

HW 19 Solutions:

1. Find the tangent plane at the point

(a) $f(x,y) = \sqrt{x+2y}$, $(0,2)$

$$f_x(x,y) = \frac{1}{2}(x+2y)^{-1/2}$$

$$f_y(x,y) = \frac{1}{2}(x+2y)^{-1/2} \cdot (1+2) = 0$$

$$F(x,y,z) = \sqrt{x+2y} - z$$

$$F(0,2) = \sqrt{4} = 2 \quad \text{pt: } (0,2,2)$$

$$\nabla F = \langle F_x, F_y, F_z \rangle = \langle f_x, f_y, -1 \rangle$$

$$\nabla F(0,2) = \left\langle \frac{1}{2}, \frac{1}{2}, -1 \right\rangle$$

$$= \left\langle \frac{1}{4}, \frac{1}{2}, -1 \right\rangle$$

$$0 = \frac{1}{4}(x-0) + \frac{1}{2}(y-2) - (z-2)$$

$$\Rightarrow 0 = (x) + 2(y-2) - 4(z-2)$$

$$\Rightarrow \boxed{-4 = x + 2y - 4z}$$

Alternate form: $z = \frac{1}{4}x + \frac{1}{2}y + 1$

$$\boxed{z = \frac{1}{4}x + \frac{1}{2}y + 1}$$

(b.) $f(x,y) = xe^{xy+1}$, $(-1,1)$

$$f(-1,1) = -1$$

$$F(x, y, z) = xe^{xy+1} - z$$

$$\nabla F = \langle xye^{xy+1} + e^{xy+1}, x^2e^{xy+1}, -1 \rangle$$

$$\begin{aligned} \nabla F(-1, 1, -1) &= \langle -1+1, 1, -1 \rangle \\ &= \langle 0, 1, -1 \rangle \end{aligned}$$

$$0 = (y-1) - (z+1)$$

$$\boxed{z = y - 2}$$

$$(c) f(x, y) = 3x^2 + y^2, \quad (1, 2)$$

$$f(1, 2) = 3 + 4 = 7 \quad \rightarrow (1, 2, 7)$$

$$F(x, y, z) = 3x^2 + y^2 - z$$

$$\nabla F = \langle 6x, 2y, -1 \rangle$$

$$\nabla F(1, 2, 7) = \langle 6, 4, -1 \rangle$$

$$0 = 6(x-1) + 4(y-2) - (z-7) = 0$$

$$\boxed{z = 6x + 4y - 7}$$

2. Estimate the values with the given information.

$$(a) f(2, 3) = 7, f_x(2, 3) = 10, f_y(2, 3) = -2. \text{ Estimate } f(2.1, 3.2)$$

$$z = 10(x-2) - 2(y-3) + 7$$

$$f(2.1, 3.2) \approx 10(.1) - 2(.2) + 7 = \boxed{7.6}$$

$$(b) \quad f(1, -1) = 2, \quad f_x(1, -1) = -1, \quad f_y(1, -1) = 3.$$

Estimate $f(2, 0)$.

$$z = -(x-1) + 3(y+1) + 2$$

$$f(2, 0) \approx -(2-1) + 3(0+1) + 2 = -1 + 3 + 2 = \boxed{4}$$