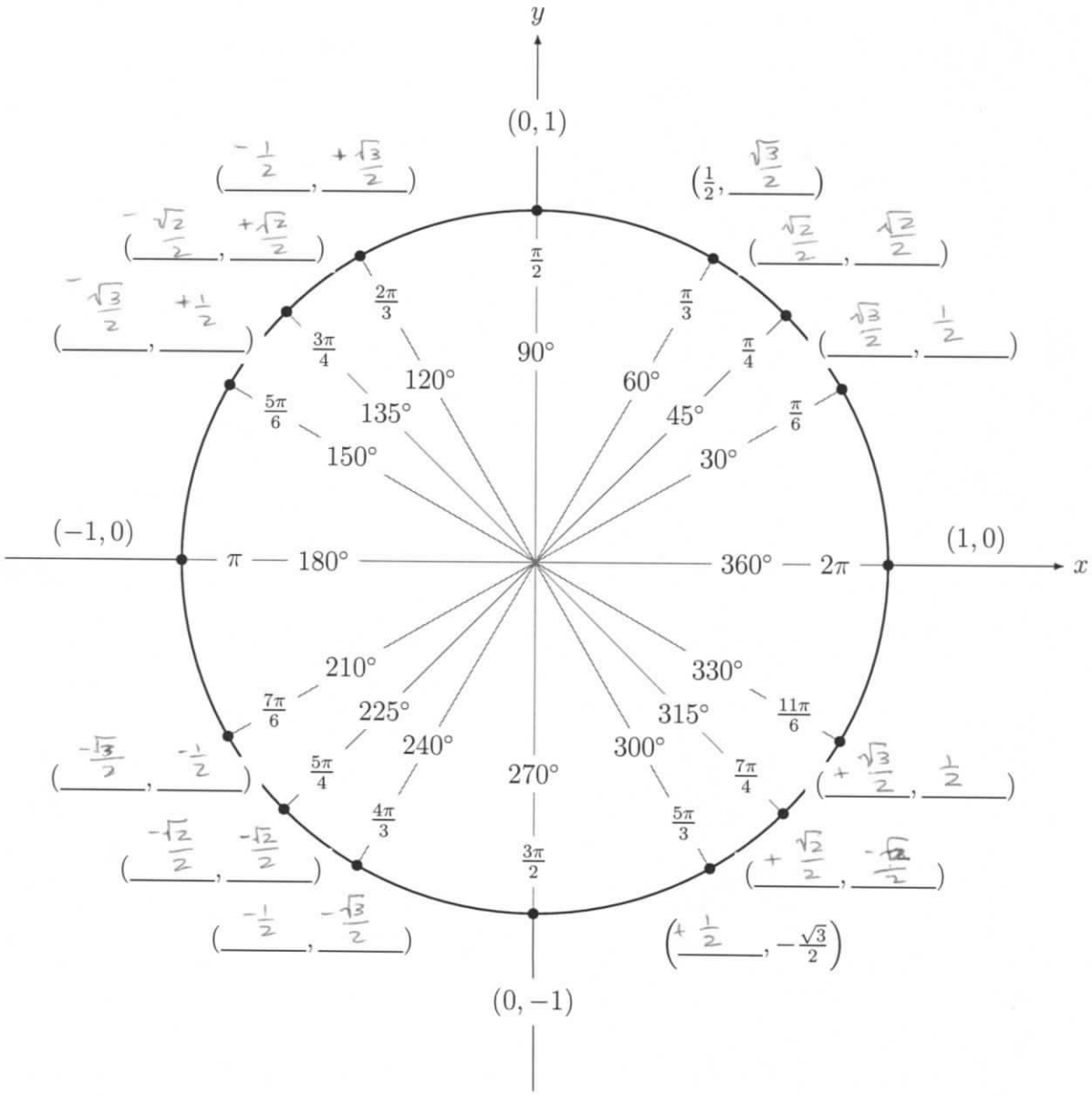
Solutions

MATH 231: Calculus of Several Variables
Section 1, 107 Ag Sc & Ind Bldg,
TR 9:05 AM - 9:55 AM

Homework 1 (Review Assignment): Due Thursday, August 29

1. Read the Notes titled "Course Philosophy & Review" posted on the website.
2. Fill in the blanks (the values of cosine and sine) for the unit circle below.



3. $\tan x \sin x + \cos x = \sec x$

$\Rightarrow \frac{\sin x}{\cos x} \sin x + \cos x = \frac{1}{\cos x}$

$\Rightarrow$ (multiply through by $\cos x$)

$\sin^2 x + \cos^2 x = 1$ ✓ TRUE

- $\tan^2\theta = \csc^2\theta \tan^2\theta - 1$

$\Rightarrow \frac{\sin^2\theta}{\cos^2\theta} = \frac{1}{\sin^2\theta} \cdot \frac{\sin^2\theta}{\cos^2\theta} - 1$

$\Rightarrow \frac{\sin^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta} - 1$ (multiply through by $\cos^2\theta$)

$\Rightarrow \sin^2\theta = 1 - \cos^2\theta$

$\Rightarrow \sin^2\theta + \cos^2\theta = 1$ ✓ True

- $\cos^2 t = \frac{\csc t \cos t}{\tan t + \cot t}$

$\Rightarrow \cos^2 t (\tan t + \cot t) = \csc t \cos t$

$\Rightarrow \cos^2 t \tan t + \cos^2 t \cot t = \csc t \cos t$

$\Rightarrow \frac{\cos^2 t \sin t}{\cos t} + \cos^2 t \frac{\cos t}{\sin t} = \frac{1}{\sin t} \cos t$

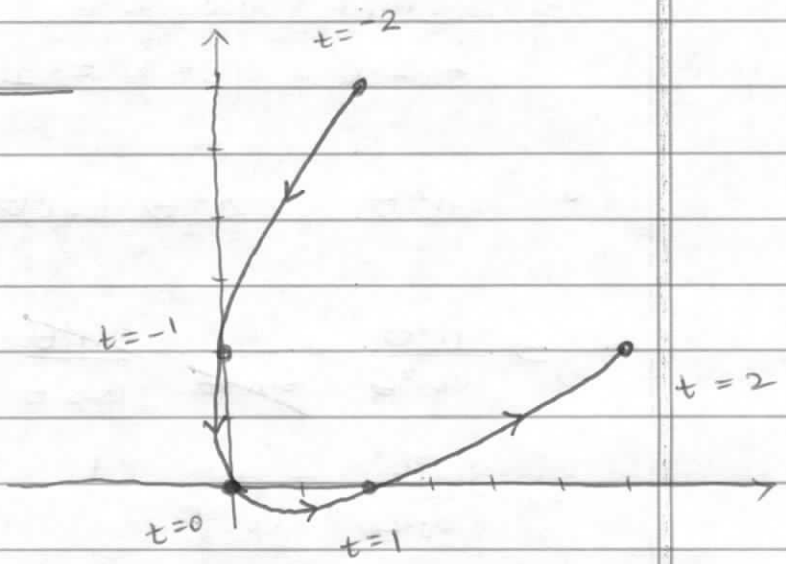
$\Rightarrow \sin t + \frac{\cos^2 t}{\sin t} = \frac{1}{\sin t}$

$\Rightarrow$ (multiply through by $\sin t$)

$\sin^2 t + \cos^2 t = 1$ ✓ **True**

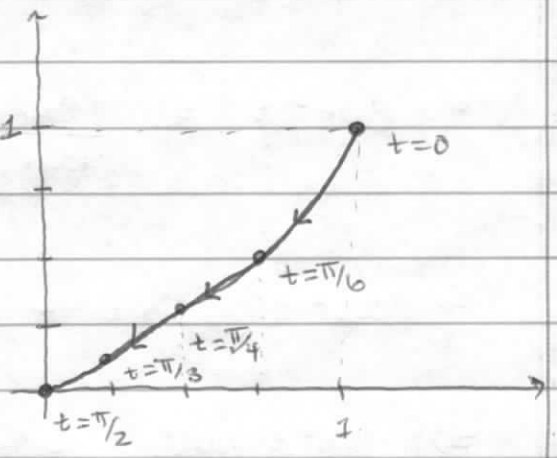
4. Graph $x = t^2 + t$, $y = t^2 - t$ $\quad -2 \le t \le 2$

t	x	y
-2	2	6
-1	0	2
0	0	0
1	2	0
2	6	2



5. Graph $x = \cos^2 t$ $\quad y = 1 - \sin t$ $\quad 0 \le t \le \pi/2$

t	x	y
0	1	1
$\pi/6$	$3/4$	$1/2$
$\pi/4$	$1/2$	$\frac{2-\sqrt{2}}{2} \approx .3$
$\pi/3$	$1/4$	$\frac{2-\sqrt{3}}{2} \approx .13$
$\pi/2$	0	0



6. Find the derivative

(a) $f(x) = \dfrac{\cos(x)}{1+\sin(x)}$

$$f'(x) = \frac{(1+\sin(x))(-\sin(x)) - (\cos(x))(\cos(x))}{(1+\sin(x))^2}$$

$$= \frac{-\sin(x) - \sin^2(x) - \cos^2(x)}{(1+\sin(x))^2}$$

$$= \frac{-\sin(x) - 1}{(1+\sin(x))^2} = \boxed{\frac{-1}{1+\sin(x)}}$$

(b) $y = \tan(\sin x)$

$$\frac{dy}{dx} = \boxed{\sec^2(\sin x) \cdot \cos(x)}$$

(c) $g(x) = \sec^3(x)$

$$g'(x) = 3\sec^2(x)\,(\sec(x)\tan(x))$$

$$= \boxed{3\sec^3(x)\tan(x)}$$