HW 9 Solutions

2. Find the derivative of the following vector functions

(a) $\vec{r}(t) = \langle 0, 1, t \rangle$

$$\boxed{\vec{r}'(t) = \langle 0, 0, 1 \rangle}$$

(b) $\vec{r}(t) = \langle \tan 2t, \sec 3t, 4/t^2 \rangle$

$$\boxed{\vec{r}'(t) = \langle 2\sec^2(2t), 3\tan(3t)\sec(3t), -8/t^{+3} \rangle}$$

(c) $\vec{r}(t) = \langle e^{2t^2}, -2, \ln(5t-2) \rangle$

$$\boxed{\vec{r}'(t) = \langle 4te^{2t^2}, 0, \frac{5}{5t-2} \rangle}$$

3. Find the integral

(a) $\vec{r}(t) = \langle 0, 1, t \rangle$

$$\int \vec{r}(t)\,dt = \langle \int 0\,dt, \int 1\,dt, \int t\,dt \rangle$$

$$= \langle C_1, t + C_2, \tfrac{1}{2}t^2 + C_3 \rangle$$

(b) $\vec{r}(t) = \langle \tan t, \sec 3t, 4/t^2 \rangle$

$\int \vec{r}(t)\,dt = \cancel{\langle -\ln|\cos t| + C_1, \int \frac{4}{t^2}}$

(u-sub)

mutiplied by $\dfrac{\sec(u) + \tan(u)}{\sec(u) + \tan(u)}$

$u = 3t \quad du = \frac{1}{3} dt$

$$\int \vec{r}(t)\,dt = \left\langle \int \frac{\sin(t)}{\cos(t)}\,dt,\ \frac{1}{3}\int \frac{\sec^2(u) + \tan(u)\sec(u)}{\sec(u) + \tan(u)}\,du,\ \int 4t^{-2}\,dt \right\rangle$$

$$= \left\langle -\int \frac{1}{u}\,du,\ \frac{1}{3}\int \frac{dv}{v},\ -2t^{-1} + C_3 \right\rangle$$

$u = \cos(t) \qquad v = \sec(u) + \tan(u)$

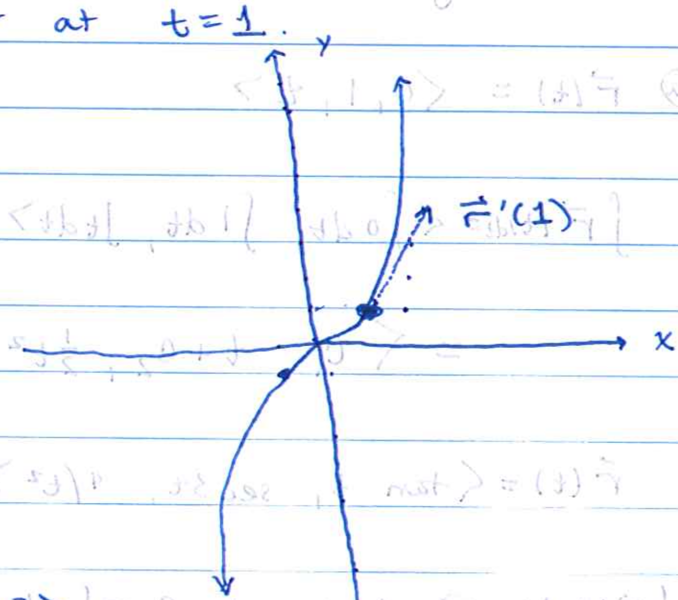
$du = -\sin(t)\,dt \qquad dv = \sec^2(u) + \tan(u)\sec(u)$

$$= \left\langle -\ln(\cos(t)) + C_1,\ \frac{1}{3}\ln(\sec(u) + \tan(u)) + C_2,\ \frac{-2}{t} + C_3 \right\rangle$$

$$= \boxed{\left\langle -\ln(\cos(t)) + C_1,\ \frac{1}{3}\ln(\sec(3t) + \tan(3t)) + C_2,\ \frac{-2}{t} + C_3 \right\rangle}$$

4. Sketch the curve $\vec{r}(t) = \langle t, t^3 \rangle$ and find the tangent vector at $t = 1$.

t	t^3
-2	-8
-1	-1
0	0
1	1
2	8



$\vec{r}\,'(t) = \langle 1, 3t^2 \rangle$

$\vec{r}\,'(1) = \langle 1, 3 \rangle$

5. $\vec{r}(t) = \langle t, t^2, t^3 \rangle$, find

(a) $\vec{r}\,'(t) = \langle 1, 2t, 3t^2 \rangle$

(b) $\vec{r}\,''(t) = \langle 0, 2, 6t \rangle$

(c) $\vec{r}\,' \times \vec{r}\,'' = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 2t & 3t^2 \\ 0 & 2 & 6t \end{vmatrix}$

$= (12t^2 - 6t^2)\vec{i} - (6t)\vec{j} + (2)\vec{k}$

$= \langle 6t^2, -6t, 2 \rangle$

Check: $\langle 1, 2t, 3t^2 \rangle \cdot \langle 6t^2, -6t, 2 \rangle = 0$ ✓
$\langle 0, 2, 6t \rangle \cdot \langle 6t^2, -6t, 2 \rangle = 0$ ✓

(d) $\vec{r}\,' \cdot \vec{r}\,'' = \langle 1, 2t, 3t^2 \rangle \cdot \langle 0, 2, 6t \rangle$

$= 0 + 4t + 18t^3$

$= \boxed{2t(2 + 9t^2)}$