HW # 3 solutions

2. Find the dot product

(a) $\vec{a} = \langle 6, -2, 3 \rangle \quad \vec{b} = \langle 2, 5, -1 \rangle$

$$\vec{a} \cdot \vec{b} = (6)(2) + (-2)(5) + (3)(-1)$$
$$= 12 - 10 - 3 = \boxed{-1}$$

(b) $\vec{a} = 2\vec{i} + \vec{j} \quad \vec{b} = \vec{i} - \vec{j} + 2\vec{k}$

$$\vec{a} \cdot \vec{b} = (2)(1) + (1)(-1) + (2)(0)$$
$$= 2 - 1 = \boxed{1}$$

(c) $\vec{a} = \langle x, y, 0 \rangle \quad \vec{b} = \langle y, -x, 0 \rangle$

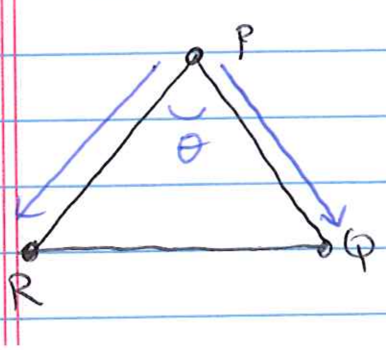
$$\vec{a} \cdot \vec{b} = (x)(y) + (y)(-x) + (0)(0)$$
$$= xy - xy = \boxed{0}$$

3. Does this make sense?

$$\vec{a} \cdot \vec{b} + \vec{c}$$

$\boxed{\text{No.}}$ $\vec{a} \cdot \vec{b}$ is a scalar and $\vec{c}$ is a vector.
You can't add a scalar and a vector.

4. Is the triangle formed by $P(1, -3, -2)$, $Q(2, 0, -4)$ and $R(6, -2, -5)$ a right triangle?



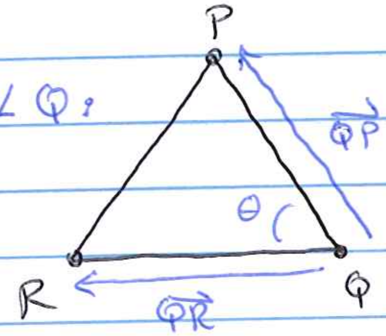
remember that if $\vec{v} \cdot \vec{w} = 0$, $\theta = 90°$

$\angle P$:

$$\vec{PQ} \cdot \vec{PR} =$$
$$= \langle 2-1, 0-(-3), -4-(-2) \rangle \cdot \langle 6-1, -2-(-3), -5-($$
$$= \langle 1, 3, -2 \rangle \cdot \langle 5, 1, -3 \rangle = 5 + 3 + 6 = 14 \neq 0$$

So $\angle P$ is not $90°$

$\angle Q$:



$\vec{QP} \cdot \vec{QR} = \langle 1-2, -3-0, -2-(-4) \rangle \cdot \langle 6-2, -2-0, -5-(-4) \rangle$

$= \langle -1, -3, 2 \rangle \cdot \langle 4, -2, -1 \rangle$

$= (-1)(4) + (-3)(-2) + (2)(-1)$

$= -4 + 6 - 2$

$= 0$

So $\angle Q$ is $90°$. Therefore, it is a right triangle.

Alternative approach:

$d(P,Q) = \sqrt{(2-1)^2 + (0+3)^2 + (-4+2)^2}$

$= \sqrt{1+9+4} = \sqrt{14}$

$d(Q,R) = \sqrt{4^2 + (-2)^2 + (-1)^2} = \sqrt{21}$

$d(R,P) = \sqrt{5^2 + 1^2 + 3^2} = \sqrt{35}$

and $\underbrace{(\sqrt{14})^2}_{\text{length of P to Q}} + \underbrace{(\sqrt{21})^2}_{\text{length of Q to R}}$

$= 14 + 21$

$= 35 = \underbrace{(\sqrt{35})^2}_{\text{length of R to P}}$

So it is a right triangle.

5. Find the scalar and vector projections of $\vec{a}$ onto $\vec{b}$:

$\vec{a} = \langle 1, 1, 1 \rangle$, $\vec{b} = \langle 1, -1, 1 \rangle$

$\text{comp}_{\vec{b}} \vec{a} = \dfrac{\vec{a} \cdot \vec{b}}{|\vec{b}|} = \dfrac{1-1+1}{\sqrt{3}} = \dfrac{1}{\sqrt{3}}$

~~project~~

$$\text{proj}_{\vec{b}}\vec{a} = \text{comp}_{\vec{b}}\vec{a} \cdot \frac{\vec{b}}{|\vec{b}|}$$

$$= \left(\frac{1}{\sqrt{3}}\right)\left(\frac{\langle 1,-1,1\rangle}{\sqrt{3}}\right)$$

$$= \left(\frac{1}{\sqrt{3}}\right)\left\langle \frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right\rangle$$

$$= \boxed{\left\langle \frac{1}{3}, -\frac{1}{3}, \frac{1}{3}\right\rangle}$$

6.) Find ~~the angle between~~ x such that the angle between

$\langle 2,1,-1\rangle$ an $\langle 1, x, 0\rangle$

is $90°$.

Two ways to see this solution:

I. $\langle 2,1,-1\rangle \cdot \langle 1,x,0\rangle = 2 + x + 0 = 0$

$\Rightarrow \boxed{x = -2}$

II. Part (c) of #2 tell's us that

$\langle x, y, 0\rangle \perp \langle y, -x, 0\rangle$

But the same is true for

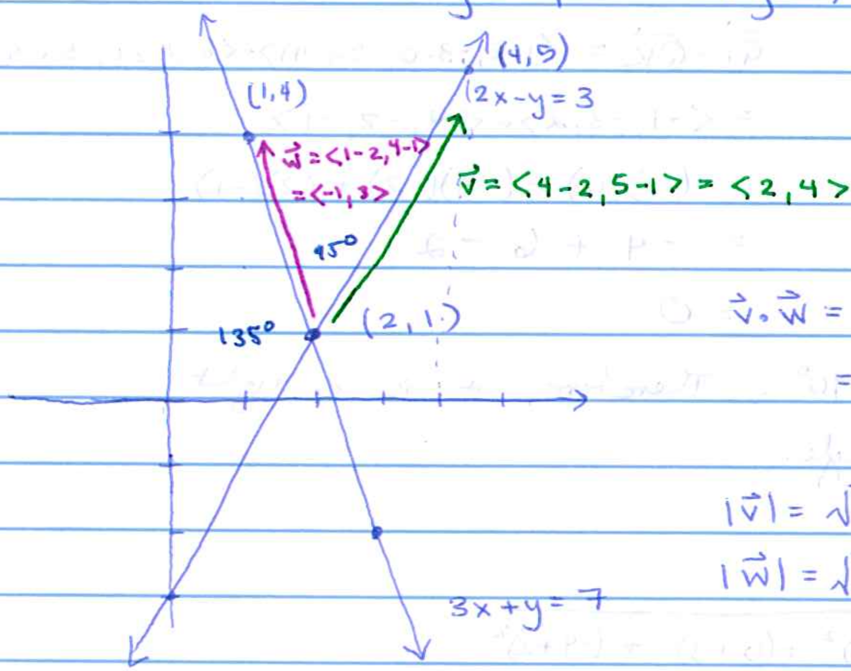
$\langle x, y, z\rangle \perp \langle y, -x, 0\rangle$

so $\cancel{x} = -2$

since $y = 1$ and $z = -1$.

7. What is the angle between two lines

$$2x - y = 3, \quad 3x + y = 7$$



$$\vec{v} \cdot \vec{w} = \langle 2,4 \rangle \cdot \langle -1, 3 \rangle$$
$$= (2)(-1) + (4)(3) = 10$$

$$|\vec{v}| = \sqrt{4+16} = \sqrt{20} = 2\sqrt{5}$$
$$|\vec{w}| = \sqrt{1+9} = \sqrt{10}$$

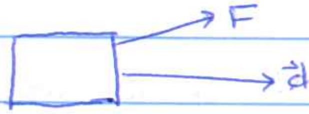
$$\cos\theta = \frac{10}{(\sqrt{20})(\sqrt{10})} = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \boxed{\theta = 45^\circ}$$

If you looked at a different orientation, you may have gotten 135°. But that's ok because

$135° + 45° = 180°$ ← straight line

8. Find the work done by $F = \langle 8,6,9 \rangle$ moves an object from $(0,10,8)$ to $(6,12,20)$.



displacement: $\vec{d} = \langle 6-0, 12-10, 20-8 \rangle$
$= \langle 6, 2, 12 \rangle$

Work (a scalar measurement) is the amount ~~___~~ Force and dispacement move together:

$$\vec{F} \cdot \vec{d} = \langle 8,6,9 \rangle \cdot \langle 6,2,12 \rangle = 48 + 12 + 108 = \boxed{168}$$