Hw 23 solutions

(a) $f(x,y) = y^2 + x^2$ s.t. $xy = 1$

$\nabla f = \langle 2x, 2y \rangle$
$\nabla g = \langle y, x \rangle$

$\nabla f = \lambda \nabla g \rightarrow$
(i) $2x = \lambda y$
(ii) $2y = \lambda x$
(iii) $xy = 1$

By the constraint, $y = 1/x$ (if $x \neq 0$)
so

(i) $2x = \lambda\left(\frac{1}{x}\right) \Rightarrow 2x^2 = \lambda$

(ii) $2\left(\frac{1}{x}\right) = \lambda x \Rightarrow 2 = \lambda x^2$

so $2 = (2x^2)x^2 = 2x^4$
$\Rightarrow 1 = x^4$
$\Rightarrow \boxed{x = \pm 1} \Rightarrow \boxed{y = \pm 1}$

The sign of x and y is the same here.

So our points are $(1,1)$ and $(-1,-1)$

Could $x = 0$? Then $2y = 0 \Rightarrow y = 0$
and $(0)(0) = 1$ from (iii)
This is false, so it can't!

$f(1,1) = 2$, $f(-1,-1) = 2$

Are these points maximums or minimums?

Let's consider another point on our constraint $xy=1$, like $(2, 1/2)$.

$$f(2, 1/2) = (2)^2 + (1/2)^2 = 4 + \frac{1}{4} = \frac{17}{4} > 2$$

So $(1,1)$ and $(-1,-1)$ are minimums.

ⓑ $f(x,y) = e^{xy}$ s.t. $x^3 + y^3 = 16$

$$\left.\begin{aligned} \nabla f &= \langle ye^{xy}, xe^{xy} \rangle \\ \nabla g &= \langle 3x, 3y \rangle \end{aligned}\right\} \longrightarrow \begin{aligned} &\text{(i) } ye^{xy} = 3x\lambda \\ &\text{(ii) } xe^{xy} = 3y\lambda \\ &\text{(iii) } x^3 + y^3 = 16 \end{aligned}$$

(i) $\lambda = \dfrac{ye^{xy}}{3x}$, (ii) $\lambda = \dfrac{xe^{xy}}{3y}$ (if $x, y \neq 0$)

so $\dfrac{y\cancel{e^{xy}}}{\cancel{3}x} = \dfrac{x\cancel{e^{xy}}}{\cancel{3}y}$

$$\Rightarrow y^2 = x^2 \Rightarrow y = \pm x$$

by (iii) $x^3 \pm x^3 = 16$

$y=x$ $2x^3 = 16 \Rightarrow x^3 = 8 \Rightarrow \boxed{x=2}$

$y=-x$ $0 = 16$ nothing!

pt:

$(2,2)$ $f(2,2) = e^4$

Could $x = 0$?

If so, then $ye^{xy} = 0 \Rightarrow y = 0$

So $0^3 + 0^3 = 16$ <u>Not</u> possible.

Is $(2,2)$ a max or a min?

$(\sqrt[3]{16}, 0)$ also satisfies the constraint.

$$f(\sqrt[3]{16}, 0) = e^0 = 1 < e^4$$

So $(2,2)$ is a max.