=1}^{m} (n_{t,k} + 1) - 1$ and let $\langle x_{ij} : i \leq n_{t,j} \rangle$ be an enumeration of $\prod_{k=1}^{m} \{x_{it,k} : i \leq n_{t,k}\}$. Then $\langle \langle x_{ij} : i \leq n_j \rangle : j \in \mathbb{N} \rangle$ attests to the compactness of $\hat{A}$.

  In the case of a countably infinite product $\hat{A} = \prod_{k \in \mathbb{N}} \hat{A}_k$, for each $j \in \mathbb{N}$ let $l_j$ be smallest $l$ such that $(j+2) \cdot 2^{-l} \leq 2^{-j-1}$. Put $n_j = \prod_{k=0}^{j+1} (n_{t,j} + 1) - 1$ and let $\langle x_{ij} : i \leq n_{j} \rangle$ be an enumeration of $\prod_{k=0}^{j+1} \{x_{it,k} : i \leq n_{t,k}\}$. Again $\langle \langle x_{ij} : i \leq n_j \rangle : j \in \mathbb{N} \rangle$ attests to the compactness of $\hat{A}$. This completes the proof of the lemma.

- Proof of Theorem III.3.2, main paragraph, replace $h : K \to \mathbb{Q}$ by $g : K \to \overline{\mathbb{Q}}$, and replace $h(a) = b$ by $g(a) = b$. Also, replace all three occurrences of $K_1$ by $M$.

- Proof of Theorem III.4.3, last line of third paragraph, replace

  \[
  \forall n \ (q_{2n} \neq 0 \to f(q_{2n+1}/q_{2n}) = n)
  \]

  by

  \[
  \forall n \ (q_{2n} \neq 0 \to f(q_{2n+1}/q_{2n}) = n)
  \]
\[ \forall n \left( q_{2n} \neq 0 \rightarrow f(q_{2n+1}/q_{2n}) = n \right) \] and \[ \forall n \left( q_{2n} = 0 \rightarrow q_{2n+1} = 0 \right). \]

- Proof of Theorem III.6.5, middle of page 120, replace \( y_{ij} = p_{f(i)z_{i,j}} \) by \( y_{ij} = p_{f(i)z_{i,j}+1} \).

- Proof of Theorem III.6.5, first line of last paragraph on page 120, replace \( h_i : A \rightarrow D, i = 1, 2 \) by \( h_1, h_2 : A \rightarrow D \).

- Proof of Lemma IV.1.4, end of proof, replace \( f^*(\sum_{i=0}^{j-1} f(i) + k) = 1 \) by \( f^*(\sum_{i=0}^{j-1} g(i) + k) = 1 \).

- Exercise IV.2.10, replace “closed set” by “closed set \( C \)”.

- Definition V.4.5, replace \( A \) by \( A \).

- Remark V.10.1, replace \( \Sigma_1^0-\text{AC}_0 \) by \( \Sigma_1^1-\text{AC}_0 \).

- Theorem VI.2.6, replace “Over” by “over”.

- Proof of Sublemma VI.3.3, in the definition of \( C_0 \), replace \( X_1 \oplus X_1 \) by \( X_0 \oplus X_1 \).

- Definition VII.3.2.4, replace \( u - (v - u) \) by \( u - (u - v) \).

- Proof of Theorem VII.3.31, first line, replace \( T \) by \( T \).

- Proof of Theorem VII.6.9, first paragraph, second last sentence, replace “The” by “Then”.

- Remark VIII.1.16, replace “as do \( \text{ACA}_0 \) and \( \Sigma_1^1-\text{AC}_0 \)” by “while \( \text{ACA}_0 \) and \( \Sigma_1^1-\text{AC}_0 \) prove the same \( \Pi_2^1 \) sentences.”

- Proof of Lemma VIII.2.16, first line of page 323, replace \( Y \) by \( Z \).

- Proof of Lemma IX.2.4, second line, replace \( |M| \cup S_M \) by \( |M| \cup S_M \).

- Theorem X.2.9, items 5, 6, and 7, replace \( X \) by \( X^* \).

- Section X.3, just before Definition X.3.1, replace “Ramsey’s theorem” by “Ramsey’s theorem for exponent 3”.

- Definition X.4.1, replace “atomic formula” by “numerical term”.

- Bibliography, item 268, replace “borel” by “Borel”.

- Index, atomic formula, replace “2, 410” by “2”.

- Index, formula, atomic, replace “2, 410” by “2”.

- Index, GKT basis theorem, replace “325–326” by “325–326, 354–356”.

- Index, numerical term, replace “23” by “2, 23, 410”.

- Index, term, numerical, replace “2, 23” by “2, 23, 410”.

- Index, add index entry, weak \( \Sigma_1^1-\text{AC}_0 \), 342–347.

- Index, universal \( \Sigma_1^1 \), replace “333, 356” by “252, 333, 356”.

2
2 Errors in the second edition


- The proof of Theorem IV.8.2 contains two typographical errors. In displayed equation (15), $< 2^{-3(n+2)}$ should be $< -2^{-3(n+2)}$. In line 7 from the bottom of the page, $> -2^{-3(n+2)}$ should be $> 2^{-3(n+2)}$.

- David Madore has pointed out that part 18 of Lemma VII.3.7 is incorrect. The error is not typographical but mathematical. Namely, $\mathcal{B}^\text{set}_0$ does not include the Axiom of Regularity, so for instance it is consistent with $\mathcal{B}^\text{set}_0$ that there are proper-class-many sets $x$ such that $x = \{x\}$, and of course all such sets are hereditarily finite.

  There are two ways to repair this.

  1. Preferred way: Move the Axiom of Regularity from Definition VII.3.8 (the axioms of $\text{ATR}^\text{set}_0$) to Definition VII.3.3 (the axioms of $\mathcal{B}^\text{set}_0$).

  2. Another way: Change part 6 of Definition VII.3.6 to read as follows:

      $$\text{Trans}(u) \leftrightarrow u \text{ is transitive, i.e., } \forall x \forall y ((x \in y \wedge y \in u) \rightarrow x \in u) \wedge \forall v ((v \subseteq u \wedge v \neq \emptyset) \rightarrow \exists x (x \in v \wedge \forall y (y \in v \rightarrow y \notin x))$$

      Thus regularity is incorporated into the definitions of transitivity and hereditary finiteness.

Under either 1 or 2, part 7 of Definition VII.3.6 can be simplified by omitting the clause which begins with $\forall v$.

- In line 2 of the proof of Theorem VII.3.9, “Then” should be “The”.

- David Madore has pointed out another mathematical error. Namely, in the Notes for §VII.5 on page 293, the stated characterization of $\alpha_{k+2}$ is correct for $k \geq 1$ but incorrect for $k = 0$.

  A correct characterization of $\alpha_2$ reads as follows. $\alpha_2$ is the least admissible ordinal which is the limit of smaller admissible ordinals.

- Andre Kornell has pointed out a typo in Definition VII.4.22. In the displayed formula, $u \subseteq \text{rng}(f)$ should be $u \in \text{rng}(f)$.