

## HW #16 Solutions

#1.  $f(x,y) = \cos(xy+y)$

$$f_x(x,y) = -y \sin(xy+y)$$

$$f_{xy}(x,y) = -(x+1)y \cos(xy+y) - \sin(xy+y)$$

$$f_y(x,y) = -(x+1) \sin(xy+y)$$

$$f_{yx}(x,y) = -y(x+1) \cos(xy+y) - \sin(xy+y)$$

same

(1)

#2. (a)  $f(x,y) = x^3 + x^2y + xy^2 + y^3$

$$f_x(x,y) = 3x^2 + 2xy + y^2$$

$$f_{xx}(x,y) = 6x + 2y$$

$$f_y(x,y) = x^2 + 2xy + 3y^2$$

$$f_{yy}(x,y) = 2x + 6y$$

$$f_{yxy} = f_{yyx} = 2$$

(b)  $f(x,y) = e^{xy+1}$

$$f_x(x,y) = ye^{xy+1}$$

$$f_{xx}(x,y) = y^2 e^{xy+1}$$

$$f_y(x,y) = xe^{xy+1}$$

$$f_{yy}(x,y) = x^2 e^{xy+1}$$

$$f_{yxy} = f_{yyx} = x^2 y e^{xy+1} + 2x e^{xy+1}$$

$$c) f(x,y) = x \cos(y) + y \sin(x) + x^2 y^2$$

$$f_x(x,y) = \cos(y) + y \cos(x) + 2xy^2$$

$$f_{xx} = -y \sin(x) + 2y^2$$

$$f_y = -x \sin(y) + \sin(x) + 2x^2 y$$

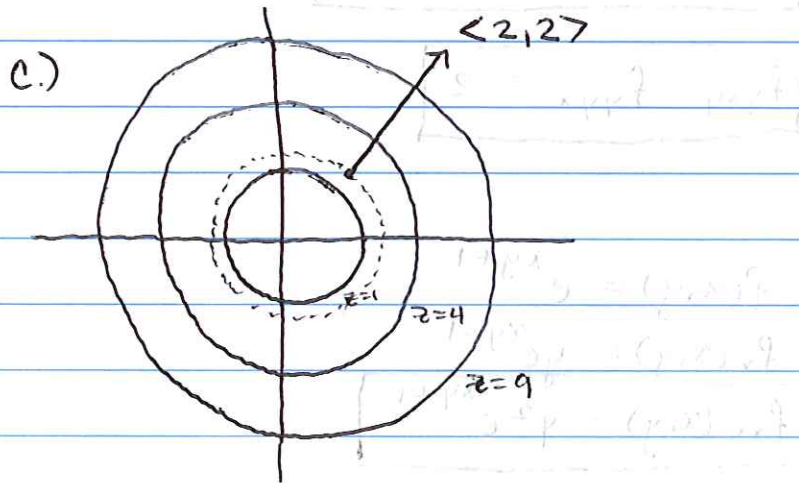
$$f_{yy} = -x \cos(y) + 2x^2$$

$$f_{yxy} = f_{yyx} = -\cos(y) + 4x$$

3) a.) paraboloid

$$b.) f_x = 2x$$

$$f_y = 2y$$

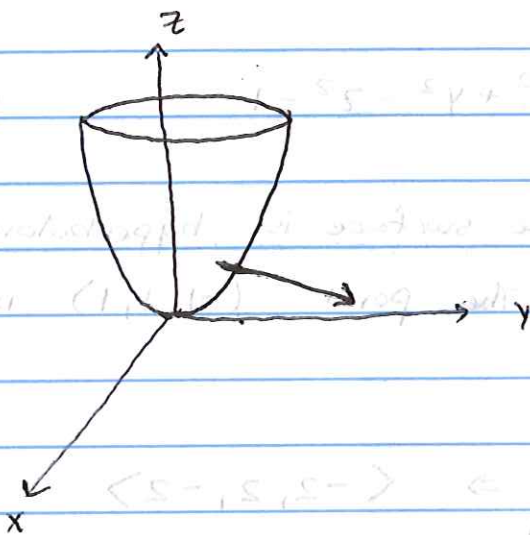


d.) The vector points toward  $z$  increasing.

$$e.) \left. \begin{array}{l} F_x = 2x \\ F_y = 2y \end{array} \right\} \text{Same as } f_x, f_y$$

$$F_z = -1$$

f.)



(g.) yes (c)

$$4.) \lim_{(x,y) \rightarrow (1,0)} \frac{4y^2 \cos(x-1)}{(1-(x-1)^2 + y^4)}$$

path 1  $x-1=y$

$$\lim_{x \rightarrow 1} \frac{4(x-1)^4 \cos(x-1)}{2(x-1)^4} = \frac{4}{2} = 2$$

path 2  $y=0$

$$\lim_{x \rightarrow 1} \frac{0}{(x-1)^4} = 0$$

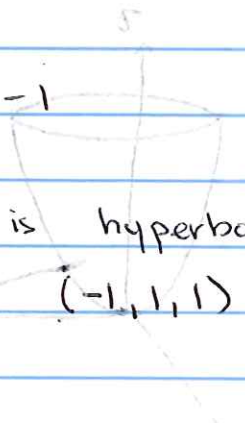
Because they do not equal, the limit does not exist.

$$5.) \lim_{(x,y) \rightarrow (1,0)} \frac{5xy}{\sqrt{x^2 + y^2}} = \frac{0}{1} = 0$$

The limit exists and is zero.



b.)  $F(x, y, z) = x^2 + y^2 - z^2 - 1$

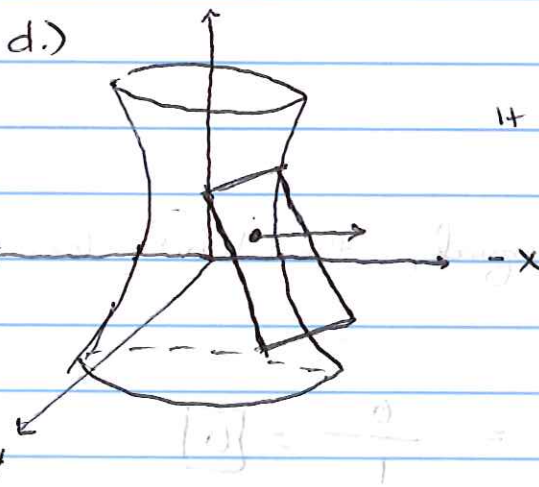


a.) At  $F=0$ , the surface is hyperboloid of one sheet. The point  $(-1, 1, 1)$  is on the surface.

b.) 
$$\left. \begin{aligned} F_x &= 2x \\ F_y &= 2y \\ F_z &= -2z \end{aligned} \right\} \Rightarrow \langle -2, 2, -2 \rangle$$

c.)  $-2(x+1) + 2(y-1) - 2(z-1) = 0$

$$\Rightarrow \boxed{-x + y - z = 1}$$
 ↖ Either form is fine.



It is a tangent plane.

$$\left[ \frac{0}{1} \right] = \frac{0}{1} = \frac{0}{1}$$