1. Given an ordinal $\delta > 0$, show that the following assertions are pairwise equivalent.

   (a) $\alpha + \delta = \delta$ for all $\alpha < \delta$.
   (b) $\alpha + \beta < \delta$ for all $\alpha, \beta < \delta$.
   (c) $\delta = \omega^\gamma$ for some $\gamma \leq \delta$.

   An ordinal with these properties is said to be \textit{additively indecomposable}.

2. Given an ordinal $\gamma$, prove that there is one and only one way to write $\gamma$ in the form

   $$\gamma = \delta_1 + \cdots + \delta_n$$

   where $n \in \mathbb{N}$ and $\delta_1 \geq \cdots \geq \delta_n > 0$ are additively indecomposable.

3. Prove that $\text{card}(\mathbb{R}^\mathbb{N}) = 2^\aleph_0$ and $\text{card}(\mathbb{R}^\mathbb{R}) = 2^{2^\aleph_0}$.

4. Prove that there exist arbitrarily large uncountable cardinals $\lambda$ such that $\lambda = \aleph_\lambda$.

5. Given an uncountable cardinal $\lambda$, define the \textit{cofinality} of $\lambda$, written $\text{cf}(\lambda)$, to be the smallest cardinal $\kappa$ such that $\lambda = \sum_{i \in I} \lambda_i$ for some indexed set of cardinals $\lambda_i < \lambda$, $i \in I$, where $|I| = \kappa$.

   Prove the following.

   (a) $\text{cf}(\lambda) \leq \lambda$. $\text{cf}(\lambda)$ is regular.
   (b) $\lambda$ is regular if and only if $\text{cf}(\lambda) = \lambda$.
   (c) $\lambda$ is singular if and only if $\text{cf}(\lambda) < \lambda$.
   (d) $\lambda^{\text{cf}(\lambda)} > \lambda$. (Hint: Use König’s Theorem.)
   (e) For any infinite cardinal $\kappa$ we have $\text{cf}(\mu^\kappa) > \kappa$ for all $\mu > 1$. In particular, $\text{cf}(2^\kappa) > \kappa$.

6. Let $\kappa$ be an infinite regular cardinal. Prove that there exist arbitrarily large strong limit cardinals $\lambda$ such that $\text{cf}(\lambda) = \kappa$. Moreover, for all such $\lambda$ we have $\lambda^\mu = \lambda$ for all $\mu$ in the interval $1 \leq \mu < \kappa$. 