This is a 50-minute exam. While taking this exam, you may not consult any books, notes, or electronic devices.

1. Write a register machine program which computes the 1-place number-theoretic function $F(n) =$ the $n$th Fibonacci number. The precise definition of $F$ is as follows:

$$F(0) = F(1) = 1, \quad F(n + 2) = F(n) + F(n + 1).$$

2. Consider the 1-place partial number-theoretic function $\psi(n) = n/p$ where $p$ is the largest prime divisor of $n$. Prove that $\psi$ is partial computable.

Notes:

(a) There is no need to write a register machine program.

(b) In your proof that $\psi$ is partial computable, you may use known lemmas and theorems, for instance our lemmas concerning bounded quantification, the least number operator, etc.

(c) $\psi(0)$ and $\psi(1)$ are undefined.

3. Write a sentence $A$ such that the spectrum of $A$ is the set of powers of two. In other words, your sentence should have the property

$$\text{spec}(A) = \{2^n \mid n = 0, 1, 2, \ldots\} = \{1, 2, 4, 8, 16, \ldots\}.$$