Homework 18 Solutions

1. Find $D_v f$

(a) $f(x,y) = e^x \sin y \qquad (0, \pi/3), \quad \vec{v} = \langle -6, 8 \rangle$

$$\nabla f(x,y) = \langle e^x \sin y, \; e^x \cos y \rangle$$

$$|\vec{v}| = \sqrt{36+64} = 10$$

$$D_v f = \frac{\nabla f \cdot \vec{v}}{|\vec{v}|} = \frac{e^x \langle \sin y, \cos y \rangle \cdot \cancel{2} \langle -3, 4 \rangle}{\cancel{10} \; 5}$$

$$= \frac{e^x}{5} \left(-3 \sin y + 4 \cos y \right)$$

$$D_v f(0, \pi/3) = \frac{1}{5} \left(-3 \left(\frac{\sqrt{3}}{2} \right) + 4 \left(\frac{1}{2} \right) \right)$$

$$= \boxed{\frac{-3\sqrt{3}+4}{10}}$$

(b.) $g(r,s) = \tan^{-1}(rs), \quad (1,2), \quad \vec{v} = \langle 3, 5 \rangle$

$$\nabla g = \left\langle \frac{s}{1+(rs)^2}, \frac{r}{1+(rs)^2} \right\rangle ; \quad |\vec{v}| = \sqrt{9+25} = \sqrt{34}$$

$$D_v g = \frac{\nabla g \cdot \vec{v}}{|\vec{v}|} = \frac{\frac{1}{(1+(rs)^2)} \langle s, r \rangle \cdot \langle 3, 5 \rangle}{\sqrt{34}}$$

$$= \frac{3s + 5r}{\sqrt{31}\,(1 + r^2 s^2)}$$

$$D_v g(1,2) = \frac{3(2) + 5(1)}{\sqrt{31}\,(5)} = \boxed{\frac{11}{5\sqrt{34}}}$$

(c.) $f(x,y,z) = \sqrt{xyz}$, $(3,2,6)$, $\vec{v} = \langle -1,-2,2 \rangle$

$$f_x = \frac{yz}{2\sqrt{xyz}}, \quad f_y = \frac{xz}{2\sqrt{xyz}}, \quad f_z = \frac{xy}{2\sqrt{xyz}}$$

$$\nabla f(x,y,z) = \frac{1}{2\sqrt{xyz}} \langle yz, xz, xy \rangle$$

$$|\vec{v}| = \sqrt{1+4+4} = 3$$

$$D_v f(x,y,z) = \frac{\frac{1}{2\sqrt{xyz}} \langle yz, xz, xy \rangle \cdot \langle -1,-2,2 \rangle}{3}$$

$$= \frac{1}{6\sqrt{xyz}} \left(-yz - 2xz + 2xy \right)$$

$$D_v f(3,2,6) = \frac{1}{6\sqrt{36}} \left(-12 - 36 + 12 \right) = \frac{-36}{36} = \boxed{-1}$$

d.) $H(x,y,z) = x$, $(1,0,0)$, $\vec{v} = \langle 1,1,1 \rangle$

$$\nabla H(x,y,z) = \langle 1,0,0 \rangle$$

$$|\vec{v}| = \sqrt{3}$$

$$D_v f = \frac{\langle 1,0,0 \rangle \cdot \langle 1,1,1 \rangle}{\sqrt{3}} = \frac{1}{\sqrt{3}}$$

$$D_v f(1,0,0) = \boxed{\frac{1}{\sqrt{3}}}$$

2.) $f(x,y) = x^3y + y^2x^2$

$\nabla f = \langle 3x^2y + 2xy^2,\ x^3 + 2x^2y \rangle$

$\nabla f(1,2) = \langle 6+8,\ 1+4 \rangle = \langle 14, 5 \rangle$

$$D_\theta f = \frac{|\langle 14,5\rangle|\,(1)\cos(\pi/6)}{(1)} = \cos(\pi/6)\sqrt{221}$$

$$= \boxed{\sqrt{221}\ \cos(\pi/6)}$$

3. $f(x,y) = x^3y + y^2x^2$

$\nabla f(x,y) = \langle 3x^2y + 2xy^2,\ x^3 + 2x^2y \rangle$

$\nabla f(1,2) = \langle 14, 5 \rangle$

$$\vec{v} = \frac{\nabla f(1,2)}{|\nabla f(1,2)|} = \boxed{\left\langle \frac{14}{\sqrt{221}}, \frac{5}{\sqrt{221}} \right\rangle}$$

4.