Combinatorial applications of the Compactness Theorem.

1. Application to graph coloring — Exercise 1.7.2 in the lecture notes. Show that a graph is $k$-colorable if and only if each finite subgraph is $k$-colorable.

2. Application to Dilworth’s Theorem — Exercise 1.7.6 in the lecture notes. Show that a partial ordering is covered by $k$ chains if and only if each finite subordering is covered by $k$ chains. Assuming Dilworth’s Theorem for finite partial orderings, deduce Dilworth’s Theorem for arbitrary partial orderings.