MATH 231: Calculus of Several Variables
Section 1, 107 Ag Sc & Ind Bldg,
TR 9:05 AM - 9:55 AM

Homework 11: Due Tuesday, Oct 10

Do each problem without any aids. There may be more than one solution for any problem. Then try to check your work to determine if you were correct. If you were not, try to find your mistake.

1. Which of the following surfaces is a hyperboloid of two sheets?

   (a) $x^2 - 4x + 2y^2 - 4y - 4z^2 = -6$
   (b) $-x^2 - 2x + 2y^2 - 4y + 4z^2 = -5$
   (c) $-x^2 + 2x - 2y^2 + 4y - 4z^2 = 7$
   (d) $-x^2 + 2x + 2y^2 - 4y + 4z^2 = 3$
   (e) None of the above

2. Which of the following could be a tangent vector for the equation below?

$$x = t, \quad y = e^{-t}, \quad z = 2t - t^2$$

   (a) $\vec{v} = \langle 1, -1, 1 \rangle$
   (b) $\vec{v} = \langle 0, -1, 2 \rangle$
   (c) $\vec{v} = \langle 0, e, 0 \rangle$
   (d) $\vec{v} = \langle 1, -e^{-2}, 6 \rangle$
   (e) None of the above

3. Consider the three lines:

   I. $\vec{q}(t) = \langle t - 1, 4t + 2, 3t - 12 \rangle$
   II. $\vec{r}(t) = \langle 2t + 1, 8t, 6t - 1 \rangle$
   III. $\vec{s}(t) = \langle 4t - 2, -t, 12 \rangle$

   Which of the following is true?

   (a) $\vec{s}$ is perpendicular to $\vec{r}$ but not $\vec{q}$
   (b) $\vec{s}$ is parallel to $\vec{r}$ but not $\vec{q}$
   (c) $\vec{q}$ is perpendicular to $\vec{r}$ but not $\vec{s}$
   (d) $\vec{q}$ is parallel to $\vec{r}$ but not $\vec{s}$
   (e) None of the above

4. Which of the following functions has the same domain as

$$\vec{r}(t) = \left\langle \frac{\ln(t)}{t^2-2}, \frac{t^2-9}{t+3}, \frac{t-1}{2} \right\rangle$$

(a)
$$\vec{r}(t) = \left\langle \frac{\sqrt{t}}{t-\sqrt{2}}, \frac{t-3}{t-\sqrt{2}}, \frac{7-t}{2t} \right\rangle$$

(b)
$$\vec{r}(t) = \left\langle \frac{\ln(t)}{\sqrt{t}}, t-3, t-1 \right\rangle$$

(c)
$$\vec{r}(t) = \left\langle \frac{\ln(t-3)}{t+\sqrt{2}}, \frac{t^2-9}{t}, \frac{t-1}{2} \right\rangle$$

(d)
$$\vec{r}(t) = \left\langle \frac{\ln(t)}{t}, \frac{t}{t-\sqrt{2}}, 7 \right\rangle$$

(e) None of the above

5. At which point(s) does the plane intersect the line?

$$x = t, \quad y = 2+t, \quad z = 9+7t; \qquad x - 8y + z = 0$$

(a) Only $(0,0,0)$
(b) Only $(0,2,16)$
(c) The line and plane do not intersect
(d) The line is contained in the plane
(e) None of the above

6. Zero is the answer to which of the following expressions?

(a) The cross product of perpendicular vectors
(b) The sum of two perpendicular vectors
(c) The dot product of the slope vector of a line with the normal vector of a plane, where the line and plane are perpendicular.
(d) The dot product of the slope vector of a line with the normal vector of a plane, where the line and plane are parallel.
(e) None of the above

HW 11 Solutions

1. Which of the following surfaces is a hyperboloid of two sheets?

(a) $x^2 - 4x + 2y^2 - 4y - 4z^2 = -6$

$\Rightarrow (x^2 - 4x + 4) + 2(y^2 - 2y + 1) - 4z^2 = -6 + 4 + 2$

$\Rightarrow (x-2)^2 + 2(y-1)^2 - 4z^2 = 0$

$$\Rightarrow \quad z^2 = \frac{(x-2)^2}{4} + \frac{(y-1)^2}{2}$$

This is a cone ✗

(b) $-x^2 - 2x + 2y^2 - 4y + 4z^2 = -5$

$\Rightarrow -(x^2 + 2x + 1) + 2(y^2 - 2y + 1) + 4z^2 = -5 - 1 + 2$

$\Rightarrow -(x+1)^2 + 2(y-1)^2 + 4z^2 = -4$

$$\Rightarrow \quad \frac{(x+1)^2}{4} - \frac{(y-1)^2}{2} - z^2 = 1$$

This is a hyperboloid of two sheets. ✓

(c) $-x^2 + 2x - 2y^2 + 4y - 4z^2 = 7$

$\Rightarrow -(x^2 - 2x + 1) - 2(y^2 - 2y + 1)^2 - 4z^2 = 7 - 1 - 2$

$\Rightarrow -(x-1)^2 - 2(y-1)^2 - 4z^2 = 4$

$$\Rightarrow \quad -\frac{(x-1)^2}{4} - \frac{(y-1)^2}{2} - z^2 = 1$$

This is an impossible surface ✗
(3 negatives cannot sum to 1)

(d) $-x^2 + 2x + 2y^2 - 4y + 4z^2 = 3$

$\Rightarrow -(x^2 - 2x + 1) + 2(y^2 - 2y + 1) + 4z^2 = 3 - 1 + 2$

$\Rightarrow -(x-1)^2 + 2(y-1)^2 + 4z^2 = 4$

$$\Rightarrow \frac{-(x-1)^2}{4} + \frac{(y-1)^2}{2} + z^2 = 1$$

This is a hyperboloid of one sheet X

Answer: (b)

2. Which could be tangent to

$x = t,\ y = e^{-t},\ z = 2t - t^2$

Solution: If $\vec{r}(t) = \langle t, e^{-t}, 2t - t^2 \rangle$

then $\vec{r}\,'(t) = \langle 1, -e^{-t}, 2 - 2t \rangle$

So tangent vectors will be of this form

so we can eliminate

(b) $\vec{v} = \langle 0, -1, 2 \rangle$

(e) $\vec{v} = \langle 0, e, 0 \rangle$

Cannot be solutions since the x-value has to be nonzero.

(a): $\vec{v} = \langle 1, -1, 1 \rangle$

$\Rightarrow -e^{-t} = -1 \Rightarrow t = 0$

So $z(0) = 2 - 2(0) = 2 \neq 1$ X

(d) $\vec{v} = \langle 1, -e^{-2}, 6 \rangle$

$\Rightarrow -e^{-2} = -e^{-t} \Rightarrow t = 2$

Then $z(2) = 2 - 2(2)$

$= -2 \neq 6$ X Not possible

Solution: (e)

3. Consider the 3 lines

I. $\vec{q}(t) = \langle t-1, 4t+2, 3t-12 \rangle$

II. $\vec{r}(t) = \langle 2t+1, 8t, 6t-1 \rangle$

III. $\vec{s}(t) = \langle 4t-2, -t, 12 \rangle$

Which is true?

(a) $\vec{s} \perp \vec{r}$ and $\vec{s} \not\perp \vec{q}$

vector of $\vec{s}(t) = \vec{V}_s = \langle 4, -1, 0 \rangle$

vector of $\vec{r}(t) = \vec{V}_r = \langle 2, 8, 6 \rangle$

Vector of $\vec{q}(t) = \vec{V}_q = \langle 1, 4, 3 \rangle$

$\vec{V}_s \cdot \vec{V}_r = (4)(2) + (-1)(8) + (12)(0)$

$= 8 - 8 + 0 \neq 0$ $\boxed{\text{False}}$

But $\vec{V}_s \cdot \vec{V}_q = 4 - 4 = 0$ so $\vec{s} \perp \vec{q}$

(b) $\vec{s} \parallel \vec{r}$ but not $\vec{q}$

If $\vec{s} \parallel \vec{r}$, then $\vec{V}_s = c\vec{V}_r$ for some constant c

Then $4 = 2c \Rightarrow c = 2$

But $-1 = 8c \Rightarrow c = -1/8$

Not possible!

$\boxed{\text{False}}$

(c) $\vec{q} \perp \vec{r}$ but not $\vec{s}$

$\vec{V}_q \cdot \vec{V}_r = (2)(1) + (8)(4) + (6)(3) \neq 0$ $\boxed{\text{False}}$

(d) $\vec{q} \parallel$ to $\vec{r}$ but not $\vec{s}$

If $\vec{q} \parallel \vec{r}$ then $\vec{V}_q = c\vec{V}_r$ for some constant c.

This is clearly true:

$$\langle 1,4,3 \rangle = c \langle 2,8,6 \rangle$$

If $c = 1/2$, this expression is true.

Now we want to check that

$\vec{q} \not\parallel \vec{s}$.

This is also easy to see since

$$\vec{V}_q \cdot \vec{V}_s = \langle 1,4,3 \rangle \cdot \langle 4,-1,0 \rangle$$
$$= 4 - 4 = 0$$

so these lines are actually $\perp$. $\boxed{\text{TRUE}}$

(4) Which of the following has the same domain as

$$\vec{r}(t) = \left\langle \frac{\ln(t)}{t^2-2}, \frac{t^2-9}{t+3}, \frac{t-1}{2} \right\rangle$$

$$D = (0,\sqrt{2}) \cup (\sqrt{2},\infty)$$

(a) $\vec{r}(t) = \left\langle \frac{\sqrt{t}}{t-\sqrt{2}}, \frac{t \cdot 3}{t-\sqrt{2}}, \frac{7-t}{2t} \right\rangle$

- t must be positive, $t \geq 0$
- $t \neq \sqrt{2}$
- $t \neq 0$

$D = (0,\sqrt{2}) \cup (\sqrt{2},\infty)$ ✓ $\boxed{\text{SAME}}$

(b) $\vec{r}(t) = \left\langle \frac{\ln(t)}{\sqrt{t}}, t \cdot 3, t \cdot 1 \right\rangle$

$D = (0,\infty)$ $\boxed{\text{Not the same}}$ X

(c) $\vec{r}(t) = \left\langle \frac{\ln(t-3)}{t+\sqrt{2}}, \frac{t^2-9}{t}, \frac{t-1}{2} \right\rangle$

$t > 3, \quad t \neq -\sqrt{2}, \quad t \neq 0$

$D = (3, \infty)$ $\boxed{\text{Not same}}$ X

(d) $\vec{r}(t) = \left\langle \frac{\ln(t)}{t}, \frac{t}{t-\sqrt{2}}, 7 \right\rangle$

$t > 0, \quad t \neq 0, \quad t \neq \sqrt{2}$

$D = (0, \sqrt{2}) \cup (\sqrt{2}, \infty)$ ✓ $\boxed{\text{same}}$

5.) At which point(s) does the plane intersect the line? $\vec{r}(t) = \langle t, 2+t, 9+7t \rangle$, $x - 8y + z = 0$

(a) $(0,0,0)$

False: $(0,0,0)$ is not on the line!

If so, then $t=0$

But $2+t=0 \Rightarrow t=-2$

t can't both be 0 and -2!

Only

(b) $(0,2,16)$

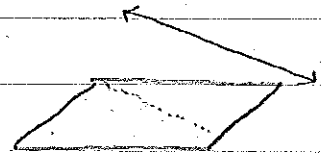
False: This is also not a point on the line.

If it were, $t=0$, But then $z = 9+7(0) = 9 \neq 16$.

(c) The line and plane do not intersect.

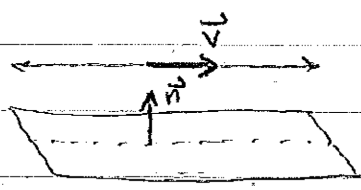
True:

This is a tricky thing to figure out. If the line and plane do not intersect, then the line is $\parallel$ to the plane. Think:



where the line is at a constant distance from the plane.

If a line is parallel to a plane then the direction vector of the line and the normal vector of a plane are $\perp$



That is, $\vec{v} \cdot \vec{n} = 0$.

So let's check:

$$\vec{v} = \langle 1, 1, 7 \rangle$$

$$\vec{n} = \langle 1, -8, 1 \rangle$$

$$\vec{v} \cdot \vec{n} = 1 - 8 + 7 = 0 \checkmark.$$

Last, we need to check that the line is not <u>inside</u> the plane. pick a point on the line, like at $t=0$, $(0, 2, 9)$ is a point on our line. Is $(0, 2, 9)$ a point on the plane $x - 8 \cdot y + z = 0$?

Plug in!

$$0 - 8(2) + 9 = 7 \neq 0$$ So it's not on the plane!

(d) The line and the plane do not intersect.

<u>False</u>: See (c).

6. Zero is the answer to which of the following?

(a) Cross product of $\perp$ vectors

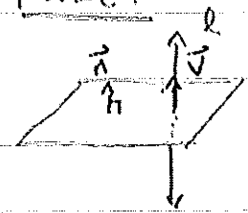
<u>False</u>: Cross product is zero when two vectors are parallel.

(b) Sum of two $\perp$ vectors

<u>False</u>: Two vectors add to zero when one is the negative of the other.

(c.) The dot product of the slope vector of a line and the normal vector of a plane, where the line and plane are $\perp$.

<u>False</u>:

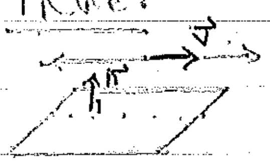


$\vec{n} \cdot \vec{v} \neq 0$.

In fact, $\vec{n} \cdot \vec{v} = |\vec{n}||\vec{v}|$

(d.) The dot product of the slope vector of a line with the normal vector of a plane, where the line and plane are parallel.

<u>TRUE!</u>



$\vec{n} \cdot \vec{v} = 0$. ✓