SOLUTIONS

1. Formulate the following argument as a propositional formula.

If it has snowed, it will be poor driving. If it is poor driving, I will be late unless I start early. Indeed, it has snowed. Therefore, I must start early to avoid being late.

Solution. Use the following atoms.

  s: it has snowed
  p: it is poor driving
  l: I will be late
  e: I start early

The argument can be translated as follows: s → p, p → (l ∨ e), s, therefore (¬l) → e. Written as a single propositional formula, this becomes:

(∗) ((s → p) & (p → (l ∨ e)) & s) → ((¬l) → e)

2. Use the tableau method to demonstrate that this formula is logically valid.

Solution. The following closed tableau shows that (∗) is logically valid.

\[
\begin{array}{c}
F ((s → p) & (p → (l ∨ e)) & s) → ((¬l) → e) \\
T (s → p) & (p → (l ∨ e)) & s \\
F (¬l) → e \\
T s → p \\
T (p → (l ∨ e)) & s \\
T p ⇒ (l ∨ e) \\
T s \\
T ¬l \\
F e \\
F l \\
/ \ \ \ / \\
F s \ T p \\
/ \ \ \ / \\
F p \ T l ∨ e \\
/ \ \ \ / \\
T l \ T e
\end{array}
\]
3. Brown, Jones, and Smith are suspected of a crime. They testify as follows:

Brown: Jones is guilty and Smith is innocent.
Jones: If Brown is guilty then so is Smith.
Smith: I’m innocent, but at least one of the others is guilty.

Let b, j, and s be the statements “Brown is innocent,” “Jones is innocent,” “Smith is innocent.” Express the testimony of each suspect as a propositional formula. Write a truth table for the three testimonies.

**Solution.** The testimonies are:

- \( B : \neg j \land s \)
- \( J : \neg b \Rightarrow \neg s \)
- \( S : s \land (\neg b \lor \neg j) \)

The truth table is:

<table>
<thead>
<tr>
<th>b</th>
<th>j</th>
<th>s</th>
<th>B</th>
<th>J</th>
<th>S</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>T</td>
<td>T</td>
<td>T</td>
<td>F</td>
<td>T</td>
</tr>
<tr>
<td>2</td>
<td>T</td>
<td>T</td>
<td>F</td>
<td>F</td>
<td>F</td>
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<tr>
<td>3</td>
<td>T</td>
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<tr>
<td>4</td>
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<td>F</td>
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<td>5</td>
<td>F</td>
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<td>F</td>
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<tr>
<td>7</td>
<td>F</td>
<td>F</td>
<td>T</td>
<td>T</td>
<td>F</td>
</tr>
<tr>
<td>8</td>
<td>F</td>
<td>F</td>
<td>F</td>
<td>F</td>
<td>T</td>
</tr>
</tbody>
</table>

4. Use the above truth table to answer the following questions:

(a) Are the three testimonies consistent?

**Solution.** Yes, by line 3 of the table.

(b) The testimony of one of the suspects follows from that of another. Which from which?

**Solution.** The table shows that \( S \) is a logical consequence of \( B \). In other words, Smith’s testimony follows from Brown’s.

(c) Assuming everybody is innocent, who committed perjury?

**Solution.** If everybody is innocent, we are in line 1 of the table. Hence \( B \) and \( S \) are false, i.e., Brown and Smith lied.

(d) Assuming all testimony is true, who is innocent and who is guilty?

**Solution.** If all the testimony is true, we are in line 3 of the table. Thus Brown and Smith are innocent, while Jones is guilty.

(e) Assuming that the innocent told the truth and the guilty told lies, who is innocent and who is guilty?

**Solution.** Our assumption is \( v(b) = v(B) \), \( v(j) = v(J) \), \( v(s) = v(S) \). Hence we are in line 6 of the table. Thus Jones is innocent, and Brown and Smith are guilty.