“Fuchsian Groups” by S. Katok

Errata

p.7 l.10: Replace “Theorem 1.1.2” by “Theorem 1.2.1”.
p.20 l.4: Delete an extra cos in (iii).
p.21 l.4: Replace “imaginary axis” by “positive imaginary axis”.
p.24 l.7: Replace “1.2.4” by “1.2.5”.

Besides being a group, $\text{PSL}(2, \mathbb{R})$ is also a topological space. More precisely, $\text{SL}(2, \mathbb{R})$ can be identified with the subset of $\mathbb{R}^4$:

$$X = \{(a, b, c, d) \in \mathbb{R}^4 \mid ad - bc = 1\}.$$  

The norm on $\text{SL}(2, \mathbb{R})$ is induced from $\mathbb{R}^4$: for $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ with $ad - bc = 1$, we define

$$\|A\| = \sqrt{a^2 + b^2 + c^2 + d^2}, \quad (0.1)$$

and $\text{SL}(2, \mathbb{R})$ is topologized with respect to the metric

$$d(A, B) = \|A - B\|. \quad (0.2)$$

Since $A \sim -A$ is an equivalence relation on $\text{SL}(2, \mathbb{R})$, the factor space $\text{SL}(2, \mathbb{R})/\sim = \text{PSL}(2, \mathbb{R})$, is topologized with the factor topology. Exercise 2.1 shows that, in fact, $\text{PSL}(2, \mathbb{R})$ is a topological group. Since by (1.3.2), orientation-reversing isometries are given by matrices in $\text{GL}(2, \mathbb{R})$ with determinant $-1$, the whole group of isometries $\text{Isom}(\mathcal{H})$ can be topologized using the same distance. Notice that since $\|A\| = \|-A\|$, the norm (2.1.3) is a well-defined function on $\text{PSL}(2, \mathbb{R})$ while the metric (2.1.4) is not. One natural way to introduce a metric on $\text{PSL}(2, \mathbb{R})$ is to represent it as a matrix group $\text{SO}_0(2, 1)$, the other is through a so-called chord metric on the unit disc obtained from the Euclidean metric on the unit sphere via stereographic projection, but both are beyond the scope of this book.

Convergence in $\text{PSL}(2, \mathbb{R})$ can be expressed in matrix language as follows. If $g_n \rightarrow g$ in $\text{PSL}(2, \mathbb{R})$, this means that there exist matrices $A_n, A \in \text{SL}(2, \mathbb{R})$ representing $g_n$ and $g$ such that $\lim_{n \rightarrow \infty} \|A_n - A\| = 0$.

p.27 l.9: Replace “metric” by “locally compact metric”
p.27 l.10: Replace “homeomorphisms” by “isometries”.
p.27 l.5-l.3: Replace “It is clear from the definition that a group $G$ acts properly discontinuously on $X$ if and only if each orbit is discrete and the stabilizer of each point is finite.” by “Since $X$ is locally compact, a group $G$ acts properly discontinuously on $X$ if and only if each orbit has no accumulation point in $X$, and the order of
the stabilizer of each point is finite. The first condition, however, is
equivalent to the fact that each orbit of $G$ is discrete. For, if $g_n(x) \to s \in X$, then for any $\epsilon > 0$, $\rho(g_n(x), g_{n+1}(x)) < \epsilon$ for sufficiently large $n$, but since $g_n$ is an isometry, we have $\rho(g_n^{-1}g_{n+1}(x), x) < \epsilon$, which implies that $x$ is an accumulation point for its orbit $Gx$, i.e. $Gx$ is not discrete."
p.30 l.7: In Lemma 2.2.4 replace $w_0$ by $z_0$.
p.32 l.8-l.19: In the proof of Theorem 2.2.6 replace those lines by:
We use Lemma 2.2.4 to see that $\{T \in \Gamma \mid T(z) \in K\} \cap \Gamma$ is a finite set (it is the intersection of a
compact and a discrete set), and hence $\Gamma$ acts properly discontinuously. Conversely, suppose $\Gamma$ acts properly discontinuously, but it is
not a discrete subgroup of PSL(2, $\mathbb{R}$). Then there exists a sequence
$\{T_k\}$ of distinct elements of $\Gamma$ such that $T_k \to \text{Id}$ as $k \to \infty$. Let
$s \in \mathcal{H}$ be a point not fixed by any of $T_k$. Then $\{T_k(s)\}$ is a sequence
of points distinct from $s$ and $T_k(s) \to s$ as $k \to \infty$. Hence every
closed hyperbolic disc centered at $s$ contains infinitely many points
of the $\Gamma$-orbit of $s$, i.e. $\Gamma$ does not act properly discontinuously, a
contradiction.
p.37 l.11: Delete "$\in \Gamma$".
p.37 l.7: In Theorem 2.4.1 delete “a cyclic group.”.
p.38 l.4: Replace “2.3.5” by “2.3.2” and “a finite cyclic group, and
hence abelian” by “an abelian group”.
p.38 l.7-l.4: Replace “Since $\lambda > 1$... upto “distinct terms” by
“Since $\lambda > 1$, the sequence $g_n \circ h \circ g^{-n} \to \text{Id}”$.
p.42: 1 -12: Replace “finite cyclic and therefore elementary” by
“elementary by Theorem 2.4.1”.
p.49 l.5: Replace “homeomorphism” by “isometry”.
p.50 l.8: Replace “hence” by “since $\mu$ is PSL(2, $\mathbb{R}$)–invariant”.
p.65 l.8: Replace “a finite cyclic group (Corollary 2.4.2)” by “an
elementary group (Theorem 2.4.1)”.
p.66 l.1: Delete “limit”.
p.69 l.10: Replace “$\varnothing$” by “$\rho(p, z)$”.
p.70 l.17: Replace “2.3.7” by “2.2.3”.
p.71 l.2: Replace “2.2.6” by “2.2.3”.
p.73 l.4: Replace “$\Gamma$” by “$\mathbb{T} \setminus \{\text{Id}\}$”.
p.73 l.2: Replace “$T_1^{-1}(F) \cap T_1^{-1}(F) \neq \varnothing$” by “$T_1^{-1}(F) \equiv T_1^{-1}(F)$”.
p.74 l.1: Replace “there exists a” by “any”.
p.75 l.1: Replace “intersecting” by “intersects”.
p.75 l.10: Delete “with $\mu(\Gamma \setminus \mathcal{H}) < \infty$”.
p.87 l.1: Replace “$5$” by “$S_0$”.
p.89 l.1: Replace “1.2.8” by “1.2.6”.
p.95 l.2: Replace “Theorem 4.3.2” by “Theorem 3.5.4”.
p.105 l.3 replace “3.2.4” by “3.3.4”.
p.113 l.3: Replace “each element” by “each non–zero element”.
p.117 l.4: Replace “have” by “are integers and have”.
p.119 after l.9 insert “In what follows $A$ will be a quaternion algebra satisfying (5.2.10)”.  
p.119 l.-9: Replace “division” by “quaternion”.
p.119 l.-5: Replace “$\text{SL}(2,\mathbb{R})$” by “$\text{M}(2,\mathbb{R})$”.
p.120 l.4: Replace “$b > 1$, we have $|x_2 - x_3 \sqrt{a}| < \frac{1}{2b} \leq \frac{1}{2}$” by “$|b| \geq 1$, we have $|x_2 - x_3 \sqrt{a}| < \frac{1}{2|b|} \leq \frac{1}{2}$”.
p.123 The end of the proof of Lemma 5.3.3 was modified according to M.Katz’s suggestion.
p.128 In Cor. 5.3.10 added “over $\mathbb{Q}$”.
p.142 Replace 3.6 by 3.5
p.155 l.2: In the matrix replace “$a$” by “$\alpha$”.
p.156 l.7: Replace “1.9” by “1.10”.
p.157 Ex. 2.4 made changes
p.158 l.8: Replace “2.10” by “2.11”.
p.158 l.14: Change “$Q$ and $T$” to “$Q$ and $T$ (or $Q$ and $T_1$)”.
p.160 l.12: Replace “. So … this” by “, which”.