Homework #4 solutions

#2. Find the cross product

(a) $\vec{a} = \langle 1,1,2 \rangle$, $\vec{b} = \langle 0,0,1 \rangle$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 1 & 2 \\ 0 & 0 & 1 \end{vmatrix} = \vec{i}(1-0) - \vec{j}(1-0) + \vec{k}(0-0)$$

$$= \vec{i} - \vec{j} = \boxed{\langle 1, -1, 0 \rangle}$$

(b) $\vec{a} = \langle t, \cos t, \sin t \rangle$, $\vec{b} = \langle 1, -\sin t, \cos t \rangle$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ t & \cos t & \sin t \\ 1 & -\sin t & \cos t \end{vmatrix} = \vec{i}(\cos^2 t + \sin^2 t) - \vec{j}(t\cos t - \sin t) + \vec{k}(-t\sin t - \cos t)$$

distribute

$$= \vec{i} + (\sin t - t\cos t)\vec{j} - (t\sin t + \cos t)\vec{k}$$

$$= \boxed{\langle 1, \sin t - t\cos t, -t\sin t - \cos t \rangle}$$

#3. Do these statements make sense?

(a) $(\vec{a} \cdot \vec{b}) \times (\vec{c} \cdot \vec{d})$

No. $\vec{a} \cdot \vec{b}$ and $\vec{c} \cdot \vec{d}$ produce scalar values. You cannot ~~cross~~ take the cross product of two scalar values.

(b) $(\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d})$

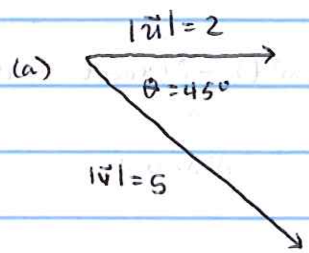
Yes. $\vec{a} \times \vec{b}$ is a vector as is $\vec{c} \times \vec{d}$. We can always take the cross product of two vectors

(c) $(\vec{a} \times \vec{b}) \cdot (\vec{c} \times \vec{d})$

Yes. Both $\vec{a} \times \vec{b}$ and $\vec{c} \times \vec{d}$ are vectors. We can take a dot product of two vectors.

4.) Find $|\vec{u} \times \vec{v}|$ and determine if the vector is goining into or out of the page

(a)

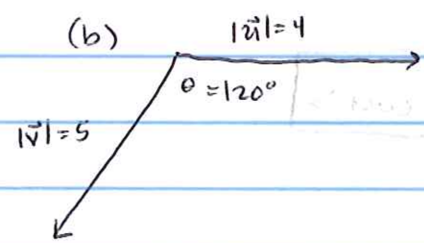


$|\vec{u} \times \vec{v}| = 10 \sin(45°)$

$= \boxed{5\sqrt{2}}$

$\vec{u} \times \vec{v}$ goes into the page

(b)

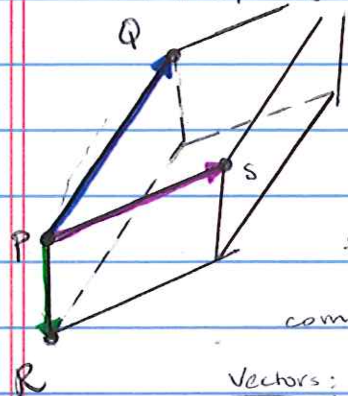


$|\vec{u} \times \vec{v}| = 20 \sin(120°)$

$= \boxed{10\sqrt{3}}$

$\vec{u} \times \vec{v}$ goes into the page

5. Find the volume of the parallelepiped formed by $P(3,0,1)$, $Q(-1,2,5)$, $R(5,1,-1)$, and $S(0,4,2)$



For this problem, I chose P to be my point in common but you are allowed to pick any point. Your final answer will come out to the same value.

Vectors:

$$\overrightarrow{PQ} = \langle -1-3,\ 2-0,\ 5-1 \rangle = \langle -4, 2, 4 \rangle$$

$$\overrightarrow{PS} = \langle 0-3,\ 4-0,\ 2-1 \rangle = \langle -3, 4, 1 \rangle$$

$$\overrightarrow{PR} = \langle 5-3,\ 1-0,\ -1-1 \rangle = \langle 2, 1, -2 \rangle$$

$$\overrightarrow{PQ} \cdot (\overrightarrow{PS} \times \overrightarrow{PR}) = \det \begin{vmatrix} -4 & 2 & 4 \\ -3 & 4 & 1 \\ 2 & 1 & -2 \end{vmatrix} = -4(-8-1) - 2(6-2) + 4(-3-8)$$

$$= -4(-9) - 2(4) + 4(-11)$$

$$= 36 - 8 - 44$$

$$= -16$$

$$\text{Volume} = |-16| = \boxed{16}$$