Homework 15 Solutions

1.) $$\lim_{(x,y)\to(1,2)} 5x^3 + y$$

Since $(1,2)$ is in the domain, we just plug in.

$$\lim_{(x,y)\to(1,2)} 5x^3 + y = \boxed{7}$$

2.) $$\lim_{(x,y)\to(0,0)} \frac{x^4 - 4y^4}{x^2 + 2y^2}$$

Here we can simplify and then plug in

$$\lim_{(x,y)\to(0,0)} \frac{x^4 - 4y^4}{x^2 + 2y^2} = \lim_{(x,y)\to(0,0)} \frac{(x^2 - 2y^2)\cancel{(x^2 + 2y^2)}}{\cancel{(x^2 + 2y^2)}}$$

$$= \lim_{(x,y)\to(0,0)} x^2 - 2y^2 = \boxed{0}$$

3.) $$\lim_{(x,y)\to(1,1)} \frac{x^2 + y^2 - 2}{\sqrt{x^2 + y^2 - 1} - 1}$$

Here, we need to multiply by the conjugate.

$$\lim_{(x,y)\to(1,1)} \frac{\cancel{(x^2 + y^2 - 2)}\left(\sqrt{x^2 + y^2 - 1} + 1\right)}{\cancel{(x^2 + y^2 - 1) - 1}} = \lim_{(x,y)\to(1,1)} \sqrt{x^2 + y^2 - 1} + 1 = \boxed{2}$$

4) $$\lim_{(x,y)\to(0,0)} \frac{2xy}{x^2+y^2}$$

This cannot be simplified, so we may expect this to not have a solution.

First, let's consider $x=0$

$$\lim_{y\to 0} \frac{0}{y^2} = \boxed{0}$$

Now, we want a path where the numerator and denominator grow at the same rate.

$2xy \sim x^2+y^2$ so

If we pick $x=y$, then we get $2x^2$ as the numerator and $2x^2$ as the denominator.

$$\lim_{x\to 0} \frac{2x^2}{2x^2} = \boxed{1}$$

Since $1 \neq 0$, the limit does not exist.

5.) $$\lim_{(x,y)\to(1,0)} \frac{7y^3(x-1)}{(x-1)^4+y^4}$$

If we pick $y=0$, then

$$\lim_{x\to 1} \frac{0}{(x-1)^4} = \boxed{0}$$

Our "smart" path needs to be one that makes the bottom and top similar.
If we pick $y = x - 1$, we get just that

$$\lim_{x \to 1} \frac{7(x-1)^3(x-1)}{(x-1)^4 + (x-1)^4} = \boxed{\frac{7}{2}}$$

There are two different limits, so the limit does not exist.

6.) $$\lim_{(x,y) \to (0,0)} \frac{5y^4 \cos^2(x)}{x^4 + y^4}$$

let's evaluate two paths that will make elements of this problem easier.

First $x = 0$, $y \to 0$:

$$\lim_{y \to 0} \frac{5y^4}{y^4} = \boxed{5}$$

Second $x = y$

$$\lim_{x \to 0} \frac{5\cancel{x^4} \cos(x)}{2\cancel{x^4}} = \lim_{x \to 0} \frac{5}{2} \cos(x) = \boxed{\frac{5}{2}}$$