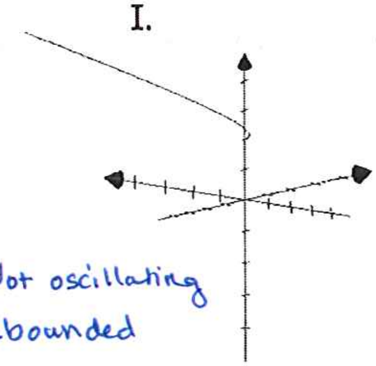
MATH 231: Calculus of Several Variables
Section 1, 107 Ag Sc & Ind Bldg,
TR 9:05 AM - 9:55 AM

Homework 8: Due Tuesday, Oct 1 Solutions

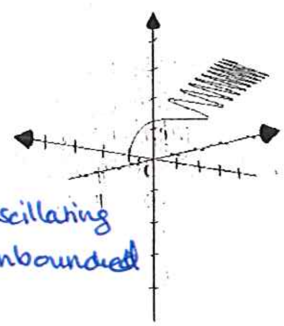
1. Read the notes titled "Vector Functions and Space Curves"
2. Sketch the vector equation $\vec{r}(t) = \langle t, 1, t^2 \rangle$
3. Match the correct functions to the correct graphs. Write the domain of each function

I.



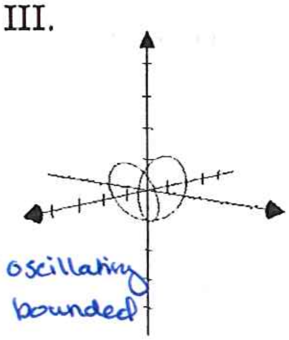
Not oscillating
unbounded

II.



Oscillating
unbounded

III.



oscillating
bounded

(a) $\vec{r}(t) = \langle \cos(t^3), t, \sqrt{t} \rangle$ (II.)
(b) $\vec{r}(t) = \langle t^2, t, 2 \rangle$ (I.)
(c) $\vec{r}(t) = \langle \cos t, \sin 2t, \cos 2t \rangle$ (III.) ← bounded since each entry is between -1 and 1.

4. Find the domain of the following functions

(a) $\vec{r}(t) = \left\langle \frac{\cos(\sqrt{t})}{1-t}, \frac{t^2-1}{(t-3)(t+1)}, \ln(t) \right\rangle$
(b) $\vec{r}(t) = \left\langle \frac{1}{t}, \sqrt{t}, 1 \right\rangle \times \left\langle \frac{\sqrt{t}}{t-1}, t, 0 \right\rangle$ (this is a cross product!)

Some Review Questions

5. Which of the following pairs of planes are orthogonal to each other? (There may be more than one.)

   (a) $x - 2y + z = 4$ and $-x + y - z = 12$

   (b) $2x + y + z = 0$ and $-x + 2z = 1$

   (c) $x + 2y + 3z = 1$ and $x + y - z = 1$

   (d) $x + y + z = 1$ and $x - y + z = -1$

6. Consider the planes

$$P1 : 3x + ay + 9z = 3 \qquad P2 : x + 2y + 3z = 1$$

$\boxed{a = 6}$

   where a is a constant. For what value of a will the two planes be parallel to each other?

7. Let m_1 and m_2 be two masses located at $\vec{r}_1$ and $\vec{r}_2$, respectively. Select which vector represents the center of mass

$$\frac{m_1\vec{r}_1 + m_2\vec{r}_2}{m_1 + m_2}$$

   if $\vec{r}_1 = \langle 1, 1, 0\rangle$, $\vec{r}_2 = \langle 2, 0, -1\rangle$, $m_1 = 2$, and $m_2 = 3$.

   (a) $\langle 7/5, 3/5, -2/5\rangle$

   (b) $\langle 8/5, 2/5, -3/5\rangle$

   (c) $\langle 3/5, 1/5, -1/5\rangle$

   (d) You can't calculate this expression

8. Which of the following best describes the surface defined by $x^2 + 20y^2 - 4z^2 + 4 = 0$

   (a) An ellipsoid

   (b) A sphere

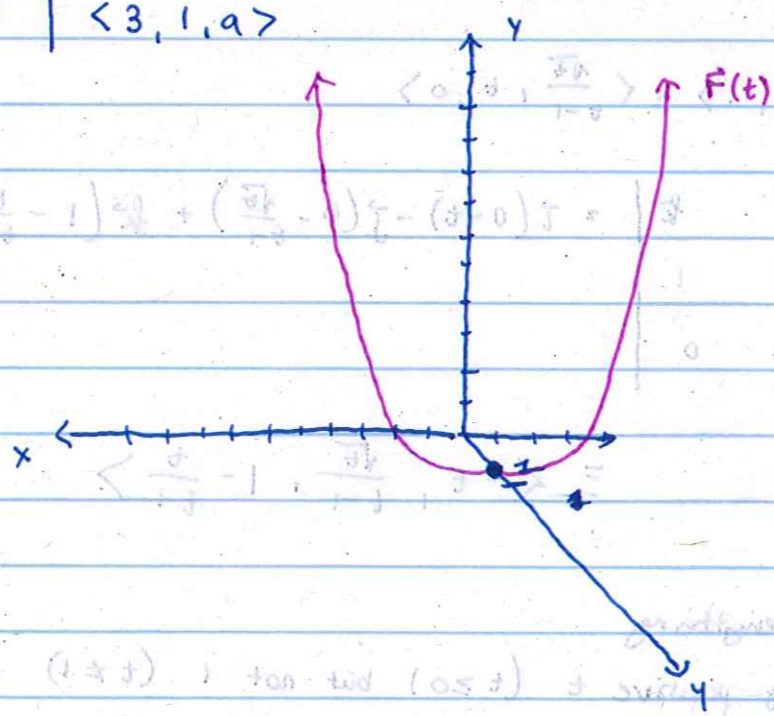
   (c) A hyperboloid of one sheet

   (d) A hyperboloid of two sheets

2. $\vec{r}(t) = \langle t, 1, t^2 \rangle$

t	$\vec{r}(t) = \langle t, 1, t^2 \rangle$
-3	$\langle -3, 1, 9 \rangle$
-2	$\langle -2, 1, 4 \rangle$
-1	$\langle -1, 1, 1 \rangle$
0	$\langle 0, 1, 0 \rangle$
1	$\langle 1, 1, 1 \rangle$
2	$\langle 2, 1, 4 \rangle$
3	$\langle 3, 1, 9 \rangle$

Notice that y doesn't change at all. All the "action" is happening on the xz-plane. So we should graph this function in an orientation where we can see that plane best.



3. (a.) II

(b.) I.

(c.) III.

4. (a) $\vec{r}(t) = \left\langle \frac{\cos\sqrt{t}}{1-t}, \frac{t^2-1}{(t-3)(t+1)}, \ln(t) \right\rangle$

- Domain of x: $[0,1) \cup (1, \infty)$

- Domain of y: $(-\infty,-1)\cup(-1,3)\cup(3,\infty)$
- Domain of z: $(0,\infty)$

Most restrictive domain is x: $[0,1)\cup(1,\infty)$

But notice that the domain of z does not contain 0 and the domain of y does not contain 3. So we combine these restrictions to get our answer:

$$\boxed{D=(0,1)\cup(1,3)\cup(3,\infty)}$$

b. $\vec{r}(t)=\langle \frac{1}{t}, \sqrt{t}, 1\rangle \times \langle \frac{\sqrt{t}}{t-1}, t, 0\rangle$

$$=\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1/t & \sqrt{t} & 1 \\ \frac{\sqrt{t}}{t-1} & t & 0 \end{vmatrix} = \vec{i}(0-t)-\vec{j}\left(0-\frac{\sqrt{t}}{t-1}\right)+\vec{k}\left(1-\frac{t}{t-1}\right)$$

$$=\left\langle -t, \frac{\sqrt{t}}{t-1}, 1-\frac{t}{t-1}\right\rangle$$

Domain of x: Everything

Domain of y: only positive t $(t\geq 0)$ but not 1 $(t\neq 1)$

Domain of z: Everything but 1, $(t\neq 1)$

Domain: $\boxed{D=[0,1)\cup(1,\infty)}$

5. a. $\langle 1, -2, 1\rangle \cdot \langle -1, 1, -1\rangle = -1-2-1 = -4 \neq 0$ <u>NO</u>

b. $\langle 2, 1, 1\rangle \cdot \langle -1, 0, 2\rangle = -2+0+2 = 0$ <u>YES</u>

c. $\langle 1, 2, 3\rangle \cdot \langle 1, 1, -1\rangle = 1+2-3 = 0$ <u>YES</u>

d. $\langle 1, 1, 1\rangle \cdot \langle 1, -1, 1\rangle = 1-1+1 = 1$ <u>NO</u>

<u>Answer</u> <u>b & c</u>

6. Two vectors are $\parallel$ if they are multiples of each other:

$\longrightarrow \vec{v}$

$\xrightarrow{\hspace{4cm}} b\vec{v}$

So,
$$\langle 3, a, 9\rangle = b\langle 1, 2, 3\rangle$$
$$\Rightarrow \quad 3 = b \text{ and } 9 = 3b$$
$$\text{So } b = 3$$
$$\text{Then } a = 3 \cdot 2 = 6$$

So $\boxed{a=6}$

7. We just plug in:

$$\frac{2\langle 1,1,0\rangle + 3\langle 2,0,-1\rangle}{2+3} = \frac{\langle 2,2,0\rangle + \langle 6,0,-3\rangle}{5}$$

$$= \frac{\langle 8, 2, -3\rangle}{5} = \left\langle \frac{8}{5}, \frac{2}{5}, \frac{-3}{5}\right\rangle$$

<u>Answer:</u> b

8. $x^2 + 20y^2 - 4z^2 + 4 = 0$

$$\Rightarrow \frac{x^2}{-4} + \frac{20y^2}{-4} - \frac{4z^2}{-4} = \frac{-4}{-4}$$

$$\Rightarrow -\frac{x^2}{4} - 5y^2 + z^2 = 1$$

This is a hyperboloid of two sheets.