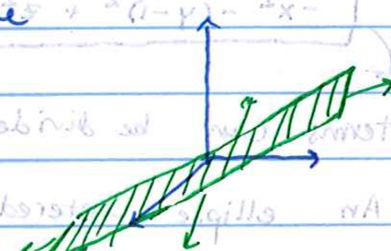
Homework 7 solutions

2. Determine if the surface is a cylinder

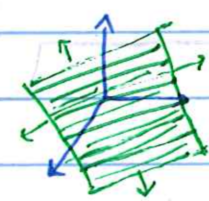
(a) $z = y$

$\boxed{\text{Yes.}}$ This is a plane



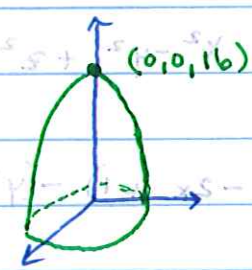
(b) $x + y + z = 12$

$\boxed{\text{Yes.}}$ This is also a plane.



(c) $x^2 + y^2 = 16 - z$

$\boxed{\text{No.}}$ This is a parabola



3. Write the equation for the following surfaces.

(a) A parabola centered at the point $(1,0,0)$ Along the y-axis

$$\boxed{\begin{array}{c} (x-1)^2 + z^2 = y \quad \text{or} \quad -(x-1)^2 - z^2 = y \\ \cancel{(x-1)^2 + y^2 = z \quad \text{or} \quad -(x-1)^2 - y^2 = z} \end{array}}$$

(or any contants in front of the terms)

(b) A hyperboloid of two sheets opening along the z-axis, centered at $(0,1,0)$

$$\boxed{-x^2-(y-1)^2+z^2=1}$$

~~multiplied by constant.~~

(terms can be divided by constants)

(c) An ellipse centered at $(1,1,1)$ where the figure is longer along the z-axis.

$$\boxed{(x-1)^2+(y-1)^2+\frac{(z-1)^2}{2}=1}$$

Any # bigger than one

4. Classify and sketch the curves

(a) $x^2-y^2+z^2-2x+4y-6z+5=0$

$$(x^2-2x+1)-(y^2-4y+4)+(z^2-6z+9)+5=1-4+9$$

$$(x-1)^2-(y-2)^2+(z-3)^2+5=9$$
$$-5 \quad -5$$

$$(x-1)^2-(y-2)^2+(z-3)^2=4$$

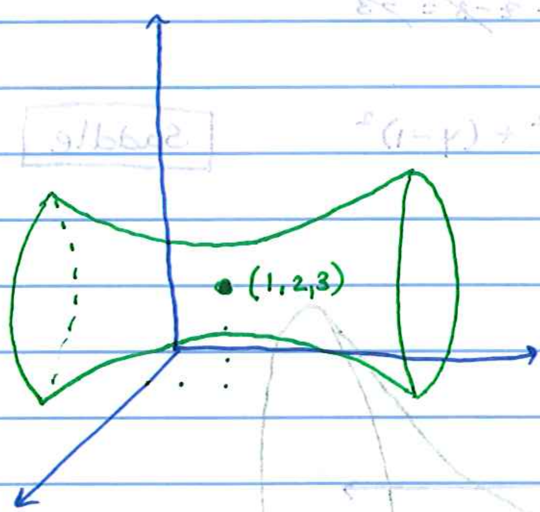
$$\Rightarrow \frac{(x-1)^2}{4}-\frac{(y-2)^2}{4}+\frac{(z-3)^2}{4}=1$$

$$\boxed{\text{Hyperboloid of One Sheet}}$$

(4a continued)

center: $(1, 2, 3)$

opens along y-axis



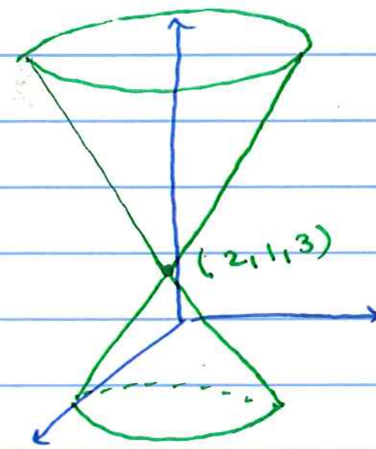
4b) $-x^2 - y^2 + z^2 + 4x + 2y - 6z + 4 = 0$

$$-(x^2 - 4x + 4) - (y^2 - 2y + 1) + (z^2 - 6z + 9) + 4 = -4 - 1 + 9$$

$$-(x-2)^2 - (y-1)^2 + (z-3)^2 + 4 = 4$$

$$-(x-2)^2 - (y-1)^2 + (z-3)^2 = 0$$

Cone



centered at $(2, 1, 3)$

opening along z-axis

4e.) $-x^2+y^2+4x-2y-z-3=0$

$$-(x^2-4x+4)+(y^2-2y+1)-z-3=-4+1$$

$$-(x-2)^2+(y-1)^2-z-\cancel{3}=\cancel{-3}$$

$$z=-(x-2)^2+(y-1)^2 \qquad \boxed{\text{Saddle}}$$