Homework 5 Solutions

1) ⓐ $\vec{v} = \langle 1,0,1 \rangle$, $\vec{x} = \langle 2,2,0 \rangle$

$\vec{v} \cdot \vec{x} = (2)(1) + (2)(0) + (0)(1) = \boxed{2}$

ⓑ $|\vec{v}| = \sqrt{1+0+1} = \boxed{\sqrt{2}}$

$|\vec{x}| = \sqrt{4+4+0} = \boxed{2\sqrt{2}}$

ⓒ $\cos\theta = \dfrac{2}{(\sqrt{2})(2\sqrt{2})} = \boxed{\dfrac{1}{2}}$

ⓓ If $\cos\theta = 1/2$, then $\theta = 60°$

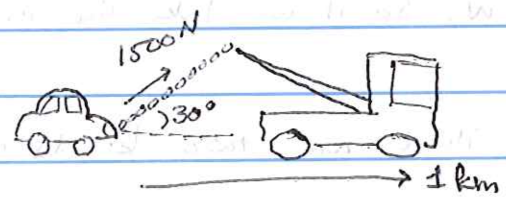
since $60° > 45°$, the vectors are more $\perp$ than $\parallel$.

2) ⓐ Work $= \langle 4,2,-5 \rangle \cdot \langle 1,1,-1 \rangle$

$= (4)(1) + (2)(1) + (-5)(-1)$

$= 4 + 2 + 5 = \boxed{11}$

ⓑ



Work $= |1||1500| \cos 30° = 1500\left(\dfrac{\sqrt{3}}{2}\right) = \boxed{750\sqrt{3}}$

3) ⓐ $2\vec{i} + 3\vec{j} - \vec{k} = 2\langle 1,0,0 \rangle + 3\langle 0,1,0 \rangle - \langle 0,0,1 \rangle$

$= \langle 2,0,0 \rangle + \langle 0,3,0 \rangle - \langle 0,0,1 \rangle$

$= \boxed{\langle 2,3,-1 \rangle}$

$\vec{i} \cdot \vec{j} = \langle 1,0,0 \rangle \cdot \langle 0,1,0 \rangle = (1)(0) + (0)(1) + (0)(0) = 0$

$\vec{j} \cdot \vec{k} = \langle 0,1,0 \rangle \cdot \langle 0,0,1 \rangle = (0)(0) + (1)(0) + (0)(1) = 0$

$\vec{k} \cdot \vec{i} = \langle 0,0,1 \rangle \cdot \langle 1,0,0 \rangle = (0)(1) + (0)(0) + (1)(0) = 0$

(c) $\vec{i} \cdot \vec{i} = \langle 1,0,0 \rangle \cdot \langle 1,0,0 \rangle = (1)(1) + (0)(0) + (0)(0) = 1$

$\vec{j} \cdot \vec{j} = \langle 0,1,0 \rangle \cdot \langle 0,1,0 \rangle = (0)(0) + (1)(1) + (0)(0) = 1$

$\vec{k} \cdot \vec{k} = \langle 0,0,1 \rangle \cdot \langle 0,0,1 \rangle = (0)(0) + (0)(0) + (1)(1) = 1$

(d) $\vec{i} \times \vec{j} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{vmatrix} = \vec{i}(0) - \vec{j}(0) + \vec{k}(1) = \boxed{\langle 0,0,1 \rangle}$

$\vec{j} \times \vec{k}$ and $\vec{i} \times \vec{k}$ will both be $\vec{i}$ and $\vec{j}$ respectively.

(e) Find a unit vector $\perp$ to $\vec{v} = \vec{i} + \vec{j}$, $\vec{w} = \vec{i} + \vec{j}$

Notice that $\vec{v} = \vec{w}$, So if we take the cross product, it won't work.

$\vec{v} = \langle 1,1,0 \rangle$. There are two kinds of vectors perpendicular to it:

$\vec{u}_1 = \langle 0,0,1 \rangle$ ← both acceptable answers

$\vec{u}_2 = \langle -1,1,0 \rangle$

We take the dot product to verify

4.) $P(0,-1,1)$ $\quad \overrightarrow{PQ} = \langle 2,4,3 \rangle$

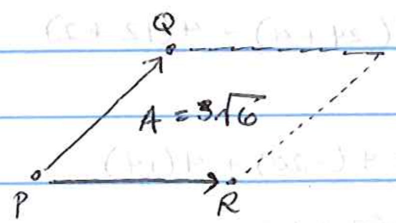
$Q(2,3,4)$ $\quad \overrightarrow{PR} = \langle -1,1,0 \rangle$

$R(-1,0,1)$

$$\overrightarrow{PQ} \times \overrightarrow{PR} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 4 & 3 \\ -1 & 1 & 0 \end{vmatrix} = -3\vec{i} - (3)\vec{j} + 6\vec{k}$$

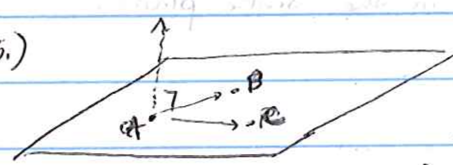
$$|\overrightarrow{PQ} \times \overrightarrow{PR}| = \sqrt{9+9+36} = \sqrt{54} = 3\sqrt{6}$$

This is the area of the parallelogram



So the area of the triangle is $\boxed{\frac{3\sqrt{6}}{2}}$

5.)



$A(1,0,1)$, $B(-2,1,3)$, $C(4,2,5)$

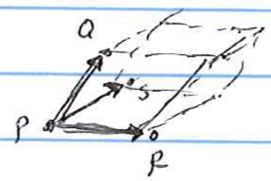
$\overrightarrow{AB} = \langle -3,1,2 \rangle$

$\overrightarrow{AC} = \langle 3,2,4 \rangle$

$$\overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -3 & 1 & 2 \\ 3 & 2 & 4 \end{vmatrix} = (4-4)\vec{i} - (-12-6)\vec{j} + (-6-3)\vec{k}$$

$$= \boxed{\langle 0, 18, -9 \rangle}$$

6.) We Find the volume of the parallelepiped. If it is zero, it's because the figure is flat. If the figure is flat, then all 3 vectors are in the same plane and, therefore, the four points are in the same plane.



$\vec{PQ} = \langle 2, -4, 4 \rangle$

$\vec{PR} = \langle 4, -1, -2 \rangle$

$\vec{PS} = \langle 2, 3, -6 \rangle$

$$V = \begin{vmatrix} 2 & -4 & 4 \\ 4 & -1 & -2 \\ 2 & 3 & -6 \end{vmatrix} = 2(6+6) + 4(-24+4) + 4(12+2)$$

$$= 2(12) + 4(-20) + 4(14)$$

$$= 24 + -80 + 56$$

$$= 80 - 80 = \boxed{0}$$

So, yes, the four points lie in the same plane.